SPINFOAM COSMOLOGY

Francesca Vidotto
Laboratoire de Physique Subatomique et Cosmologie
Grenoble, France

1107.2633

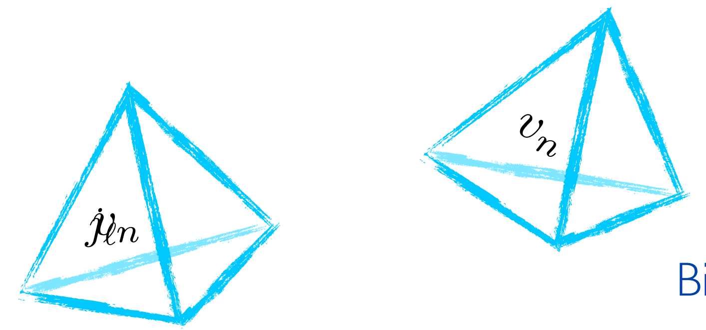
PLAN OF THE TALK

- SPINFOAM *I present the details of the calculation for a* ***generic regular graph****.*
- COSMOLOGY *I discuss the simplified framework provided by cosmology.*
- OPEN ISSUES *New numerics validate the previous results, but also demand us to rethink about some steps taken.*

- ACHTUNG *technical talk! for a general presentation, see my talk at QG4 session on Friday*

COHERENT STATES

- **Spinnetwork states** $|\Gamma, j_\ell, v_n\rangle$



- **Coherent states** $|\Gamma, z_\ell, \vec{n}_\ell, \vec{n}'_\ell\rangle$ Bianchi, Magliaro, Perini

- **Geometrical interpretation** *for the labels* $(z_\ell, \vec{n}_\ell, \vec{n}'_\ell)$*:* Freidel, Speziale

  $\vec{n}_\ell, \vec{n}'_\ell$ *are the 3d normals to the faces of the cellular decomposition;*

  $Im(z_\ell) \leftrightarrow$ *curvature at the faces and* $Re(z_\ell) \leftrightarrow$ *area of the face* $Re(z_\ell) = \theta(\gamma K + \Gamma)$

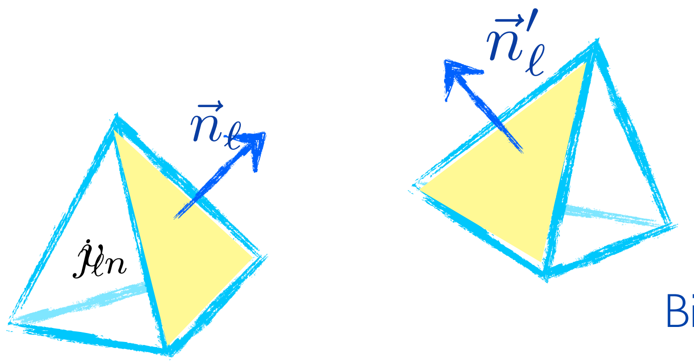
- **Hom&Iso coherent states** $|\Gamma, z\rangle$ Marcianò, Magliaro, Perini, Rovelli, FV

  $\vec{n}_\ell, \vec{n}'_\ell$ *fixed by requiring a regular cellular decomposition*

- *in terms of the scale factor* $Re(z) \sim \dot{a}$ *and* $\sqrt{Im(z)} \sim a$

COHERENT STATES

- **Spinnetwork states** $|\Gamma, j_\ell, v_n\rangle$
- **Coherent states** $|\Gamma, z_\ell, \vec{n}_\ell, \vec{n}'_\ell\rangle$



Bianchi, Magliaro, Perini

- **Geometrical interpretation** *for the labels* $(z_\ell, \vec{n}_\ell, \vec{n}'_\ell)$*:* Freidel, Speziale

  $\vec{n}_\ell, \vec{n}'_\ell$ *are the 3d normals to the faces of the cellular decomposition;*

  $Im(z_\ell) \leftrightarrow$ *curvature at the faces and* $Re(z_\ell) \leftrightarrow$ *area of the face* $Re(z_\ell) = \theta(\gamma K + \Gamma)$

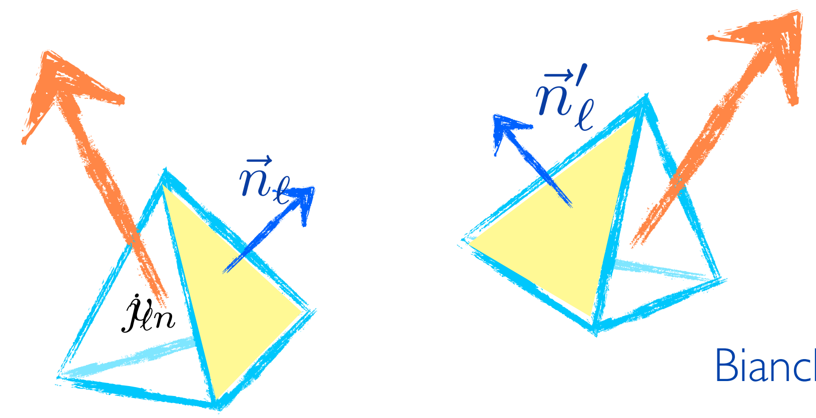
- **Hom&Iso coherent states** $|\Gamma, z\rangle$ Marcianò, Magliaro, Perini, Rovelli, FV

  $\vec{n}_\ell, \vec{n}'_\ell$ *fixed by requiring a regular cellular decomposition*

- *in terms of the scale factor* $Re(z) \sim \dot{a}$ *and* $\sqrt{Im(z)} \sim a$

COHERENT STATES

- **Spinnetwork states** $|\Gamma, j_\ell, v_n\rangle$



- **Coherent states** $|\Gamma, z_\ell, \vec{n}_\ell, \vec{n}'_\ell\rangle$

Bianchi, Magliaro, Perini

- **Geometrical interpretation** *for the labels* $(z_\ell, \vec{n}_\ell, \vec{n}'_\ell)$: Freidel, Speziale

  $\vec{n}_\ell, \vec{n}'_\ell$ *are the 3d normals to the faces of the cellular decomposition;*

  $Im(z_\ell) \leftrightarrow$ *curvature at the faces and* $Re(z_\ell) \leftrightarrow$ *area of the face* $Re(z_\ell) = \theta(\gamma K + \Gamma)$

- **Hom&Iso coherent states** $|\Gamma, z\rangle$ Marcianò, Magliaro, Perini, Rovelli, FV

  $\vec{n}_\ell, \vec{n}'_\ell$ *fixed by requiring a regular cellular decomposition*

- *in terms of the scale factor* $Re(z) \sim \dot{a}$ *and* $\sqrt{Im(z)} \sim a$

VERTEX AMPLITUDE

$$\langle (j,\gamma j); j', m' | Y | j, m\rangle = \delta_{p,\gamma j}\delta_{kj}\delta_{jj'}\delta_{mm'}$$

$$W_v(h_\ell) = \int_{SL(2,\mathbb{C})} \prod_{n=1}^{N-1} dG_n \prod_{\ell=1}^{L} P(h_\ell, G_\ell)$$

kernel of the map:

$$Y: \quad \mathcal{H}^{(j)} \longrightarrow \mathcal{H}^{(j,\gamma j)}$$
$$|j,m\rangle \qquad |(j,\gamma j); j, m\rangle$$

$$P(h_\ell, G_\ell) = \sum_{j_\ell} (2j_\ell + 1)\ D^{(j_\ell)}(h_\ell)_m^{m'}\ D^{(\gamma j_\ell, j_\ell)}(G_\ell)_{jm'}^{jm}$$

- coherent states

$$\psi_{H_\ell}(h_\ell) = \int_{SU(2)^N} dg_n \prod_{\ell=1}^{L} \sum_{j_\ell} (2j_\ell+1)\ e^{-2t\hbar\, j_\ell(j_\ell+1)}\ \mathrm{Tr}\,[D^{(j_\ell)}(h_\ell)]$$

$$H_\ell \in SL(2,\mathbb{C})$$

$$P_t(H_\ell, G_\ell) = \int dh_\ell\ K_t(h_\ell, H_\ell)\ P(h_\ell, G_\ell)$$
$$= \sum_{j_\ell} (2j_\ell + 1)\ e^{-2t\hbar j_\ell(j_\ell+1)}\ \mathrm{Tr}\,[\ D^{(j_\ell)}(H_\ell) Y^\dagger D^{(\gamma j_\ell, j_\ell)}(G_\ell) Y\ ]$$

SADDLE POINT

- Coherent states introduces $D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\; D^{(j)}(e^{-iz\frac{\sigma_3}{2}})\; D^{(j)}(R^{-1}_{\vec{n}_n})$

$$D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) = \sum_m e^{-izm}\; |m\rangle\langle m|$$

Im(z) large: $D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) \approx e^{izj}\; |j\rangle\langle j|$

$$D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\; e^{-izj}|j,+j\rangle\langle j,+j|D^{(j)}(R^{-1}_{\vec{n}_n}) = e^{-izj}|j,\vec{n}_\ell\rangle\langle j,\vec{n}_\ell|$$

SADDLE POINT

- Coherent states introduces $D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\; D^{(j)}(e^{-iz\frac{\sigma_3}{2}})\; D^{(j)}(R^{-1}_{\vec{n}_n})$

$$D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) = \sum_m e^{-izm}\ |m\rangle\langle m| \qquad Im(z)\ large\!:\ \ D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) \approx e^{izj}\ |j\rangle\langle j|$$

$$D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\ e^{-izj}|j,+j\rangle\langle j,+j|D^{(j)}(R^{-1}_{\vec{n}_n}) = e^{-izj}|j,\vec{n}_\ell\rangle\langle j,\vec{n}_\ell|$$

- $\mathrm{Tr}\,[\,D^{(j_\ell)}(H_\ell)Y^\dagger D^{(\gamma j_\ell,j_\ell)}(G_\ell)Y\,]$ $\qquad e^{-izj}\langle j,\vec{n}_\ell|\ Y^\dagger D^{(\gamma j_\ell,j_\ell)}(G_\ell)Y\,|j,\vec{n}_\ell\rangle$

$$\int_{SL(2,\mathbb{C})}\prod_{n=1}^{N-1}dG_n\sum_{j_\ell}\prod_{\ell=1}^{L}\ (2j_\ell+1)\ e^{-2t\hbar j_\ell(j_\ell+1)}\ e^{-izj}\,\langle(\gamma j,j);j,\vec{n}_\ell|\ D^{(\gamma j_\ell,j_\ell)}(G_\ell)\,|(\gamma j,j);j,\vec{n}_\ell\rangle$$

SADDLE POINT

- Coherent states introduces $D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\ D^{(j)}(e^{-iz\frac{\sigma_3}{2}})\ D^{(j)}(R^{-1}_{\vec{n}_n})$

$$D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) = \sum_m e^{-izm}\ |m\rangle\langle m| \qquad Im(z)\ large:\ \ D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) \approx e^{izj}\ |j\rangle\langle j|$$

$$D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\ e^{-izj}|j,+j\rangle\langle j,+j|D^{(j)}(R^{-1}_{\vec{n}_n}) = e^{-izj}|j,\vec{n}_\ell\rangle\langle j,\vec{n}_\ell|$$

- $\mathrm{Tr}\,[\,D^{(j_\ell)}(H_\ell)Y^\dagger D^{(\gamma j_\ell, j_\ell)}(G_\ell)Y\,]$ $\qquad e^{-izj}\langle j,\vec{n}_\ell|\ Y^\dagger D^{(\gamma j_\ell,j_\ell)}(G_\ell)Y\,|j,\vec{n}_\ell\rangle$

$$\sum_{j_\ell}\prod_{\ell=1}^{L}\ (2j_\ell+1)\ e^{-2t\hbar j_\ell(j_\ell+1)}e^{-izj}\int_{SL(2,\mathbb{C})}\ \prod_{n=1}^{N-1}dG_n\prod_{\ell=1}^{L}\langle(\gamma j,j);j,\vec{n}_\ell|\ D^{(\gamma j_\ell,j_\ell)}(G_\ell)\,|(\gamma j,j);j,\vec{n}_\ell\rangle$$

- $j_\ell \to \alpha j_\ell$ and $\alpha \gg 1$ $\qquad \Omega(j_\ell) \approx \dfrac{1}{\alpha^3\sqrt{\det\ Hess(j_\ell)}}\ e^{-\frac{1}{2}ij_\ell\,\theta}$

EVALUATION OF THE AMPLITUDE

$$W(z) = \sum_{j_\ell} \prod_{\ell=1}^{L} \frac{1}{\alpha^3 \sqrt{\det \ Hess(j_\ell)}} (2j_\ell + 1)\, e^{-2t\hbar j_\ell (j_\ell+1) - izj_\ell} \ e^{-\frac{1}{2} i j_\ell \, \theta}$$

$$\theta\,(\gamma K + 1) - \theta = 0$$

- **Gaussian sum** *peaked at* j_o *for all* j_ℓ $\quad j \sim j_o + \delta j$
- *max (real part of the exponent) gives where the gaussian is peaked;* $\quad j_o \sim Im\, \tilde{z}/4t\hbar$
- *imaginary part of the exponent=2kπ gives where the gaussian is not suppressed.* $\quad Re\, \tilde{z} = 0$

$$\dot{a} \sim 0$$

- *We obtain Minkowski space!*

$$W(z) = \left(\sqrt{\frac{\pi}{t}}\, e^{-\frac{\tilde{z}^2}{8t\hbar}} \ 2j_o \right)^L \frac{N_\Gamma}{j_o^3}$$

EVALUATION OF THE AMPLITUDE

- **Cosmological constant**

$$Z_C = \sum_{j_f, \mathrm{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda \mathrm{v}_e} \prod_v A_v(j_f, \mathrm{v}_e)$$

intertwiner $\mathrm{v}_e \sim \mathrm{v}_o\, j^{3/2}$

$$W(z) = \sum_j (2j+1)\, \frac{N_\Gamma}{j^3}\; e^{-2t\hbar j(j+1) - izj - i\lambda \mathrm{v}_o j^{\frac{3}{2}}}$$

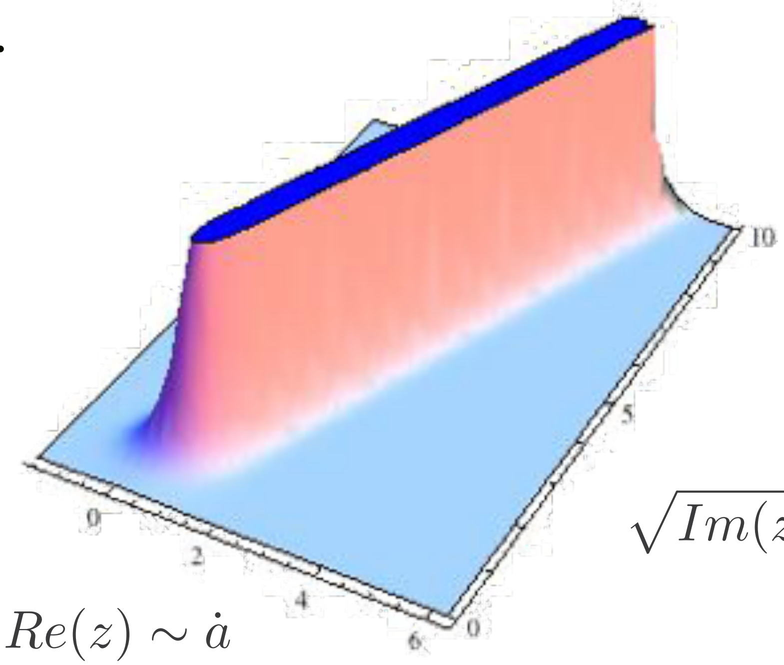
$$i\lambda \mathrm{v}_o j^{\frac{3}{2}} \sim i\lambda \mathrm{v}_o j_o^{\frac{3}{2}} + \frac{3}{2} i\lambda \mathrm{v}_o j_o^{\frac{1}{2}}\, \delta j$$

- *the gaussian is peaked on* $\quad j_o = \dfrac{Im(z)}{4t\hbar}$

- *the gaussian is not suppressed for* $\quad Re(z) + \lambda \mathrm{v}_o j^{\frac{1}{2}} = 0.$

- $\dfrac{Re(z)^2}{Im(z)} = \dfrac{\lambda^2 \mathrm{v}_o^2}{4t\hbar} \longrightarrow \left(\dfrac{\dot a}{a}\right)^2 = \dfrac{\Lambda}{3}$

$$\Lambda = const\, \lambda^2 G^2 \hbar^2$$



$\sqrt{Im(z)} \sim a$

$Re(z) \sim \dot a$

RESULTS and OPEN ISSUES

- *There are approximations in the quantum theory that yield cosmology.*
- *Coherent states for cosmology.*
- *There is a simple way to add the cosmological constant to LQG dynamics.*
- *The theory recover general relativity in the semiclassical limit, also for non-trivial solutions.*
- *Connecting canonical and covariant in loop cosmology.*
- *We can couple fermions in the full theory.*

- *Are these approximations viable?*
- *Is there any relation between coherent states in a truncation and embedding?*
- *There is a complicated way to add the cosmological constant using q-deformed groups.*
- *We want to go beyond semiclassicality, studying quantum correction.*
- *Derivation of $\bar{\mu}$-scheme in the covariant theory.*
- *Matter in Spinfoam Cosmology?*