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Oscillations in the ratio of factorial to cumulant multiplicity moments in Z decays at LEP

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Abstract

The ratio of the cumulant factorial to factorial moments of the charged particle multiplicity distribution is known to show a quasi-oscillatory behaviour when plotted versus the order of the moments. This peculiar behaviour is also predicted by the NNLLA of perturbative QCD assuming the validity of the LPHD hypothesis. Using the subjet multiplicity distribution obtained from both the Durham and the Cambridge jet algorithms, instead of the charged particle multiplicity distribution, in order to vary the dependence on the LPHD hypothesis, we find that the oscillations appear only for non-perturbative energy scales.

Introduction

The shape of a distribution can be analyzed in terms of its moments. For studying the shape of the charged particle multiplicity distribution, the normalized factorial moments are often used [1]. The normalised factorial moment of order q is defined in terms of the multiplicity distribution, P_n, by:

$$F_q = \frac{\sum_{n=q}^{\infty} n(n-1)....(n-q+1)P_n}{\left(\sum_{n=1}^{\infty} nP_n\right)^q}$$

The factorial moments reflect correlations in particle production. If particles are produced independently the particle distribution is Poissonian, for which all factorial moments are equal to one. If the particles are correlated, the distribution is broader than Poisson and the factorial moments are greater than one. If the particles are anti-correlated, the distribution is narrower than Poisson and the factorial moments are smaller than one.

The cumulant factorial moments are obtained from the factorial moments by:

$$K_q = F_q - \sum_{m=0}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m} F_m$$

The cumulant factorial moment, K_q, expresses the phase space integral over the genuine q-particle correlation function, i.e., the function which describes the correlations between q particles. The behavior of F_q and H_q with moment order q is shown in fig. 1 for the Poisson distribution and for the negative binomial distribution, which has often been used to fit the charged particle multiplicity distribution [2].

Since $|K_q|$ and F_q increase rapidly with their moment order, it is useful to define the ratio $H_q = \frac{K_q}{F_q}$, which has the same order of magnitude over a large range of q. One can view this ratio as a cumulant factorial moment normalised to the factorial moment. It reflects the genuine q-particle correlation integral relative to the global correlation integral.

The H_q have been calculated for the soft gluon multiplicity distribution at different orders of perturbative QCD (pQCD) [3]:

- For the Double Leading Logarithm Approximation (DLLA), H_q decreases to 0 as q^{-2}.
- For the Modified Leading Logarithm Approximation (MLLA), H_q decreases to a negative minimum at $q = 5$, and then rises to approach 0 asymptotically.
- For the Next to Leading Logarithm Approximation (NLLA), H_q decreases to a positive minimum at $q = 5$ and then increase to a positive constant value for large moment rank.
- For the Next to Next to Leading Logarithm Approximation (NNLLA), H_q decreases to a negative first minimum for $q = 5$, and for $q > 5$, H_q shows quasi-oscillations about 0.

These behaviors are illustrated in figure 2.

Assuming the validity of the Local Parton Hadron Duality hypothesis (LPHD), such behavior is also expected for the charged particle multiplicity distribution.

The H_q of the charged particle multiplicity distribution have been measured by the SLD collaboration [4] for $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} = M_Z$. They observed (figure 3) a negative

minimum for $q = 5$ followed by quasi-oscillations about 0 for higher q. This result has been interpreted as confirmation of the NNLLA prediction.

The H_q moments have also been calculated from published charged particle multiplicity distributions of ALEPH, DELPHI, OPAL [5,6] and in each case, similar behavior with a negative first minimum followed by quasi-oscillation about 0, has been seen. Furthermore, this result is not limited to e^+e^- physics; this oscillatory behavior of the H_q moments has also been observed in hadron collisions [7] between 30 and 900 GeV.

While it is tempting to conclude that the observed behavior is a confirmation of the NNLLA prediction, it must be realized that such a conclusion rests strongly on the validity of LPHD. Although the LPHD hypothesis is accepted for single-particle distributions and mean multiplicity, difficulties have been detected when resonances are involved [8,9]. Further, heavy quark decays will cause a modification of the multiplicity distribution which is unlikely to be properly accounted for LPHD [6]. For these reasons, we consider the assumption of LPHD as highly questionable.

Therefore, in this paper we present measurements of the ratio of cumulant to factorial moments not only of the charged particle multiplicity distribution but also of the jet multiplicity distributions. By considering jets, i.e., jets obtained with small values of the jet resolution parameter, we can, in some sense, follow the development of parton to hadron by varying the jet resolution parameter (and hence the energy scale). The variation of the mean jet multiplicity has been found [10–13] to agree well with analytic perturbative QCD calculations in the perturbative region (energy scale greater than about $1-2$ GeV) and with Monte Carlo models in both the perturbative and the non-perturbative region. These jet multiplicity distributions should therefore enable us to extend the moment-ratio analysis to the energy scales of partons and to that of perturbative QCD, avoiding (or at least lessening) the dependence on the assumption of LPHD.

Selection and correction procedures on the data

This analysis is based on data collected by the L3 detector [14] in 1994 at the energy of the Z. The data sample corresponds to approximately 1.5 million produced hadronic Z decays.

Events are selected in a two-step procedure, which begins with a pre-selection using the energy measured in the electro-magnetic and hadronic calorimeters. All calorimeter clusters are required to have an energy of at least 100 MeV. Hadronic events are then selected by requiring

$$0.5 < \frac{E^{\rm cal}}{\sqrt{s}} < 1.4 \ , \ \frac{|E_{\parallel}^{\rm cal}|}{E^{\rm cal}} < 0.4 \text{ and } \frac{|E_{\perp}^{\rm cal}|}{E^{\rm cal}} < 0.6$$

where $E^{\rm cal}$ is the total calorimeter energy, $E_{\parallel}^{\rm cal}$ is the energy imbalance along the beam direction and $E_{\perp}^{\rm cal}$ is the energy imbalance in the plane perpendicular to the beam direction. To ensure that the event lies within the full acceptance of the calorimeter, selected events are required to have $|\cos(\theta_{\rm th}^{\rm cal})| \leq 0.74$, where $\theta_{\rm th}^{\rm cal}$ is the polar angle of the event thrust axis determined from calorimeter clusters. Further, more than 14 clusters are required in order to reduce background from $e^+e^- \longrightarrow \tau^+\tau^-$.

The second step of the procedure is the selection of charged tracks from events which have passed the pre-selection. It is based on the track momenta measured in the Central Tracking Detector including the Silicon Micro-vertex Detector. In order to select well-measured tracks,

a selection is performed using tracks which have passed certain quality cuts. Hadronic events are then selected using the track momenta $\vec{p}$ of well measured tracks by requiring

$$0.15 < \frac{\sum |\vec{p}|}{\sqrt{s}} \qquad \text{and} \qquad \frac{|\sum \vec{p}_{\perp}|}{\sum |\vec{p}|} < 0.8$$

and a thrust axis direction within $|\cos(\theta_{\rm th}^{\rm TEC})| \leq 0.7$, where $\theta_{\rm th}^{\rm TEC}$ is the polar angle of the event thrust axis determined from charged tracks.

The advantage of the pre-selection is that it largely decouples the event selection from the track selection, allowing relatively weak cuts on track momenta. This reduces systematic errors in the charged particle multiplicity distribution on which our analysis is based. About 900000 events survive the selection.

The normalized factorial moments calculated from the raw data were corrected for detector acceptance and efficiency and for the selection procedure by the following method. Events are generated by the Monte Carlo programs JETSET 7.4 PS [15], HERWIG 5.9 [16] and ARIADNE 4.08 [17], which have been tuned using L3 data [18]. For each sample the factorial moments of the multiplicity distribution are measured, $F_q^{\rm gen}$. The events are passed through the simulation of the L3 detector [19] and reconstructed using the same program as for the data. After the above-described event and track selection procedure a multiplicity distribution is obtained and factorial moments, $F_q^{\rm sim}$, are calculated. The factorial moment calculated from the raw data, $F_q^{\rm raw}$, is then corrected for detector effects by $F_q = F_q^{\rm raw} \times C_q$, where $C_q = \frac{F_q^{\rm gen}}{F_q^{\rm sim}}$. From these F_q, cumulant factorial moments K_q and the ratio $H_q = \frac{K_q}{F_q}$ are calculated.

Statistical and systematic errors

To calculate the statistical errors of the H_q, 10000 multiplicity distributions are generated from the experimental particle multiplicity distribution according to a Poissonian fluctuation of each P_n. This is also done for the generated and simulated multiplicity distributions. The statistical errors on the H_q are taken as the standard deviation of the 10000 values of H_q obtained from the 10000 $F_q = F_q^{\rm raw} \times C_q$. Statistical and systematic errors were added in quadrature. The standard deviation of the values of the H_q corrected using the different Monte Carlo model is taken as a systematic error.

Charged particle multiplicity analysis

The H_q from the data corrected by the three different Monte Carlo samples do not agree perfectly and have large statistical errors. The largest multiplicities are affected by large statistical fluctuations due to the limited statistics of a sample. This strongly influences the higher-order H_q and causes the statistical errors of the H_q to be large. Since this effect appears not only in the raw data, but also in the Monte Carlo samples used to correct the data, it could explain why the H_q of the data corrected by these three Monte Carlo samples are different. To ameliorate this problem the data and the simulated multiplicity distributions are truncated at 45 (events with $N_{\rm ch} \geq 45$ are rejected) and the generator level Monte Carlo distribution at 50. The value of 45 corresponds to the first high-multiplicity bin with fewer than 35 events. The use of the truncated distributions results in good agreement among the three corrected H_q, as seen in fig. 4, as well as a reduction in statistical errors.

The corrected H_q, shown in fig. 5, have a negative first minimum at $q = 6$ and quasi-oscillations around 0 for greater q. These result are in qualitative agreement with those of the SLD collaboration. Furthermore, amplitudes and $|H_6|$ are found to increase if the multiplicity distribution is further truncated (not shown). The H_q calculated from JETSET and ARIADNE agree quite well with the data, when they are truncated in the same way as the data. The H_q calculated from HERWIG show oscillations with maxima and minima at the same values of q as for the data. However the amplitudes of the HERWIG oscillations are much larger.

Jet multiplicity analysis

Since the behavior of the H_q agrees qualitatively with that predicted in NNLLA, it is natural to interpret this agreement as a confirmation of the NNLLA prediction, and SLD made this interpretation. However, JETSET and ARIADNE agree with the data and HERWIG shows qualitatively similar behavior. But the parton-shower modelling of these programs are not at the NNLLA order of pQCD. Hence the H_q from JETSET and HERWIG should not be expected to oscillate. That they do, together with doubt about the LPHD assumption, suggests that we view the NNLLA interpretation with scepticism.

The jet analysis assumes that the jet multiplicity distribution is related to the parton multiplicity distribution at the energy scale corresponding to the jet resolution. By choosing a scale where perturbative QCD should be applicable ($\gtrsim 1$ GeV), we should be able to test the pQCD predictions for the behavior of the H_q. We define jets using both the Durham [20] and the new Cambridge [21] jet algorithms. These algorithms use $y_{ij} = \frac{2\min(E_i^2, E_j^2)}{E_{\rm vis}^2}(1 - \cos\theta_{ij})$ as the distance between particles i and j and combine them by calculating the sum of their momenta (E-scheme).

For the Durham algorithm, the pair of objects (particles or clusters) having the smallest y_{ij} ($y_{ij}^{\min}$) is checked. If $y_{ij}^{\min} < y_{\rm cut}$, the pair of objects is combined and the y_{ij} are recomputed with the new object, to find a new $y_{ij}^{\min}$. This process is repeated until no pair of objects remains with $y_{ij} < y_{\rm cut}$.

For the Cambridge algorithm, the pair of objects with $y_{ij}^{\min}$ is also checked. If $y_{ij}^{\min} < y_{\rm cut}$, the same procedure used by the Durham algorithm is repeated. If $y_{ij}^{\min} \geq y_{\rm cut}$, the object i or j with the smaller energy is stored as a jet and removed from the list of objects to be treated. This is called *soft freezing*, since the object with the softer energy is removed. The object with the higher energy is kept in the list for the check of the next $y_{ij}^{\min}$.

Both algorithms have the property of minimizing hadronization effects, since they can associate particles with large angles and small transverse energies which is a specific effect of hadronization. But the Cambridge algorithm, with its *soft freezing* allows a better treatment of the softer jets by removing them from calculation before any extra particles are associated with them, which leads to a better energy identification of gluon jets.

It has been shown [10–13] in previous analyses of hadronic Z decays (see for review [22]) that mean jet multiplicities agree well with the NLLA prediction in the perturbative region, i.e., for values of $y_{\rm cut}$ greater than 10^{-3} which corresponds to transverse energies greater than ~ 1 GeV.

Since this study starts with the charged track multiplicity distribution, we have also constructed jets using only charged tracks. The charged particle and jet multiplicities are, of course, identical for sufficiently small values of $y_{\rm cut}$ ($y_{\rm cu54485448t} \lesssim 10^{-8}$).

Using both charged and neutral particles instead of only the charged particles gives similar

results. We have checked that, for $y_{\rm cut} = 0.01$ the fractions of 2-jet and 3-jet events are comparable, 63% and 32% for charged and neutral particles and 61% and 32.5% for charged tracks only.

The H_q measured for jet multiplicity distributions obtained from both the Durham and Cambridge jet algorithms have been corrected using the same procedure as for the charged particle multiplicity distribution, again truncating the distribution at the highest multiplicities. The resulting H_q are shown in figures 6, 8, 7 and 9, for four different values of $y_{\rm cut}$ representing both non-perturbative and pertubative regions.

For each value of $y_{\rm cut}$, the H_q of the jet multiplicity distributions obtained from the two jet algorithms show the same qualitative behavior, and almost the same values of the H_q. The H_q calculated from JETSET and ARIADNE agree well with the data for each $y_{\rm cut}$, while the H_q calculated from HERWIG disagree in magnitude with the data for low values of $y_{\rm cut}$, where the jet multiplicity distributions are closer to the charged particle multiplicity distribution.

A first minimum at $q = 4$ followed by quasi-oscillation about 0, is seen for $y_{\rm cut} = 6 \times 10^{-6}$ (energy scale ≈ 200 MeV) (figure 6a and 7a). This behavior is qualitatively similar to that for the charged particle multiplicity distribution, as well as for jet multiplicity distributions with smaller $y_{\rm cut}$, but both the position of the first minimum and of subsequent maxima and minima are shifted to lower values of q.

As $y_{\rm cut}$ is increased, the behavior of the H_q changes further. The negative first minimum near 5 disappears. For $q \leqslant 5$ each H_q alternates between positive and negative values. For greater q, the oscillations begin to disappear. For $y_{\rm cut} = 1.5 \times 10^{-5}$ (energy scale ≈ 350 MeV) these oscillations have disappeared, as seen in figure 6b and 7b.

For $y_{\rm cut} > 1.5 \times 10^{-5}$ and in particular in the perturbative region $y_{\rm cut} \geqslant 10^{-4}$ (corresponding to an energy scale $\gtrsim 1$ GeV), the oscillations and the negative first minimum near 5 have completely disappeared as seen in figures 8 and 9 for $y_{\rm cut} \approx 10^{-4}$ (energy scale ≈ 1 GeV) and $y_{\rm cut} \approx 10^{-3}$ (energy scale ≈ 2.8 GeV). Instead, H_q alternates between positive and negative values for each q. Furthermore $|H_q|$ increases sharply with q.

None of the predictions made at different orders of pQCD for the H_q is observed. Even though pQCD describes the average jet multiplicity well, it seems to be unable to describe the shape of these distributions. Therefore, the minimum and oscillatory behavior of the H_q seen for the charged particle multiplicity distribution and for very low values of $y_{\rm cut}$ appears unrelated to the behavior of the H_q calculated in NNLLA.

Monte Carlo study of the oscillatory behavior of the H_q

Given the success of JETSET, ARIADNE and HERWIG and the failure of the NNLLA prediction seen in the previous section, we attempted to find that aspect of the Monte Carlo generators which is responsible for the agreement.

We have varied several options in JETSET and studied their influence on the behavior of the H_q. First we used different models of parton generation using in all cases the Lund string fragmentation model. We tried the following:

- no angular ordering in the parton shower, which makes it essentially an LLA shower with the addition of energy-momentum conservation at each branching;
- $\mathcal{O}(\alpha_{\rm s})$ and $\mathcal{O}(\alpha_{\rm s}^2)$ matrix elements instead of a parton shower;
- matrix element production of $\rm q\bar{q}$ only.

Next we considered the possibility that this behavior could come from the fragmentation model, which could simulate some hidden higher-order aspects of pQCD. We used JETSET 7.4 with the various choices of parton generation used previously but with independent fragmentation for the hadronization process. We also used HERWIG 5.9 which uses cluster fragmentation.

In all cases the H_q have a negative first minimum near $q = 5$ and quasi-oscillations for greater q. Some examples are shown in figure 10

This investigation done with Monte Carlo generators shows that we can reproduce the oscillatory behavior of the H_q without the need of the NNLLA of pQCD.

Conclusions

The ratio of cumulant factorial to factorial moments has been measured for both the charged particle and jet multiplicity distributions obtained with the Durham and Cambridge algorithms. A perturbative QCD calculation at the next to next to leading logarithm order predicts H_q to have a negative first minimum at $q \approx 5$ and quasi-oscillation around 0 for higher q. This behavior is observed for the charged particle multiplicity distribution, confirming previous results. However, our analysis of the H_q of the jet multiplicity distributions reveals that this behavior appears only for very small $y_{\rm cut}$, corresponding to energy scales $\lesssim 100$ MeV, which is far from the perturbative region. Furthermore an investigation performed on different models of parton generation, and for different fragmentation models has shown in each case oscillatory behavior of H_q.

From this we conclude, contrary to the claim of ref. [4], that the oscillations of H_q observed in the charged particle multiplicity distribution are unrelated to the behavior predicted by the NNLLA perturbative QCD calculations.

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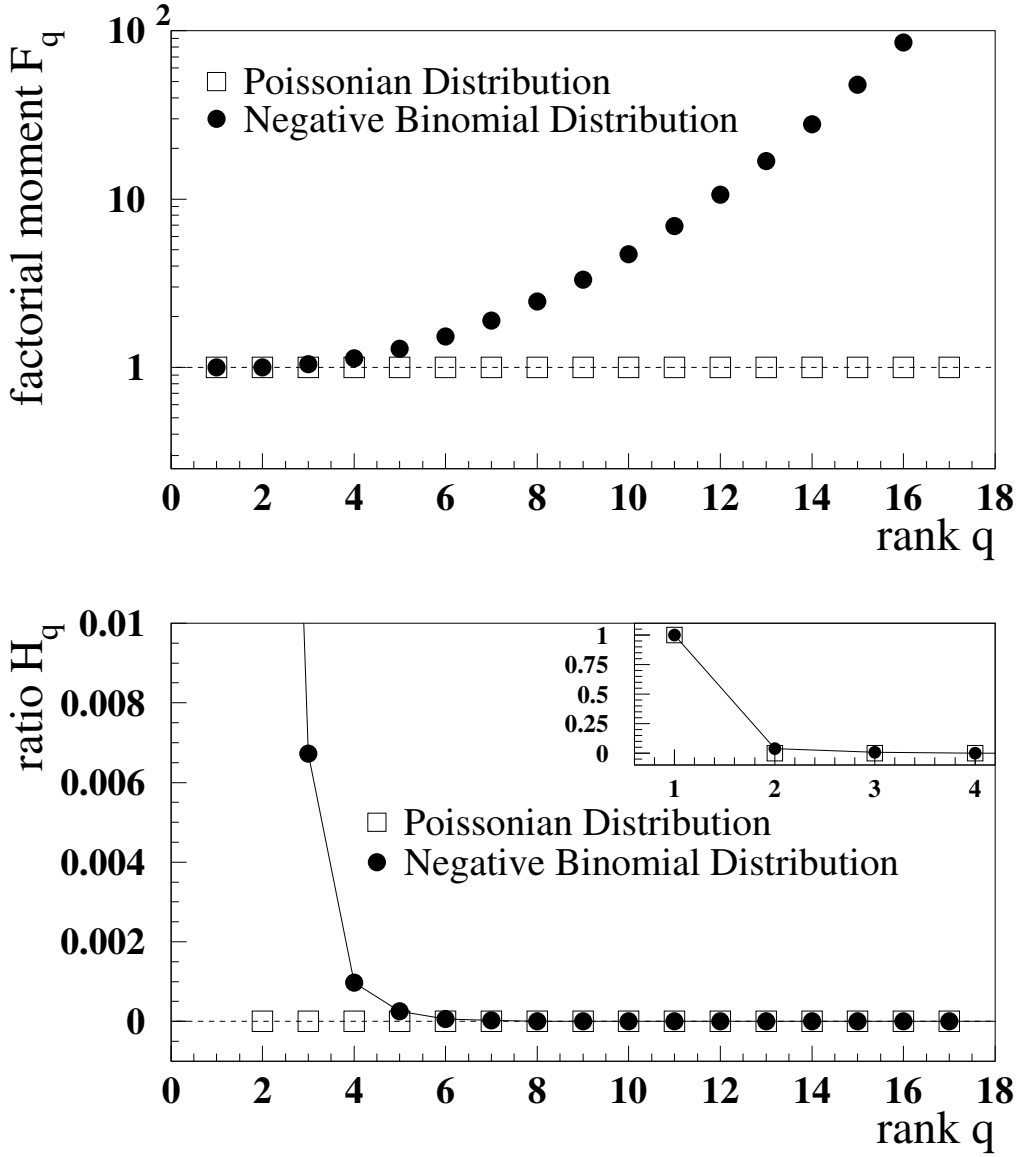
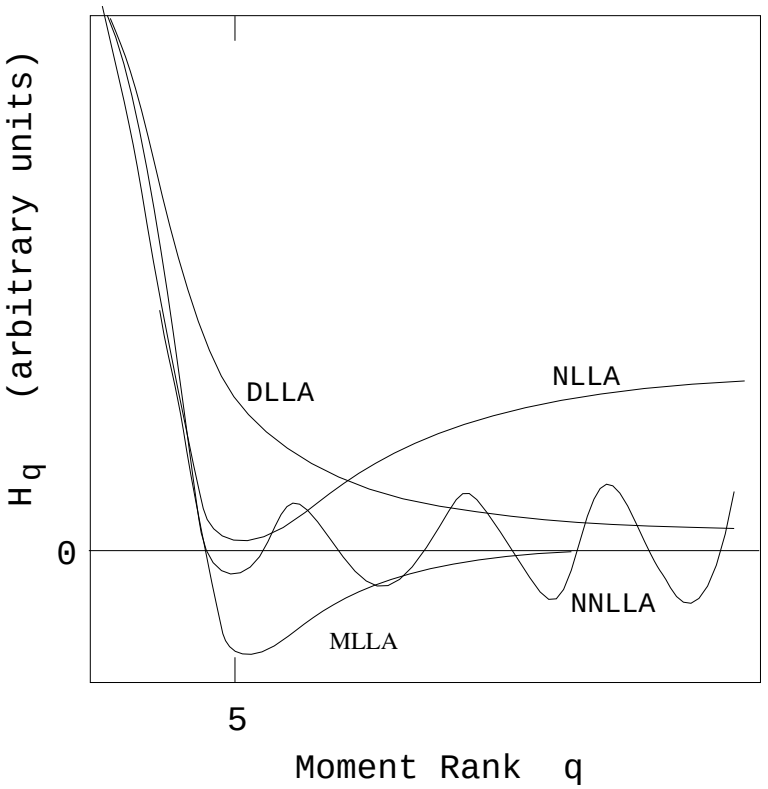


Figure 1: Factorial moments F_q and ratio of cumulant to factorial moment H_q as a function of their moment rank for a Poisson distribution and for a Negative Binomial Distribution.



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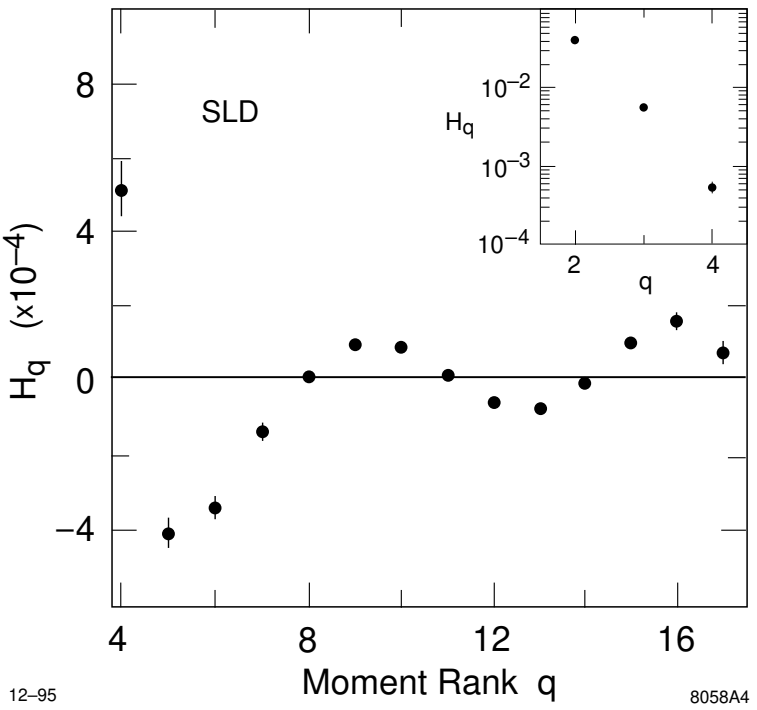

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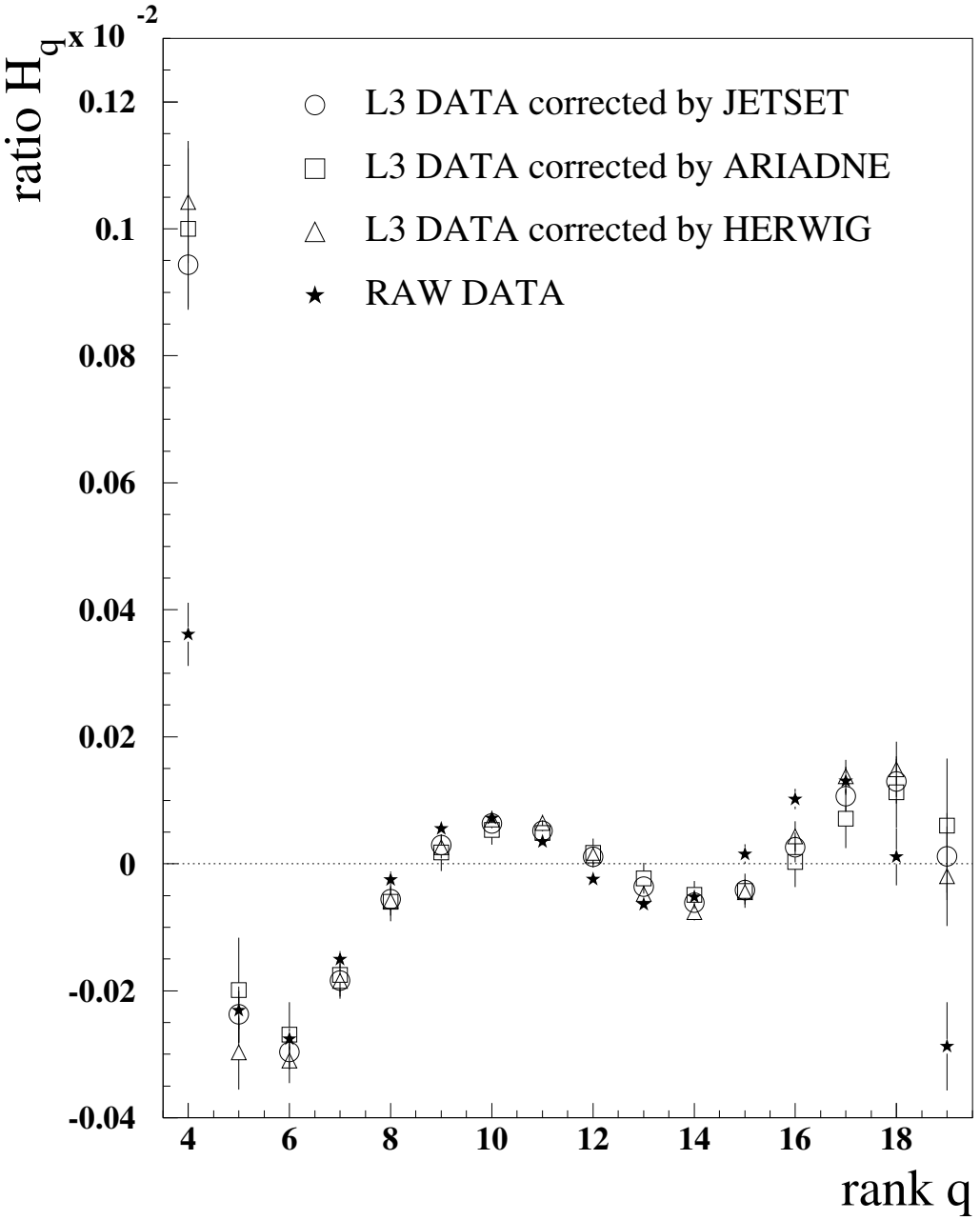

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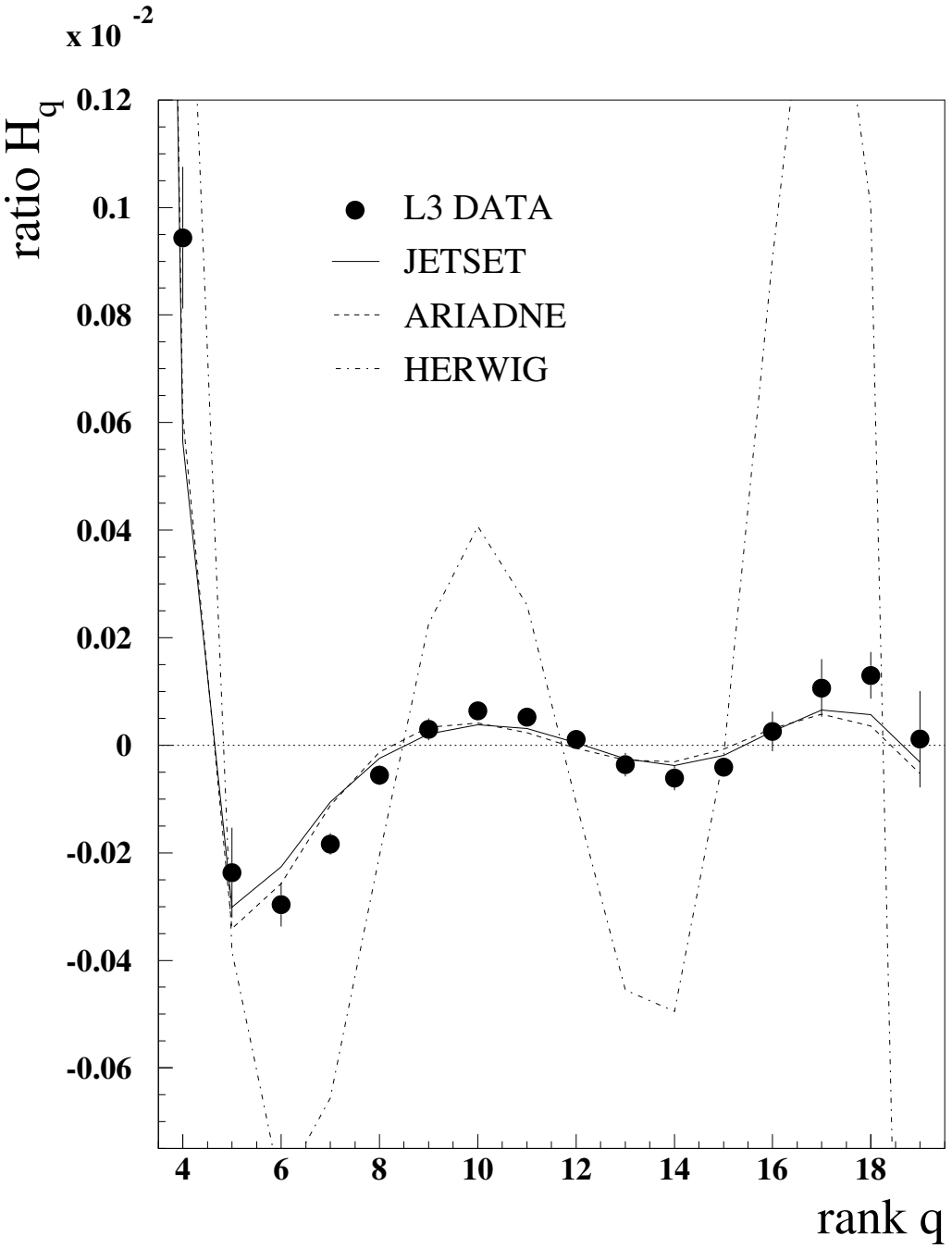

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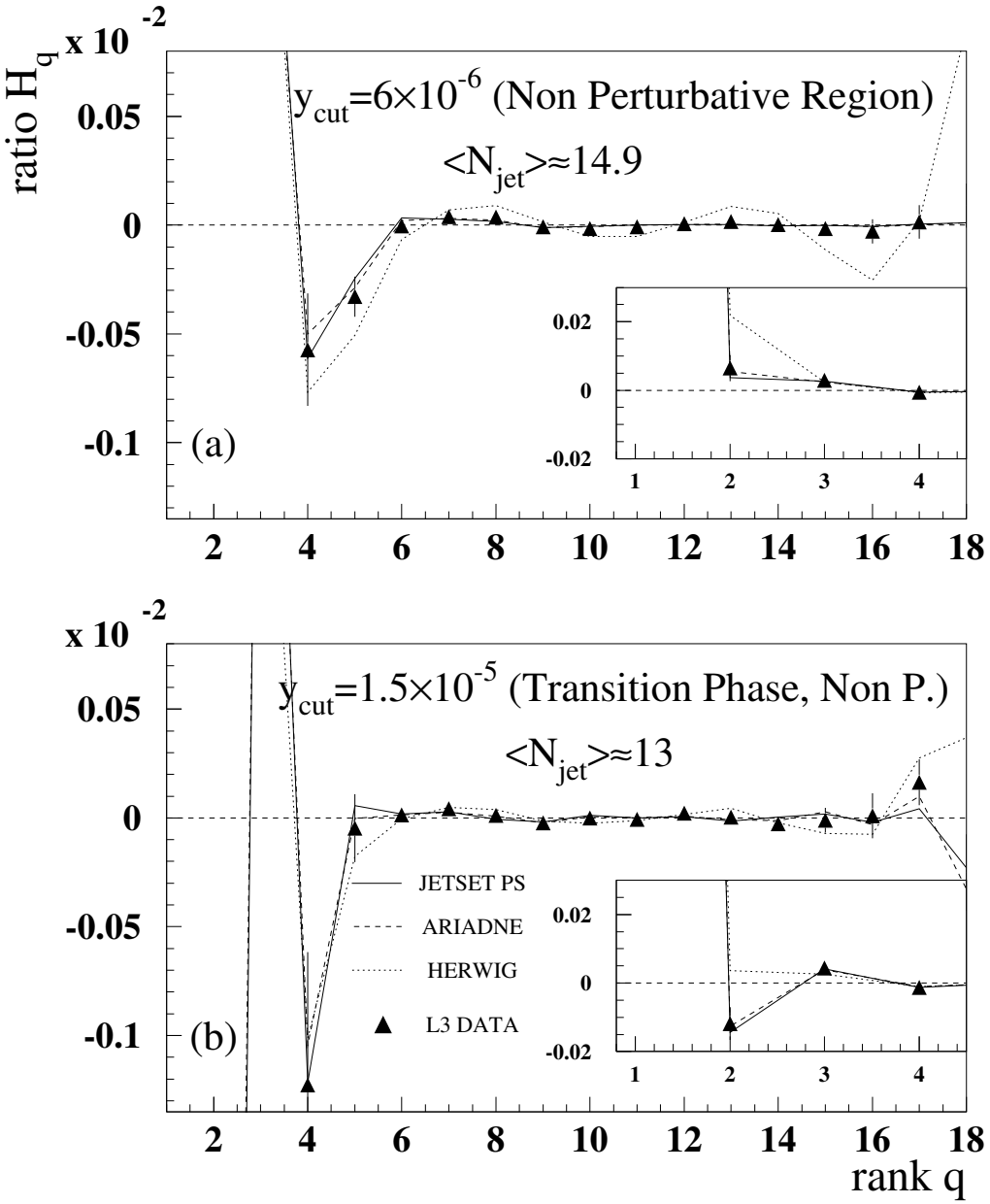

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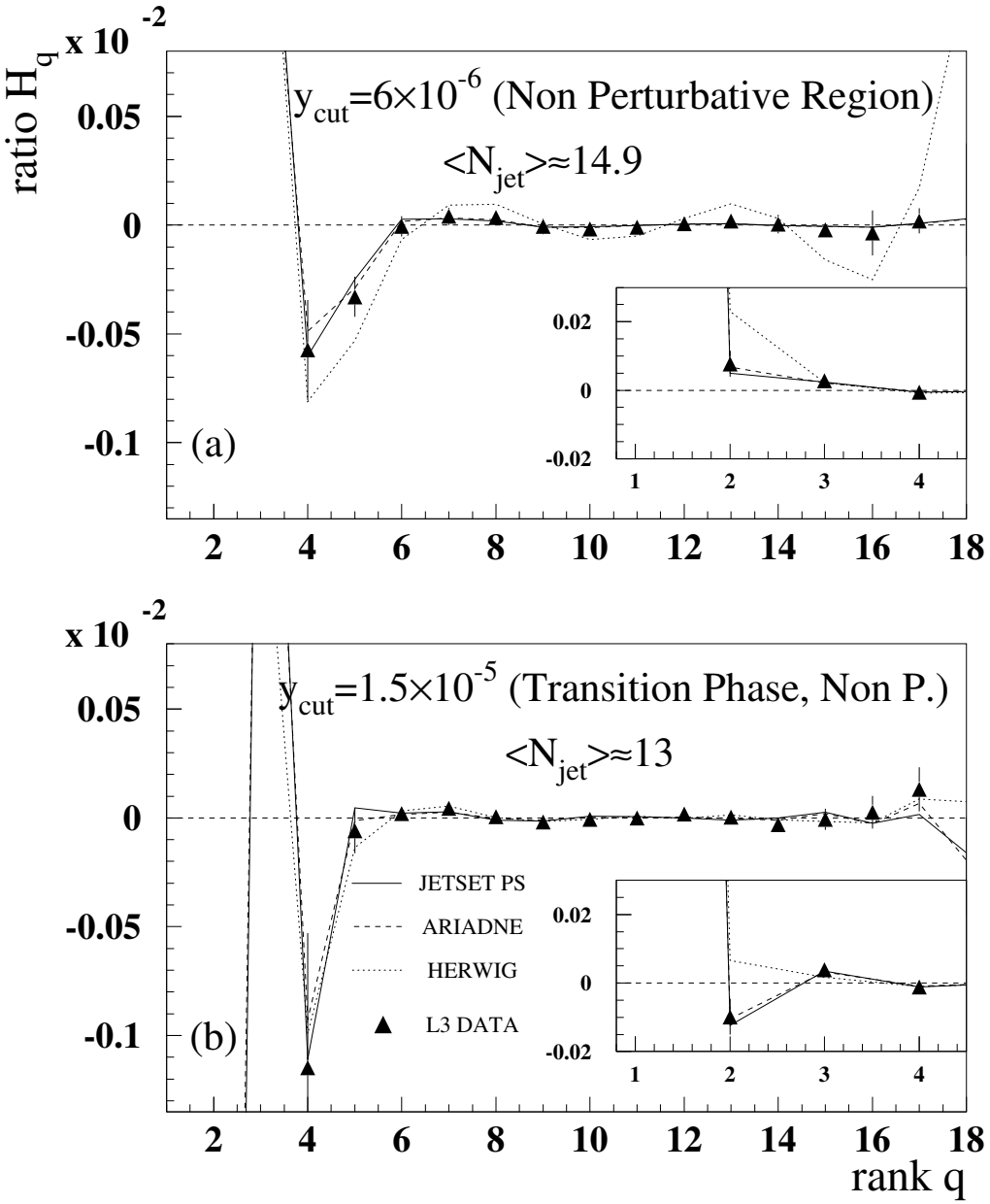
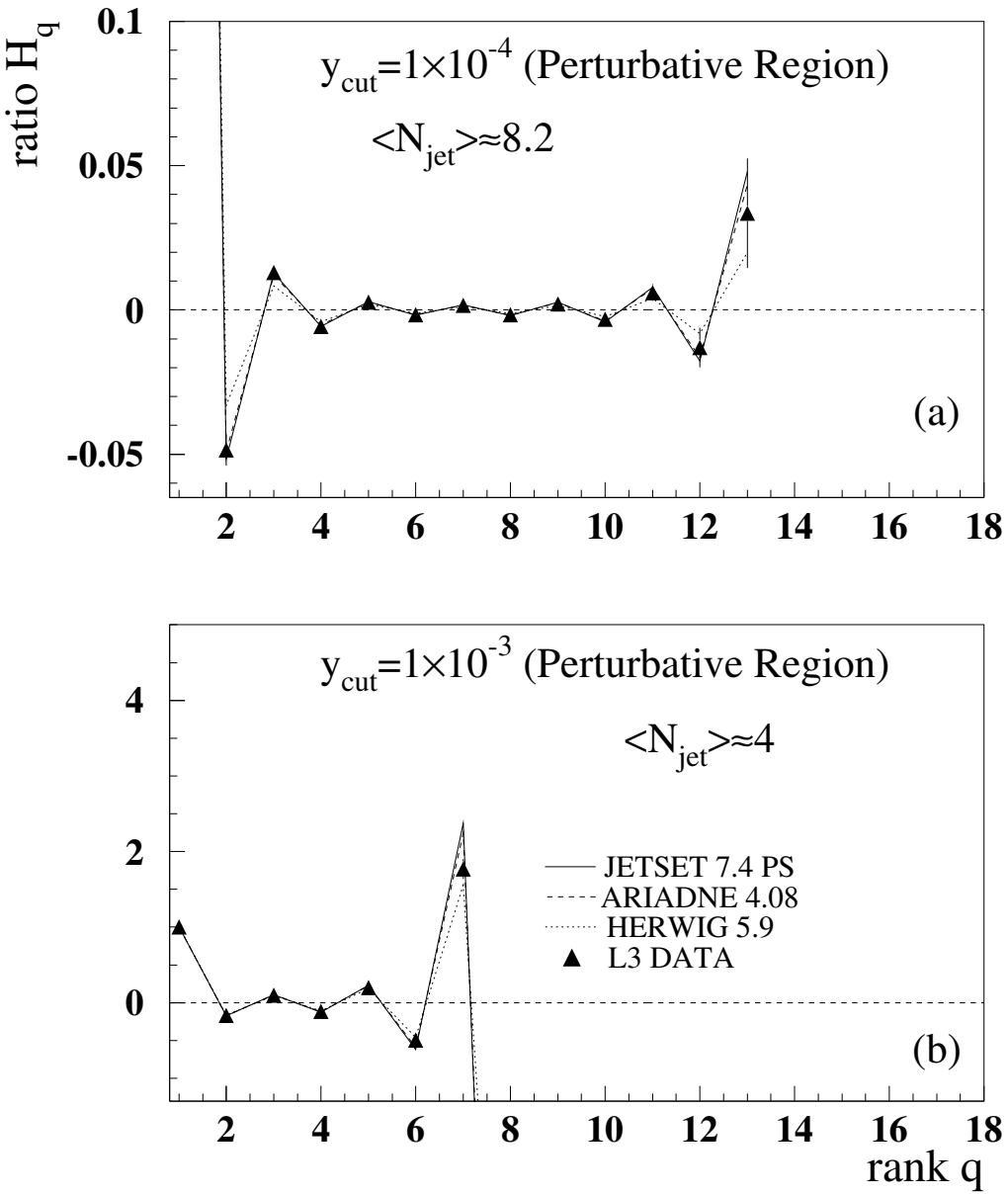


Figure 7: H_q of the jet multiplicity distributions obtained with the Cambridge jet algorithm at non-perturbative energy scales, (a) for $y_{\rm cut} = 6 \times 10^{-6}$, (b) for $y_{\rm cut} = 1.5 \times 10^{-5}$.



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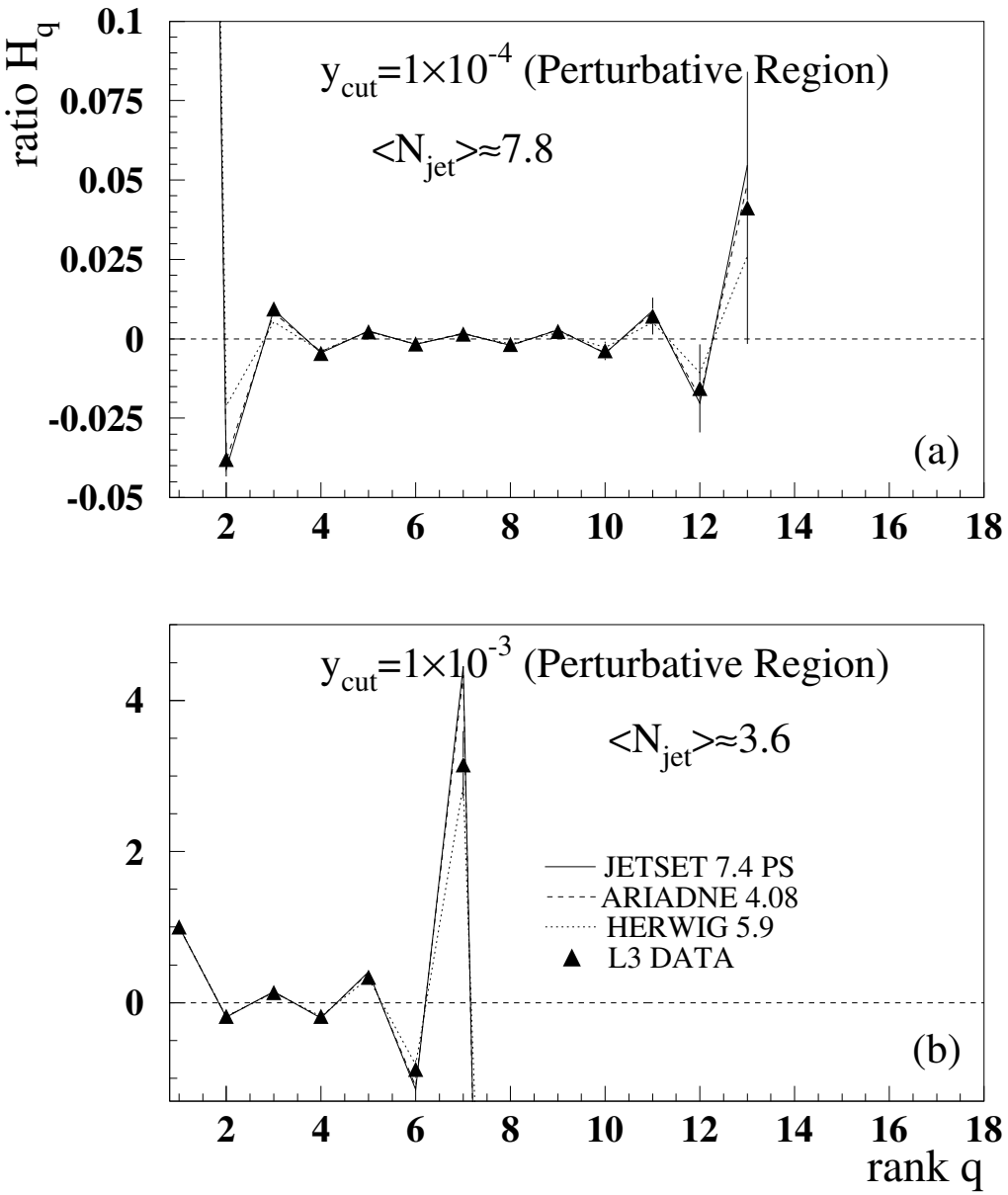

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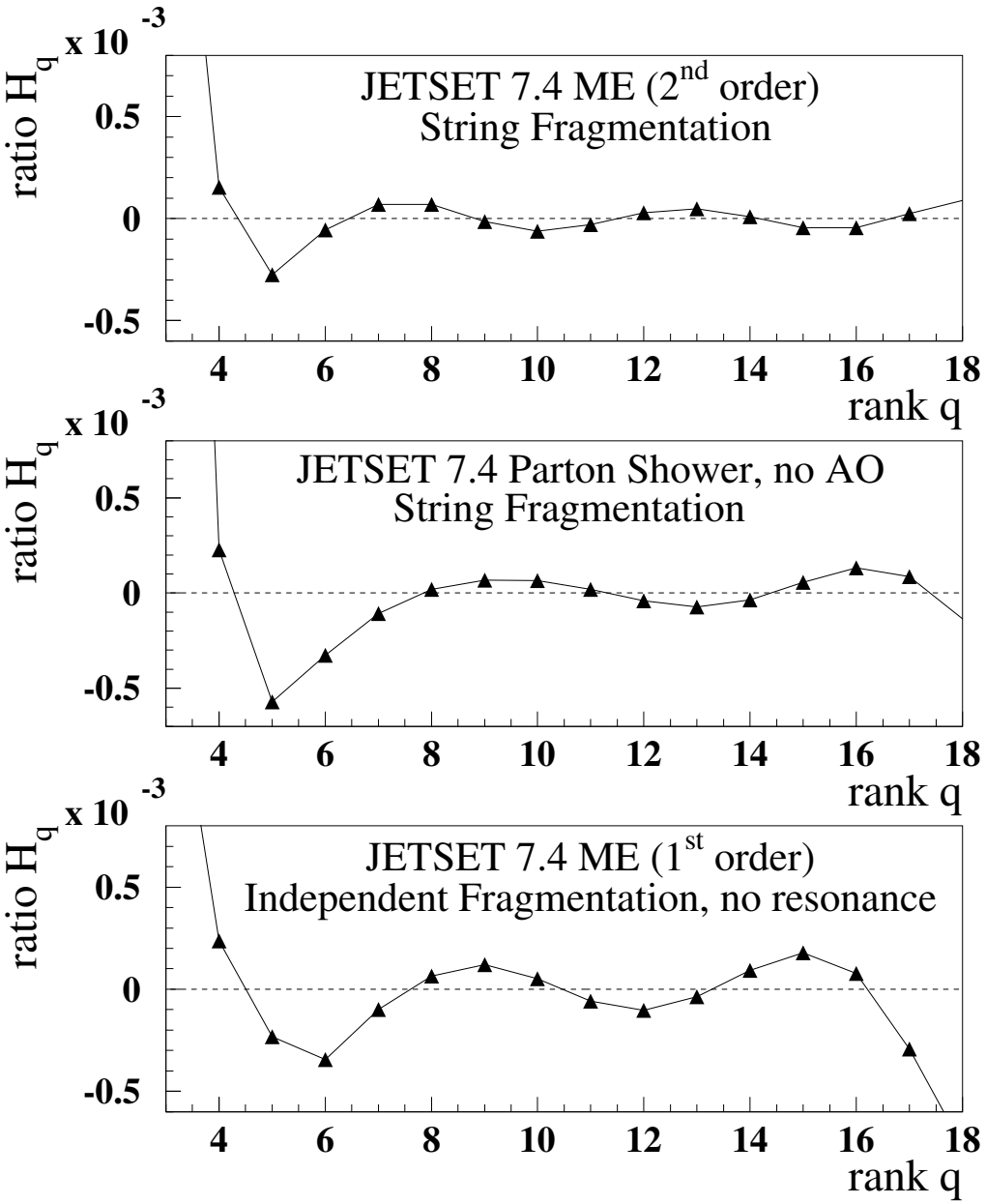

Figure 10: H_q for the charged particle multiplicity distribution obtained from Monte Carlo generators using different methods of parton generation and different models of fragmentation.