Spinfoam Cosmology

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quantum cosmology from the full theory

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What is quantum cosmology?

gravity

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$$\text{gravity} \xrightarrow{\textit{quantization}} \text{quantum gravity}$$

What is quantum cosmology?

$$
\begin{tikzcd}
\text{gravity} \arrow[r, "\text{quantization}"] \arrow[d, "\text{simmetry reduction}"'] & \text{quantum gravity} \\
\text{cosmology} \arrow[r] & \text{quantum cosmology}
\end{tikzcd}
$$

- Wheeler, DeWitt, Misner (1967)
- LOOP QUANTUM COSMOLOGY
  Bojowald (1999), *see the talks by Ashtekar and Singh*

What is quantum cosmology?

$$\begin{tikzcd}
\text{gravity} \arrow[r, "\text{quantization}"] \arrow[d, swap, "\begin{matrix}\text{simmetry}\\ \text{reduction}\end{matrix}"] & \begin{matrix}\text{quantum}\\ \text{gravity}\end{matrix} \arrow[d] \\
\text{cosmology} \arrow[r] & \begin{matrix}\text{quantum}\\ \text{cosmology}\end{matrix}
\end{tikzcd}$$

What is quantum cosmology?

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\text{gravity} \arrow[r, "\text{quantization}"] \arrow[d, "\text{simmetry reduction}"'] & \text{quantum gravity} \arrow[d] \\
\text{cosmology} \arrow[r] & \text{quantum cosmology}
\end{tikzcd}$$

- ☐ What is the relation between LQC and full LQG?
- ☐ Can we describe the full quantum geometry at the bounce?
- ☐ Can we include "naturally" inhomogeneities ?

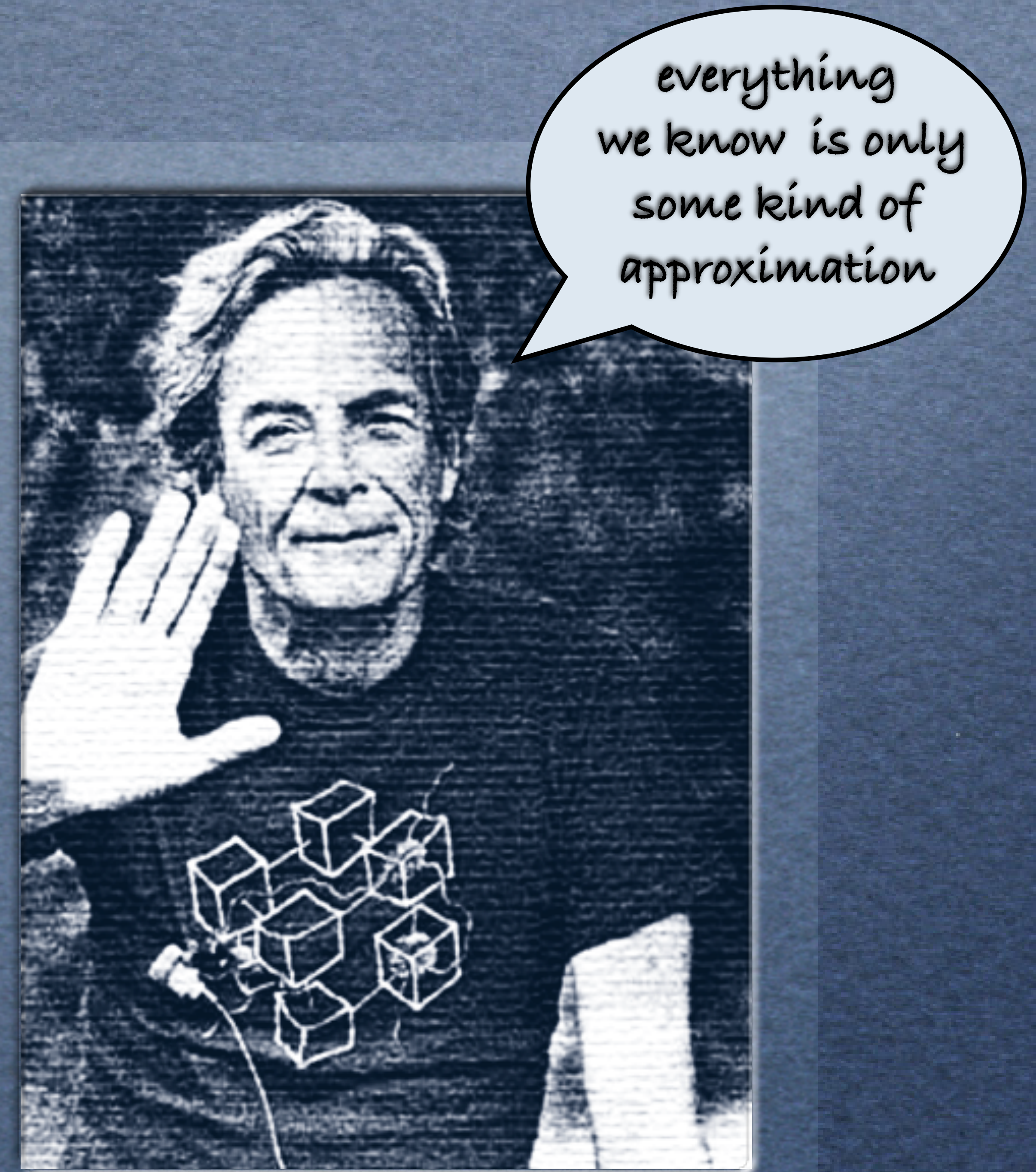
Plan of the talk

- What is Quantum Cosmology?
  Approximations in cosmology

- Definition of the theory

  1. *Kinematics*
     Graph expansion
  2. *Dynamics*
     Vertex expansion
  3. *Classical limit*
     Large volume expansion

- Summary and comments



The cosmological principle

- The dynamics of a homogeneous and isotropic space approximates well the observed universe.

- The effect of the inhomogeneities on the dynamics of its largest scale, described by the scale factor, can be neglected in a first approximation.

- This is not a large scale approximation, because it is supposed to remain valid when the universe was small! It is an expansion in $N \sim a/\lambda$!

- The full theory may be recovered by adding degrees of freedom one by one, starting from the cosmological ones.

- We can define an approximated dynamics of the universe for a finite number of degrees of freedom .

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Kinematics

Hilbert space: $\tilde{\mathcal{H}} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma$ where $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$

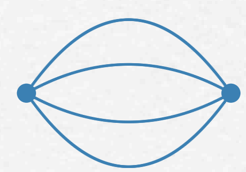
Identifications: $\tilde{\mathcal{H}}/\sim$

1. if Γ is a subgraph of Γ' then we must identify $\mathcal{H}_\Gamma$ with a subspace of $\mathcal{H}'_\Gamma$
2. divide $\mathcal{H}_\Gamma$ by the action of the discrete group of the automorphisms of Γ

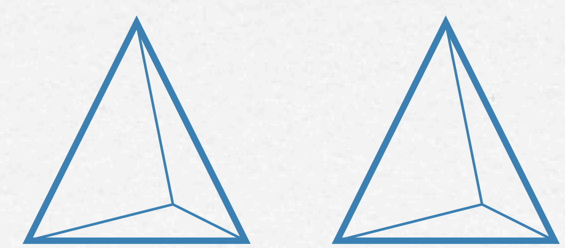
States that solve gauge constraint: $|\Gamma, j_\ell, v_n\rangle \in \tilde{\mathcal{H}} = \bigoplus_{\Gamma} \bigoplus_{j_\ell} \bigotimes_{n} \mathcal{H}_n$

Example:

the "dipole" where $N = 2$

$\Delta_2^* =$ 

so that Δ_2 is formed by two tetrahedra glued along all their faces = triangulated 3-sphere !

$\Delta_2 =$ 

Coherent states $\rightarrow$ Semiclassical States

$\mathcal{H}_\Gamma$ contains an (over-complete) basis of "wave packets" $\psi_{H_\ell} = \psi_{\vec{n}_\ell, \vec{n}'_\ell, \xi_\ell, \eta_\ell}$

Holomorphic-states: see Perini's talk

$$\psi_{H_\ell}(U_\ell) = \int_{SU(2)^N} dg_n \bigotimes_{l\in\Gamma} K_t(g_{s(\ell)} U_\ell g_{t(\ell)}^{-1} H_\ell^{-1}).$$

$$H = D^{\frac{1}{2}}(R_{\vec{n}})\, e^{-i(\xi+i\eta)\frac{\sigma_3}{2}}\, D^{\frac{1}{2}}(R_{\vec{n}'}^{-1})$$

superpositions of SN states

"group averages" on the gauge invariant states

$H_\ell \in SL(2,\mathbb{C})$

heat kernel peaks each U_ℓ on H_ℓ

Geometrical interpretation

for the $(\vec{n}, \vec{n}', \xi, \eta)$ labels:

$\vec{n}, \vec{n}'$ are the 3d normals to the faces of the cellular decomposition

$\xi \leftrightarrow$ extrinsic curvature at the faces and

$\eta \leftrightarrow$ area of the face divided by $8\pi G$.

Choose coherent states $|H_\ell\rangle$ describing a homogeneous and isotropic geometry:

$$z_\ell = \xi_\ell + i\eta_\ell$$
$$\downarrow$$
$$z = \alpha c + i\beta p \qquad \forall \ell$$

Graph Expansion

Mode expansion $\longleftrightarrow$ truncation on a graph

- **Restrict the states to a fixed graph with a finite number N of nodes.**
  This defines an approximated kinematics of the universe, inhomogeneous but truncated at a finite number of cells.

- **The graph captures the large scale d.o.f. obtained averaging the metric**
  over the faces of a cellular decomposition formed by N cells.

- **The full theory can be regarded as an expansion for growing N.**
  FRW cosmology corresponds to the lower order where there is only a regular cellular decomposition: the only d.o.f. is given by the volume.

- **Coherent states $|H_\ell\rangle$ describing a homogeneous and isotropic geometry:**

$$z = \xi_\ell + i\eta_\ell \quad \longrightarrow \quad z = \alpha c + i\beta p \qquad \forall \ell$$

  Geometry is determined by (c, p) in the past and (c′, p′) in the future.

Dynamics

The spinfoam formalism associates an amplitude to each boundary state $\psi \in \mathcal{H}$.

$$\langle W \mid \psi \rangle = \sum_{\sigma} \prod_{f} d_f(\sigma) \prod_{v} W_v(\sigma)$$

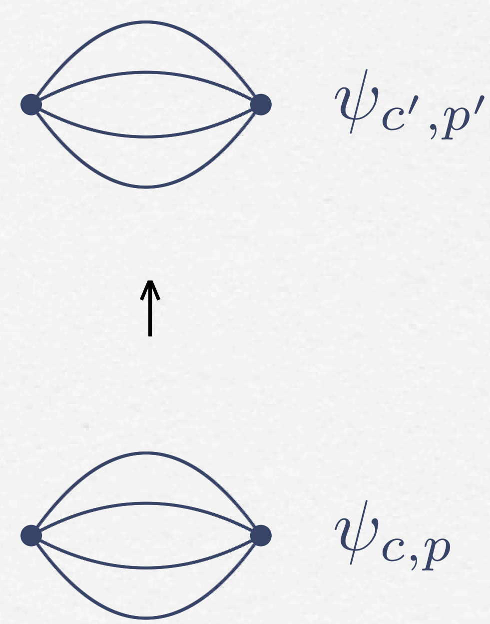
see Magliaro's talk

$$W_v(H_\ell) = \int_{SO(4)^N} dG_n \; \prod_{\ell} P_t(H_\ell \, , \, G_{s(\ell)} G^{-1}_{t(\ell)})$$

where

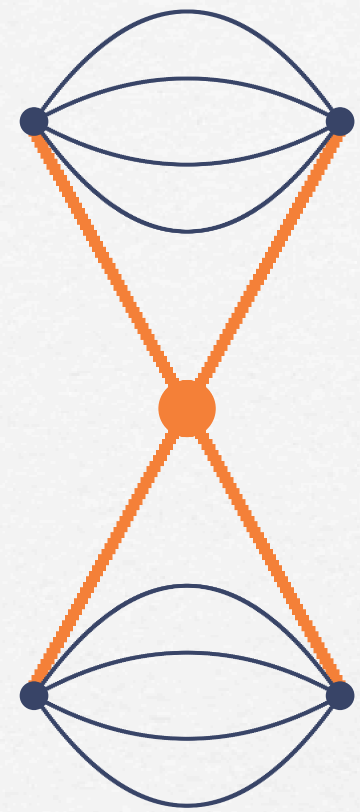
$$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)} \mathrm{Tr}\left[D^{(j)}(H) Y^\dagger D^{(j^+,j^-)}(G) Y \right]$$

In cosmology:
transition 3-sphere → 3-sphere



see next talk by Bianchi

Vertex expansion



$$\langle W|\psi_{H_{l(z,z')}}\rangle = W(z,z')$$

At 1st order in the vertex expansion,
the "dipole-dipole" amplitude
is given by the spinfoam :

Large-distance expansion:

Boundary state peaked on boundary geometry large compared with the Planck length.
Holomorphic boundary states ψ_{H_ℓ} where $\eta_\ell >> 1$ in each H_ℓ .

This can be computed explicitly! Bianchi, FV, Rovelli

$$W(z,z') = C\ zz' e^{-\frac{z^2+(z')^2}{\hbar}}$$

Classical limit 1

$z = \alpha c + i\beta p$

This resulting amplitude happens to satisfy an equation

$$\hat{H}(z, \frac{d}{dz}) W(z', z) = \frac{3}{8\pi G(4\alpha\beta\gamma)^2} \left(\hat{z}^2 - \hat{\bar{z}}^2 - 2\right)^2 W(z', z) = 0.$$

In terms of (c,p) variables:

$$H = \frac{3}{8\pi G(4\alpha\beta\gamma)^2} \left(4i\,\alpha\beta\, cp - 2\hbar\right)^2 = 0$$

In the large p limit and dividing by $Vol \sim p^{3/2} > 0$ we have:

$$H = -\frac{3}{8\pi G\gamma^2} \sqrt{p} c^2 = 0$$

This is precisely the hamiltonian constraint of a homogeneous and isotropic cosmology.

→ LQG yields the Friedmann equation.

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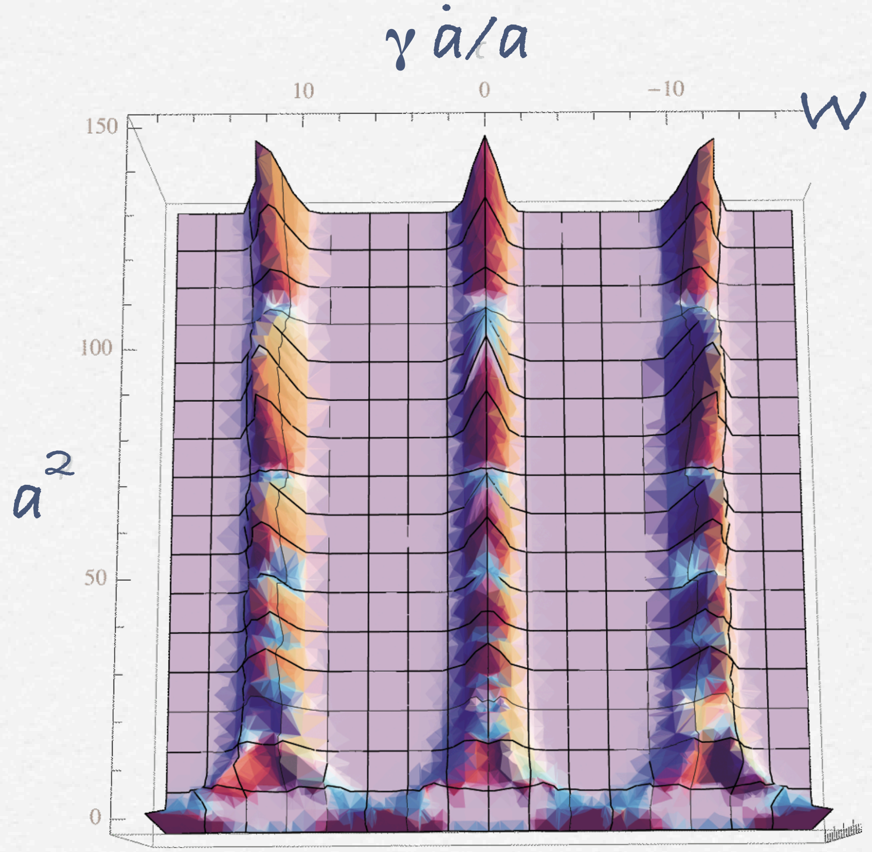
Classical limit 2

At fixed (c′,p′), the transition amplitude W is peaked on the following values of (c,p)



Work in progress !!!

The peaks correspond to the (semi) classical trajectories

Summary

1. It is possible to compute quantum transition amplitudes explicitly

   in suitable approximations:
   1 graph expansion
   2 vertex expansion
   3 large volume expansion

2. The transition amplitude computed appears to give the correct

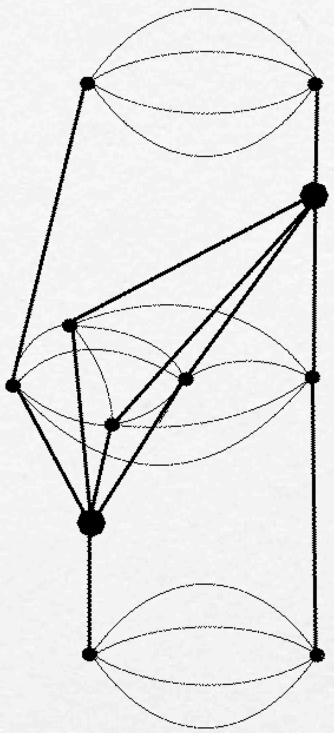
   Friedmann dynamics in the classical limit.

3. Family of models opening a systematic way

   - for describing the inhomogeneous d.o.f. in quantum cosmology,
   - for studying the fluctuations of quantum geometry at the bounce.

4. Light on LQC/LQG relation.

Work in progress



- Lorentzian version
  Cosmological Constant
  Matter

work in progress !!!

Bianchi, Magliaro, Marcianò, Perini, Rovelli, FV...

- Further order in the vertex expansion → → →

- Many nodes; Many edges ($U(N)$ simmetry) Borja, Diaz-Polo, Garay, Livine.

- Relation to Loop Quantum Cosmology

also to Ashtekar, Campiglia, Henderson SF expansion.

- Compute quantum corrections: does bounce scenario survives?

- How do quantum inhomogeneous fluctuations affect structure formation? and inflation?

... and more!