Loop Quantum Thermodynamics

Francesca Vidotto

Università di Pavia and Centre de Physique Théorique, Marseille

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Plan of the talk

1. Interaction Hamiltonian
   single quantum particle on the quantum gravitational field
   - construction and proprieties -

2. comparison with Graph Theory
   our Hamiltonian provides (up to a certain approximation)
   the same objects already studied in Graph Theory

3. Loop Quantum Thermodynamics
   this Hamiltonian is a tool to construct statistical objects in LQG

References:

2004 The Laplacian of a graph as a density matrix: a basic combinatorial approach to separability of mixed states. Braunstein, Ghosh, Severini. quant-ph/0406165

2008 The Von Neumann entropy of networks. Passerini, Severini. 0812.2597

$\Gamma = (N, L)$ undirected simple graph

Adjacency $[A(\Gamma)]_{n,m} = 1$ if $\{n, m\} \in L(\Gamma)$ and $[A(\Gamma)]_{u,v} = 0$ otherwise

Degree $[\Delta(\Gamma)]_{n,n} := d_n = \#$ links adjacent to the node n

Laplacian $L(\Gamma) := \Delta(\Gamma) - A(\Gamma)$

Density $\rho_\Gamma := \frac{L(\Gamma)}{d_\Gamma} = \frac{L(\Gamma)}{\mathrm{Tr}(\Delta(\Gamma))}$ Hermitian, positive semi-definite, trace-1

Entropy $S(\Gamma) = -\mathrm{Tr}[\rho_\Gamma \log \rho_\Gamma]$

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connection variables

Phase Space $\quad (q_{ab}(x), \pi^{ab}(x), X^a, P_a)$

Hamiltonian constraint $\quad C(x) = H_{ADM}(x) + \delta^3(x, X) P_0 \quad P_0 = \sqrt{P^2 + m^2} \sim m + \frac{P^2}{2m}$

$P^2 = q^{ab}(x) P_a P_b$

connection variables

Phase Space $\left(q_{ab}(x),\ \pi^{ab}(x),\ X^a,\ P_a \right)$

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$$P^2 = q^{ab}(x) P_a P_b$$

$$H = \int \mathrm{d}x\ \delta^3(x, X)\ N(x)\ q^{ab}(X)\ \frac{P_a\ P_b}{2m}$$

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connection variables

Phase Space $(\, q_{ab}(x),\ \pi^{ab}(x),\ X^a,\ P_a \,) \ \rightarrow\ (\, E^{ai}(x),\ A^i_a(x),\ X^a,\ P_a \,)$

Hamiltonian constraint $C(x) = H_{ADM}(x) + \delta^3(x, X) P_0$ $\quad P_0 = \sqrt{P^2 + m^2} \sim m + \frac{P^2}{2m}$

$$P^2 = q^{ab}(x) P_a P_b \qquad\qquad q^{ab}(X) = \frac{E^{ai}(X) E^{bi}(X)}{q(X)}$$

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$$P^2 = q^{ab}(x)P_aP_b \qquad\qquad q^{ab}(X) = \frac{E^{ai}(X)E^{bi}(X)}{q(X)}$$

$$H = \int \mathrm{d}x\ f_R(x,X)\,\frac{E^{ai}(x)E^{bi}(X)}{\sqrt{q(X)}}\,\frac{P_a\,P_b}{2m}$$

pointlike nature of the particle $\rightarrow$ regularization by a smearing function

$$f_R(x,X) = \begin{cases} \frac{1}{V_R} = \frac{3}{4\pi R^3} & \text{if} \quad |x-X| \leq R \\ 0 & \text{if} \quad |x-X| \geq R \end{cases}$$

Quantum States

Spin network states $\quad |\, s, x \,\rangle \equiv |\, s \,\rangle \otimes |\, x \,\rangle \subset \mathcal{H}_{\text{LQG}} \otimes \mathcal{H}_{\text{P}}$.

$$\mathbb{I}_{\text{P}} = \int \mathrm{d}x \quad |\, x \,\rangle\langle\, x \,|$$

$$\langle\, x \,|\, y \,\rangle = \quad \delta(x, y)$$

the volume operator vanishes everywhere except at the nodes

$$\sqrt{q(x)}\,|\, s \,\rangle = \sum_{n \in N(s)} \nu_n \, \delta(x, x_n)\,|\, s \,\rangle$$

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restriction to the nodes

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$$\langle\, x \,|\, y \,\rangle = \frac{1}{\sqrt{q(x)}} \;\; \delta(x, y)$$

$$\langle\, s, x_n \,|\, s', x_{n'} \,\rangle = \nu_n^{-1} \, \delta_{ss'} \delta_{nn'} \qquad \mathbb{I} = \textstyle\sum_s \sum_{n \in N(s)} \nu_n \,|\, s, x_n \,\rangle\langle\, s, x_n \,|$$

the volume operator vanishes everywhere except at the nodes

$$\sqrt{q(x)} \,|\, s \,\rangle = \textstyle\sum_{n \in N(s)} \nu_n \, \delta(x, x_n) \,|\, s \,\rangle$$

Quantum States

for later convenience... $|\, \underset{\sim}{x} \,\rangle := \sqrt[4]{q(x)}\,|\, x \,\rangle$

Spin network states $\quad |\, s, x \,\rangle \equiv |\, s \,\rangle \otimes |\, x \,\rangle \subset \mathcal{H}_{\text{LQG}} \otimes \mathcal{H}_{\text{P}}$.

$$\mathbb{I}_{\text{P}} = \int \mathrm{d}x \, \sqrt{q} \;\; |\, x \,\rangle\langle\, x \,| \;\; \rightarrow \;\; \langle\, s \,|\mathbb{I}_{\text{P}}|\, s \,\rangle = \sum_{n \in \underset{\sim}{N}(s)} \nu_n^2 \,|\, \underset{\sim}{x_n} \,\rangle\langle\, \underset{\sim}{x_n} \,|$$

$$\langle\, x \,|\, y \,\rangle = \frac{1}{\sqrt{q(x)}} \;\; \delta(x, y)$$

$$\langle\, s, \underset{\sim}{x_n} \,|\, s', \underset{\sim}{x_{n'}} \,\rangle = \nu_n^{-2}\, \delta_{ss'} \delta_{nn'} \qquad \mathbb{I} = \textstyle\sum_s \sum_{n \in N(s)} \nu_n^2 \,|\, s, \underset{\sim}{x_n} \,\rangle\langle\, s, \underset{\sim}{x_n} \,|$$

the volume operator vanishes everywhere except at the nodes

$$\sqrt{q(x)}\,|\, s \,\rangle = \textstyle\sum_{n \in N(s)} \nu_n \, \delta(x, x_n) \,|\, s \,\rangle$$

Quantum Operators

$$E^{ai}(x)\,|\,s\,\rangle \;=\kappa\hbar \sum_\ell \int_\ell \mathrm{d}t\, \dot{\ell}^a(t)\, \delta^3(x,\ell(t))\,|\,s,\tau^i\,\rangle$$

$$E^{ai}(x)E^{bi}(x)\,|\,s\,\rangle \;=$$
$$(\kappa\hbar)^2 \sum_{\ell,\ell'} \;\int_\ell \mathrm{d}t\; \int_{\ell'} \mathrm{d}t' \;\; \dot{\ell}^a(t)\, \dot{\ell}^b(t')\; \delta^3(x,\ell(t))\delta^3(x,\ell(t'))j_\ell(j_\ell+1)\,|\,s\,\rangle$$

$$P_a = -i\hbar\; D_a$$ covariant derivative

$$\langle\, s,\psi\,|H|\,s,\phi\,\rangle = \frac{\kappa^2\hbar^4}{2m} \quad \sum_\ell\ j_\ell(j_\ell+1)\int_\ell \mathrm{d}s \int_\ell \mathrm{d}t\ \overline{\partial_s\psi(\underset{\sim}{\ell(s)})}\,\partial_t\phi(\underset{\sim}{\ell(t)})\ f_R(\ell(s),\ell(t))$$

Planck scale! $\quad \Delta_\ell\psi := \int_\ell \mathrm{d}s\ \partial_s\psi(\underset{\sim}{\ell(s)}) = \psi(\underset{\sim}{\ell_f}) - \psi(\underset{\sim}{\ell_i})$

particle states

$$|\,\underset{\sim}{\ell}\,\rangle := |\,\underset{\sim}{\ell_f}\,\rangle - |\,\underset{\sim}{\ell_i}\,\rangle$$

renormalization

$$H = \frac{(8\pi\hbar)^2\ell_{\rm Pl}^4}{2m\ v_R} \quad \sum_{s,\ell\in s} j_\ell(j_\ell+1)\ |\,s,\underset{\sim}{\ell}\,\rangle\langle\, s,\underset{\sim}{\ell}\,|$$

trace-1 $\quad \mathrm{Tr}H = \frac{\hbar^2\ell_{\rm Pl}^4}{2m^*} \quad \sum_{n\in N} v_n \sum_{\ell\in n}\ j_\ell(j_\ell+1)$

$$\longrightarrow \quad \rho = \frac{H}{\mathrm{Tr}H} = \frac{\sum_{\ell\in n}\ j_\ell(j_\ell+1)}{\sum_{n\in N} v_n \sum_{\ell\in n}\ j_\ell(j_\ell+1)}\,|\,\underset{\sim}{\ell}\,\rangle\langle\,\underset{\sim}{\ell}\,|$$

$$\langle s, \psi | H | s, \phi \rangle = \frac{\kappa^2 \hbar^4}{2m} \sum_{\ell} j_\ell (j_\ell + 1) \int_\ell \mathrm{d}s \int_\ell \mathrm{d}t \, \overline{\partial_s \psi(\ell(s))} \, \partial_t \phi(\ell(t)) \, f_R(\ell(s), \ell(t))$$

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$$|\ell\rangle := |\ell_f\rangle - |\ell_i\rangle$$

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trace-1 $\mathrm{Tr} H = \frac{\hbar^2 \ell_{\mathrm{Pl}}^4}{2m^*} \sum_{n \in N} v_n \sum_{\ell \in n} j_\ell (j_\ell + 1)$

$$\longrightarrow \quad \rho = \frac{H}{\mathrm{Tr} H} = \frac{\sum_{\ell \in n} j_\ell (j_\ell + 1)}{\sum_{n \in N} v_n \sum_{\ell \in n} j_\ell (j_\ell + 1)} |\ell\rangle\langle\ell|$$

$$\langle\, s, \psi\, |H|\, s, \phi\, \rangle = \frac{\kappa^2 \hbar^4}{2m} \sum_{\ell} j_\ell (j_\ell + 1) \;\; \overline{\Delta_\ell \phi} \;\; \Delta_\ell \psi \;\; f_R(\ell(s), \ell(t))$$

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Approximation! j_ℓ all the same v_n all the same

Adjacency

$$\langle\, n \,|\rho|\, m \,\rangle_{ph} = \frac{1}{d_\Gamma} \sum_\ell (\delta_{n,\ell_f} - \delta_{n,\ell_i})(\delta_{m,\ell_f} - \delta_{m,\ell_i})$$

$$= \frac{1}{d_\Gamma}(-1) \text{ if } \{n, m\} \in S \text{ and } [A(\Gamma)]_{u,v} = 0 \text{ otherwise}$$

Degree

$$\langle\, n \,|\rho|\, n \,\rangle_{ph} = \frac{1}{d_\Gamma} \sum_\ell (\delta_{n,\ell_f} - \delta_{n,\ell_i})^2 = \frac{d_n}{d_\Gamma}$$

Entropy

$$S = -\mathrm{Tr}[\rho \log \rho] \equiv S_{BGS}$$

Approximation! j_ℓ all the same v_n all the same

Adjacency

$$\begin{aligned} \langle\, n\,|\rho|\, m\,\rangle_{ph} &= \frac{1}{d_\Gamma}\sum_\ell (\delta_{n,\ell_f}-\delta_{n,\ell_i})(\delta_{m,\ell_f}-\delta_{m,\ell_i}) \\ &= \frac{1}{d_\Gamma}(-1) \text{ if } \{n,m\}\in S \text{ and } [A(\Gamma)]_{u,v}=0 \text{ otherwise} \end{aligned}$$

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Entropy

$$S = -\mathrm{Tr}[\rho \log \rho] \equiv S_{BGS}$$

$< E > = \mathrm{Tr}_{LQG}[H' \tilde{\rho}] = -d(\ln Z)/d\mu$ mean energy of the particle on a gravitational field

from a measure of the particle (geometry known)

from a measure of the geometry (particle position known)

distribution that maximizes entropy $S = -\mathrm{Tr}_{LQG}[\tilde{\rho} \log \tilde{\rho}] = \log Z + \mu \, \mathrm{Tr}_{LQG}[H' \, e^{-\mu H'}]$

Partition function: $Z = \mathrm{Tr}_{LQG}[e^{-\mu H'}]$

$Z = \sum_s e^{-\frac{E_s}{kT}}$ where $E_s = E_0 d_s$ and take $\frac{E_0}{kT} := \mu$ $\rightarrow$ $Z(\mu) = \sum_s e^{-\mu d_s}$

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$Z = \sum_s e^{-\frac{E_s}{kT}}$ where $E_s = E_0 d_s$ and take $\frac{E_0}{kT} := \mu$ $\rightarrow$ $Z(\mu) = \sum_s e^{-\mu d_s}$

$< E > \; = \mathrm{Tr}_{\mathrm{LQG}}[H' \tilde{\rho}] = -d(\ln Z)/d\mu$ mean energy of the particle on a gravitational field

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Partition Function $Z(\mu) = \sum_s e^{-\mu \bar{d}_s}$ $\quad$ $\bar{d}_s(\mu) = \frac{\sum_n^N d_n}{N} = \frac{2\ell}{N}$

Energy Density $\rho_s(\mu) = \frac{1}{Z(\mu)} e^{-\mu \bar{d}_s} = e^{-\frac{2}{N}\ell} \left(1 + e^{-\mu \frac{2}{N}}\right)^{-L}$

$$< d > = \frac{1}{Z(\mu)} \sum_s \bar{d}_s e^{-\mu \bar{d}_s} = -\frac{1}{Z(\mu)} \frac{d}{d\mu} Z(\mu) = \frac{2}{N} L \left(1 + e^{+\mu \frac{2}{N}}\right)^{-1}$$

$$\Delta d = < d^2 > - < d >^2 = -\frac{4L}{N^2} e^{+\mu \frac{2}{N}} \left(1 + e^{+\mu \frac{2}{N}}\right)^{-2}$$

Entropy

$$S = \mu < d > - \ln Z(\mu) = \mu \frac{2}{N} L \left(1 + e^{+\mu \frac{2}{N}}\right)^{-1} - L \ln \left(1 + e^{-\mu \frac{2}{N}}\right)$$

Partition Function $$Z(\mu) = \sum_s e^{-\mu \bar{d}_s} = \left(1 + e^{-\mu \frac{2}{N}}\right)^L$$

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Conclusion and prospectives

Results: We have calculated a partition function, and from this other statistical quantities... is this a first step through a viable thermodynamics of the gravitational field?

Next steps are to extend this results to:

1. an arbitrary N (grand canonical analysis)
2. arbitrary spins
3. arbitrary intertwiners