Causal Dynamical Triangulations

Causal Dynamical Triangulations, for short CDT, is a new approach to the nonperturbative quantization of gravity. As in the case of Euclidean dynamical triangulation approaches, CDT provides a regularization of the gravitational path integral through a sum over piecewise linear geometries where the edge length of the individual building blocks serves as an ultraviolet cutoff. However, in the latter the principle of microcausality has been implemented in the path integral by only allowing for geometries which have a definite causal structure even at the smallest scales. This lead to considerable successes over the Euclidean model. This summary reviews sum of the recent results obtained in the case where spacetime is two-dimensional and analytical results can be obtained.

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Why quantum gravity?

Quantum field theory has proven to be a marvelously successful way to describe three of the four fundamental forces of nature. For gravity however we do not have a well-defined predictive quantum field theoretic description yet, but we do have a very successful classical field theoretic description in the form of Einstein's general relativity. Since the other forces are well described by quantized field theories, it seems natural that there also exists a quantum theory of Einstein's general relativity.

Another reason for believing that such a theory of quantum gravity should exist is the fact that gravity is universal in the sense that it couples to all forms of energy. Hence the energy fluctuations at small distances due to Heisenberg's uncertainty relations induce also quantum fluctuations in the gravitational field. This leads to the prediction that spacetime geometry has a highly non-trivial microstructure at extremely small scales proportional to the Planck length, $l_p = \sqrt{\hbar G_N c^{-3}} \approx 1.616 \times 10^{-35} m$.

There are however obvious problems in constructing a quantum theory of general relativity. It has already been shown in the seventies by 't Hooft and Veltman that perturbative quantum gravity is nonrenormalizable in four dimensions[1]. This does not mean that it is impossible to find a predictive theory of quantum general relativity. There are good indications that one can define a theory of quantum general relativity nonperturbatively[2,3].

There are several nonperturbative approaches to quantum general relativity of which many are studied by researchers of the ENRAGE network. Some of those attempts suggest that the ultraviolet divergences can be resolved by the existence of a minimal length scale, commonly expressed in terms of the Planck length l_p. A famous example is loop quantum gravity[4,5,6]; in this canonical quantization program the discrete spectra of area and volume operators are interpreted as evidence for fundamental discreteness. Other approaches, such as four-dimensional spin-foam models[7] or causal set theory[8,9], postulate fundamental discreteness from the outset. Unfortunately, most of these quantization programs still have problems in recovering a sensible classical limit. The latest successes in the approach of causal set theory will be presented in one of the forthcoming ENRAGE newsletters.

There are also nonperturbative approaches which do not introduce a fundamental discreteness scale from the outset. One example is the exact renormalization group flow method for Euclidean quantum gravity in the continuum[3]. Another attempt is Causal Dynamical Triangulations (CDT), a covariant path integral formulation, in which Lorentzian quantum gravity is obtained as a continuum limit of a superposition of simplicial space-time geometries [2].

The CDT approach

The CDT program is a quantization scheme for general relativity where no supersymmetry or ad hoc fundamental discreteness is assumed from the outset. The program is meant to give a rigorous nonperturbative definition of a path integral over all causal geometries (by this we mean the equivalence class of a Lorentzian metric modulo its diffeomorphisms) weighted by the Einstein-Hilbert action

$$\mathcal{Z} = \int \mathcal{D}[g_{\mu\nu}] e^{iS_{\mathrm{EH}}[g_{\mu\nu}]}. \tag{1}$$

From lattice QCD we know that discrete methods are a powerful tool to investigate nonperturbative effects in quantum field theory. In CDT one uses a specific discretization similar to Regge calculus where the geometry itself is encoded in a simplicial lattice. The advantage of using this type of discretization is that one is automatically working with gauge invariant degrees of freedom. There is no need to introduce coordinates in the construction [10]. There is however a crucial difference between Regge calculus and dynamical triangulations. In the first the dynamics is encoded in the variation of the edge lengths whereas in dynamical triangulations the edge lengths are fixed but the dynamics is encoded in the gluing of the simplicial building blocks.

In the explicit construction of CDT the path integral over all causal geometries is written as a sum over all causal triangulations T weighted by the Regge action (the simplicial analog of the Einstein-Hilbert action including a cosmological constant λ),

$$\mathcal{Z}(\lambda, G_N) = \sum_{\text{causal T}} \frac{1}{C_T} e^{iS_{\mathrm{Regge}}}, \tag{2}$$

where C_T is the discrete symmetry factor of the triangulation T. In contrast to previous attempts of dynamical triangulations the triangulations appearing in *causal* dynamical triangulations have a definite foliated structure. In this type of triangulations each $(d-1)$-dimensional spatial slice is realized as an Euclidean triangulation whose simplicial building blocks have all squared edge lengths given by $l_s^2 = a^2$. The successive spatial slices are connected by time-like edges of squared edge lengths $l_t^2 = -\alpha a^2$ with $\alpha > 0$, such that all building blocks in T are d-simplices (see Fig. 1 for an illustration in 1+1 dimensions). Here the parameter a is a cut-off length that one takes to zero in order to obtain the continuum limit of the regularized path integral (2). Note that in this limit the individual triangulations correspond to the individual histories of the path integral, which in general do not resemble smooth manifolds.

The foliated structure of the triangulations introduces a natural notion of discrete global time t given by the label of the successive spatial slices. Note that one has to be careful in attaching a physical meaning to this global time as we discuss later on. The clear distinction between space-like edges and time-like edges enables us to define a Wick rotation on each causal triangulation by analytic continuation of $\alpha \mapsto -\alpha$. It is important to realize that the set of Euclidean triangulations one obtains after the Wick rotation is strictly smaller than the set of all Euclidean triangulations.

Recent results in 3+1 dimensions

Before describing the analytic results in 1+1 dimensions let us mention some of the recent exciting successes of CDT in 3+1 dimensions as a motivation for the CDT approach to quantum gravity (see also [11] for a general overview).

In absence of an analytic solution for the 3+1 dimensional model, one uses Monte Carlo simulations to obtain numerical results. A very important nontrivial test for every nonperturbative formulation of quantum gravity is whether it can reproduce a sensible classical limit at macroscopic scales. The numerical results indicate that the scaling behavior of the spatial volume as a function of space-time volume is that of a four-dimensional universe at large scales, a first indication of sensible classical behavior [12]. Moreover, after integrating out all dynamical variables apart from the spatial volume as a function of proper time, one can derive the scale factor whose dynamics is described by the simplest minisuperspace model used in quantum cosmology [13].

Having passed the first consistency checks regarding the *macro*scopical structure of space-time it is very interesting what predictions one can make for the quantum nature of the *micro*structure of space-time. One important observable which has been measured is the spectral dimension of space-time which is the dimension a diffusion process would feel on the space-time ensemble. Surprisingly, this quantity depends on the scale at which it is measured. More precisely, one observes a dimensional reduction from four at large scales to two at small scales within measurement accuracy [14]. This gives an indication that nonperturbative quantum gravity defined through CDT provides an effective ultraviolet cut-off through a dynamical dimensional reduction of space-time.

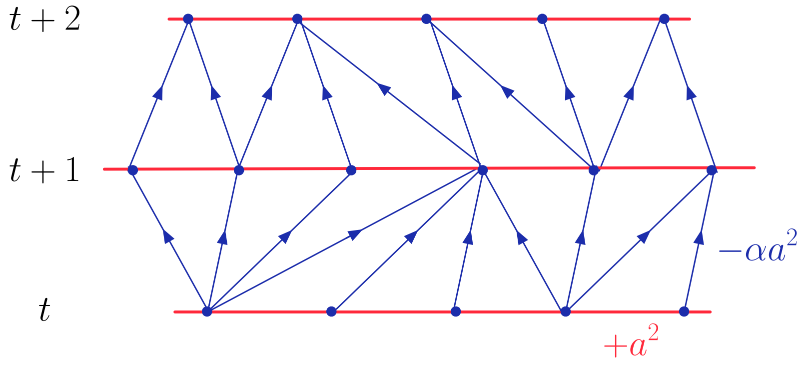

Figure 1: A section of a sequence $[t, t+2]$ of two spacetime strips of a triangulated two-dimensional spacetime contributing to the regularized path integral.

Analytic results in 1+1 dimensions

To better understand the methods used in CDT it is useful to look in more detail at the 1+1 dimensional model, since it is exactly solvable [15].

Although two-dimensional quantum gravity does not have any propagating degrees of freedom it is a fertile playground to study certain aspects of diffeomorphism invariant theories. Among the issues that have been addressed within the two-dimensional framework are the inclusion of a sum over topologies [16] and the emergence of a background geometry purely from quantum fluctuations [17] which we want to discuss in the following.

Recall that the Einstein-Hilbert action in two dimensions is given by

$$S_{\text{EH}}[g] = \int_M d^2x \sqrt{|\det g|}\, \Lambda - 2\pi K \chi(M) \qquad (3)$$

where $K = G_N^{-1}$ is the inverse Newton's constant, $\chi(M) = 2 - 2\mathfrak{g}$ the Euler characteristic of the manifold M and $\mathfrak{g}$ the genus of M. Therefore, for fixed spatial topology, the curvature term in the action contributes just a constant phase factor to the path integral. If one allows for topology changes this term becomes important for the quantum dynamics as we will see in the next section. In this section we fix the topology of space-time to be $\mathbb{R}\times S^1$. The most natural thing to calculate is the propagator from an initial geometry of length l_{in} to a final geometry of length l_{out} in time t. Using the discrete analog of the Einstein-Hilbert action (3) one can write down the propagator after Wick rotation as the path integral (2) for fixed boundaries l_{in} and l_{out},

$$G_\lambda(l_{in}, l_{out}; t) = \sum_{\substack{\text{causal T:}\\ l_{in}\to l_{out}}} e^{-\lambda a^2 N(T)}, \qquad (4)$$

where λ is the bare cosmological constant and $N(T)$ the number of triangles in the triangulation. In two dimensions the sum in (4) can be evaluated and one obtains the continuum limit after a suitable choice of renormalization, yielding the continuum propagator

$$G_\Lambda(L_{in}, L_{out}; T) = \frac{e^{-\sqrt{2\Lambda}(L_{in}+L_{out})\coth(\sqrt{2\Lambda}T)}}{\sinh(\sqrt{2\Lambda}T)} \times$$
$$\times \sqrt{\frac{2\Lambda}{L_{in}L_{out}}}\, I_1\left(2\frac{\sqrt{2\Lambda L_{in}L_{out}}}{\sinh(\sqrt{2\Lambda}T)}\right), \qquad (5)$$

where $I_1(x)$ denotes the modified Bessel function of the first kind and Λ, L_i and T are the continuum counterparts of λ, l_i and t. Physically this solution can be interpreted as a fluctuating two-dimensional "universe" (Fig. 2), where the average spatial length and its fluctuations are determined by the cosmological constant, $\langle L\rangle \sim \langle \Delta L\rangle \sim 1/\sqrt{\Lambda}$. This means that the two-dimensional "universe" is purely governed by quantum fluctuations and we have no notion of a semi-classical background. This situation changes when we will discuss the transition to non-compact space-times later on in the text.

Remarkably, the continuum propagator (5) agrees with the result of the propagator obtained from a continuum calculation in the proper-time gauge of 1+1 dimensional pure gravity [18]. This indicates that the above choice of global time is similar to the one used in the proper-time gauge. Another interesting question is whether the result obtained in (5) is independent of the choice of foliation. There are good indications that due to the broad universality class of this model one also obtains the same dynamics (5) for different choices of time slicing [19].

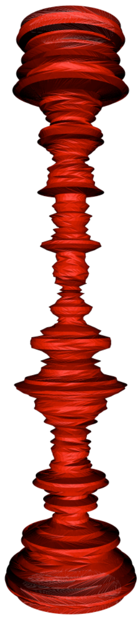

Figure 2: A typical two-dimensional Lorentzian spacetime. The compactified direction shows the spatial hypersurfaces of length L and the vertical axis labels time T. Technically, the picture was generated by a Monte Carlo simulation, where a total volume of $N = 18816$ triangles and a total time of $t = 168$ steps was used. Further, initial and final boundary has been identified.

Including topology changes

A recurring question in the history of quantum gravity approaches is whether one should allow for topology changes of space-time. In terms of path integrals this translates into whether one should include the sum over topologies in the path integral

$$\mathcal{Z} = \sum_{\text{topol.}} \int \mathcal{D}[g_{\mu\nu}] e^{iS_{\text{EH}}[g_{\mu\nu}]}. \tag{6}$$

There have been several attempts to solve the path integral (6) in the setting of Euclidean quantum gravity. The big problem however that becomes apparent even in the simplest case of two dimensions is that the full sum over topologies cannot be uniquely defined nonperturbatively, since the sum over genera is badly divergent. One of the main differences between this approach and the one we propose here is that we restrict the class of topology changes by means of imposing an (almost everywhere) causal structure. In the following we show that this restriction on the topology changes leads to a better defined path integral and we introduce a model with infinitesimal wormholes where one can perform the sum over topologies explicitly[20,21,16].

We define the sum over topologies in (6) by performing surgery moves directly on the triangulations to obtain regularized versions of higher-genus manifolds[20,21]. For the construction of these moves let us concentrate on a single space-time strip of topology $[0, 1] \times S^1$ and height $\Delta t = 1$ as illustrated in Fig. 3. The infinitesimal wormholes can be constructed by identifying two of the time-like edges and subsequently cutting open the geometry along this edge. By applying this procedure repeatedly and obeying certain causality constraints[20,21], more and more wormholes can be created.

To obtain the dynamics of the model it is sufficient to analyze the one-step propagator. Including the topological term in the action and performing the Wick rotation gives

$$G_{\lambda,\kappa}(l_{in}, l_{out}; t = 1) = \sum_{\substack{\text{causal T:} \\ l_{in} \to l_{out}}} e^{-\lambda\, a^2\, N - 2\kappa\, \mathfrak{g}(T)}, \tag{7}$$

where the sum is taken over all possible triangulations T of height $t = 1$ with fixed initial boundary l_{in} and final boundary l_{out}, but arbitrary genus $0 \leq \mathfrak{g}(T) \leq [N/2]$ (here $N = l_{in} + l_{out}$ is the number of triangles in T, which coincides with the number of time-like edges). Further, λ is the bare cosmological constant and κ is the bare inverse Newton's constant. The sum over all possible triangulations with arbitrary genus can be performed unambiguously and one can obtain the continuum limit after a suitable choice of renormalization and double scaling limit for λ and κ, yielding the continuum propagator[16] of the form of (5), where the renormalized cosmological constant Λ is replaced by the effective cosmological constant $\Lambda_{\text{eff}} = \Lambda(1 - e^{-4\pi/G_N})$ which both depends on the renormalized cosmological constant and Newton's constant.

In addition to the known geometrical observables this model possesses a new type of topological observable, namely, the density of wormholes, which can be calculated to give a finite expression

$$n = \frac{\langle N_{\mathfrak{g}} \rangle}{\langle V \rangle} = \frac{1}{e^{\frac{4\pi}{G_N}} - 1}\, \Lambda. \tag{8}$$

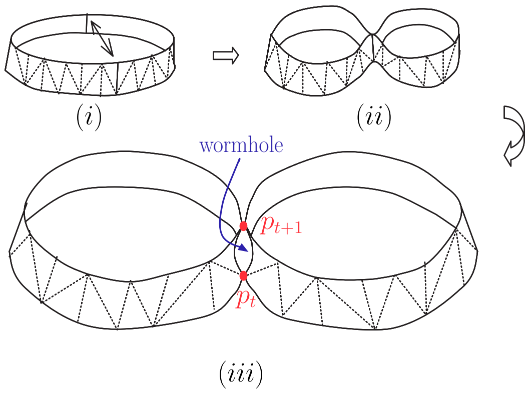

Figure 3: Construction of a wormhole: starting from a space-time strip of topology $[0,1]\times S^1$ as in the pure CDT model (i), one identifies two time-like edged (ii) and then cuts open the geometry perpendicular to this line (iii). The two resulting saddle points at time t and $t+1$ are labeled with p_t and p_{t+1}.

It is useful to reinterpret the physical system in terms of its physical quantities, namely the cosmological constant Λ and the density of wormholes in units of Λ, i.e. $\eta = \frac{n}{\Lambda}$. These two quantities can be seen to set the physical scales of the system. Whereas in the case without topology changes (5) there was only one scale, namely, the cosmological scale $\langle L\rangle \sim 1/\sqrt{\Lambda}$, in the case with topology changes there is in addition the relative scale η between cosmological and topological fluctuations. Both together define the effective fluctuations in length through $\langle L\rangle \sim 1/\sqrt{\Lambda_{\text{eff}}}$, where we can now write $\Lambda_{\text{eff}} = \Lambda/(1+\eta)$. One can observe that the presence of wormholes in spacetime leads to a decrease in the "effective" cosmological constant Λ_{eff}. This connects nicely to former attempts to derive a mechanism, the so-called Coleman's mechanism, to explain the smallness of the cosmological constant in a formal Euclidean path integral formulation of four-dimensional quantum gravity in the continuum with the presence of infinitesimal wormholes [22]. The wormholes considered in those models resemble those of our toy model in that both are non-local identifications of the spacetime geometry of infinitesimal size. The counting of the wormholes considered here is of course different, since we are working in a genuinely causal and background independent setup which enables us to actually perform the sum over topologies explicitly, without assuming any information on a background manifold.

The emergence of background geometry

In the previous sections we described the quantum geometry of the 1+1 dimensional universe in the case where the geometry was compact. We have seen that under this conditions the geometry was purely governed by quantum fluctuations. Therefore, there was no sensible notion of a semi-classical background. In the following we want to describe the transition in which the compact geometries become non-compact. We will see that in this case we find the emergence of a semiclassical background dressed with small quantum fluctuations. [17]

To determine the background geometry of the 1+1 dimensional universe we calculate the average spatial length at proper time $t \in [0,T]$

$$\langle L(t)\rangle_{X,Y,T} = \frac{1}{G_\Lambda(X,Y;T)} \times \\ \times \int_0^\infty \mathrm{d}L\, G_\Lambda(X,L;t)\, L\, G_\Lambda(L,Y;T-t). \qquad (9)$$

Here X and Y are the boundary cosmological constants which are the conjugate variables of the boundary lengths L_{in} and L_{out}. The continuum propagator $G_\Lambda(X,Y;T)$ with respect to the boundary cosmological constants can be obtained form $G_\Lambda(L_{in},L_{out};T)$, i.e. (5), by inverse Laplace transformation.

Evaluating the average length at the boundary $t=T$ and taking the limit $T\to\infty$ gives

$$\lim_{T\to\infty} \langle L(T)\rangle_{X,Y,T} = \frac{1}{Y+\sqrt{\Lambda}}. \qquad (10)$$

Interestingly, one observes that there is a special value $Y = -\sqrt{\Lambda}$ of the boundary cosmological constant for which the boundary length diverges and the geometry becomes non-compact. Using this critical value for the boundary cosmological constant Y one can obtain the boundary length for finite T

$$\begin{aligned} L^c(T) &= \langle L(T)\rangle_{X,Y=-\sqrt{\Lambda};T} \\ &= \frac{1}{\sqrt{\Lambda}}\,\frac{1}{\coth\sqrt{\Lambda}T-1}. \end{aligned} \qquad (11)$$

Instead of using boundary cosmological constants one can also fix the spatial length of the boundaries. Using the continuum propagator $G_\Lambda(L_1,L_2;T)$ we can evaluate the average spatial length $\langle L(t)\rangle_{L_{in},L_{out},T}$ for fixed lengths L_{in} and L_{out} of the boundary loops.

In the following we want to investigate the quantum geometry in the case where it becomes non-compact. Therefore we set the boundary length at $t=T$ to the critical value $L^c(T)$ as defined in (11) and for simplicity we shrink the spatial geometry at $t = 0$ to a point. In the limit $T \to \infty$ one obtains the average length of the spatial geometry at proper time $t \in [0, T]$

$$\begin{aligned} \langle L(t) \rangle &\equiv \lim_{T\to\infty} \langle L(t) \rangle_{L_{in}=0, L_{out}=L^c(T), T} \\ &= \frac{1}{\sqrt{\Lambda}} \sinh(2\sqrt{\Lambda} t). \end{aligned} \tag{12}$$

Due to the fact that L and T are defined from the continuum limit of a simplicial geometry there is a relative constant of proportionality that can only be fixed by comparing with continuum calculations [18] yielding $L_{cont}(t) = \pi \langle L(t) \rangle$. From this result the metric for the background geometry is readily obtained,

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{d}t^2 + \frac{L_{cont}^2}{4\pi^2}\, \mathrm{d}\theta^2 \\ &= \mathrm{d}t^2 + \frac{\sinh^2(2\sqrt{\Lambda} t)}{4\Lambda}\, \mathrm{d}\theta^2. \end{aligned} \tag{13}$$

This is nothing but the metric of the Poincaré disc which can be seen as a Wick rotated version of AdS_2 with constant negative curvature $R = -8\Lambda$.

To better understand the quantum nature of the geometry it is useful to compute the fluctuations of the spatial length. From expressions analogous to (9) one can determine the relative fluctuations

$$\frac{\Delta L(t)}{\langle L(t) \rangle} \equiv \frac{\sqrt{\langle L^2(t) \rangle - \langle L(t) \rangle^2}}{\langle L(t) \rangle} \sim \mathrm{e}^{-\sqrt{\Lambda} t}. \tag{14}$$

Surprisingly, the fluctuations of the spatial geometry become exponentially small for $t \gg \Lambda^{-1/2}$. Concluding from (13) and (14), one can view the quantum geometry as a version of Wick rotated AdS_2 dressed with small quantum fluctuations.

We have shown that in 2D quantum gravity defined through CDT there is a transition from compact geometry to non-compact AdS_2-like geometry for a special value of the boundary cosmological constant. This phenomenon is similar to the Euclidean case where non-compact ZZ-branes appear in a transition from compact 2D geometries in Liouville quantum gravity. [23] A surprising feature of the CDT result is that the fluctuations become exponentially small which enables us to interpret the emerging AdS_2 spacetime as a genuine semiclassical background. It is interesting that similar results have been reported in four-dimensional CDT where numerical simulations indicate the emergence of a semi-classical background from a nonperturbative and background-independent path integral. [2]

Conclusion

We have presented a summary of some of the recent results obtained within the framework of CDT in two spacetime dimensions. Among those were the inclusion of a sum over topologies and the emergence of a background geometry purely from quantum fluctuations. In particular, the possibility of obtaining analytical results in two spacetime dimensions provides us with a better understanding of some of the phenomena which have been numerically observed in the four-dimensional context. The reader who wants to learn more about those recent successes in four dimensions is referred to the following nontechnical introduction [11].

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