OSCILLATING H_Q, INTERMITTENCY INDICES, AND QCD

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Preliminary measurements by the L3 collaboration of the ratio of the cumulant factorial moments of the charged-particle multiplicity distribution to the factorial moments, H_q, and of intermittency indices in hadronic Z decays are compared to predictions of perturbative QCD under the assumption of LPHD and to JETSET.

1. H_q

The normalised factorial moments, F_q, and their cumulants, K_q, of order q are defined in terms of the multiplicity distribution, P_n, by

$$F_q = \frac{\sum_{n=q}^{\infty} n(n-1)....(n-q+1)P_n}{\left(\sum_{n=1}^{\infty} nP_n\right)^q} \tag{1}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m} F_m \tag{2}$$

Their ratio, $H_q = K_q/F_q$, has been calculated[1] for the soft gluon multiplicity distribution in different approximations of perturbative QCD (pQCD). The expected behavior of H_q depends on the approximation as shown in Fig. 1. Assuming Local Parton Hadron Duality (LPHD), similar behavior is expected for the charged particle multiplicity distribution (CPMD).

Previous measurements of the H_q in e^+e^- collisions[2,3] at 91.2 GeV and also in hadron collisions[4] have shown, qualitatively, the behavior predicted in NNLLA. This analysis uses L3 data collected in 1994 and 1995 at the Z-pole, which results in about 1 million selected hadronic events.[5]

From the definition (1) it is clear that the F_q and H_q are very sensitive to the high-multiplicity tail, just where statistics are low and detector corrections are large. The statistical uncertainties on the measured H_q can be reduced by truncating the CPMD. However, one then measures the H_q not of the CPMD, but of the truncated CPMD, which are biased estimators of the H_q of the untruncated CPMD. One sometimes speaks of natural truncation, meaning that the tail is 'naturally' truncated by the statistics of the experiment.

The effect of truncation on the L3 data is shown in Fig. 2 and compared to the SLD results.[2] It is observed that stronger truncation deepens the minimum at $q = 5$ and increases the amplitudes of the oscillations while shifting the location of the extrema slightly. Indeed, truncation of the CPMD near the maximum multiplicity, 54, reported by SLD, which was based on about 87K events compared to the 1M events of L3, results in agreement between the H_q measured by the two experiments.

It is clear that the H_q measured for a truncated CPMD are biased estimators of the H_q of the untruncated CPMD. The question is how big is the bias both for the value of a particular H_q and for the position of the extrema of the oscillations. We can even ask whether the extrema do not disappear completely in the limit of infinite statistics. Unfortunately these questions can be answered definitely only if we know the functional form of the CPMD, which we do not. We have investigated two possibilities: the negative binomial distribution (NBD) having mean and dispersion equal to that measured, and PYTHIA. Of the two, PYTHIA agrees better with the measured CPMD. The H_q of an NBD can be shown analytically not to have a minimum or oscillations. Nevertheless, when truncated such a minimum

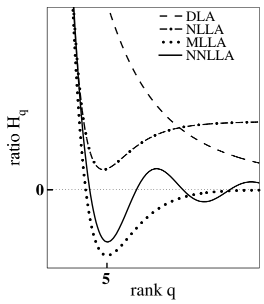

Figure 1. Predicted behavior of the H_q in various approximations of pQCD.

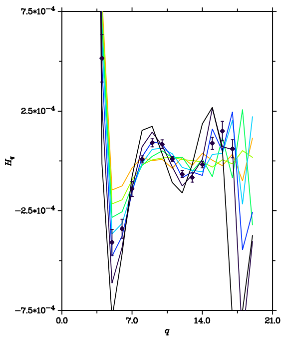

Figure 2. Effect of truncation on the H_q measured by L3. The CPMD is truncated at 67 (no truncation), 57, 55, 53, 51, 49, 47. The stronger the truncation, the deeper the minimum at $q = 5$. The points are from SLD.[2]

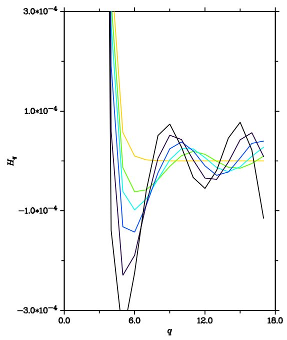

Figure 3. Effects of truncation on the H_q of an NBD with mean and dispersion as measured by L3. Truncations at 200, 55, 53, 51, 49, 47 are shown. The stronger the truncation, the deeper the minimum at $q = 5$.

and oscillations develop, which are rather similar to those of the data (see Fig. 3). We have studied this effect in PYTHIA using 25 samples of 100K events, 25 samples of 1M events, and 25 samples of 10M events. The effect of truncation is similar to that observed for the data in Fig. 2. For the untruncated CPMD, the first minimum is at $q = 5$ independent of the number of events. Also the value of H_5 depends only weakly on the statistics. The period of the oscillations increases and their amplitudes decrease as the number of events increases, the effect becoming larger for larger q. From these observations we arrive at the conclusion that for comparison with a model a truncated CPMD is best, since then the statistical uncertainties are smaller, but that to establish the existence of the oscillatory behavior the untruncated CPMD should be used. However, with 1M events the bias is sufficiently small only up to about $q = 8$.

The H_q from the untruncated CPMD are shown in Fig. 4. While a minimum at $q \approx 5$ appears to be present, nothing can be concluded about the presence of further oscillations. Thus the data cannot distinguish between the behaviors expected from MLLA and NNLLA. Note that JETSET agrees with these H_q. Good agreement is also observed for the truncated CPMD when JETSET is truncated at the same value as the data.

2. Intermittency indices, ϕ_q

To measure the ϕ_q, we (a) partition three-dimensional phase space into M bins, (b) measure $F_q(M)$, averaging over the M bins, and (c) fit

$$F_q(M) = b_q M^{\phi_q} \quad . \tag{3}$$

The coordinate system must be chosen such that it does *not* depend on QCD dynamics. Otherwise the observed dynamical fluctuations will be reduced. As z-axis we use the thrust axis, which approximates the $\mathrm{q\overline{q}}$ direction, which is determined by the electro-weak process. Per event, the x-axis is chosen at random. Using, *e.g.*, the major axis, which depends on the QCD dynamics results in lower values of F_q and ϕ_q. We use the variables rapidity (y), transverse momentum (p_t) and azimuthal angle (φ). The dynamical fluctuations are found to be isotropic, which corresponds to a self-similar fractal, by observing that (3) describes the data when $M = M_y M_{p_\mathrm{t}} M_\varphi$ with $M_y = M_{p_\mathrm{t}} = M_\varphi$ and that fits of the one-dimensional F_2 by $F_2(M) = A - BM^{-\gamma}$ yield values of γ which are equal for all three variables.[6] Consequently, the ϕ_q are determined by fitting (3) with $M_y = M_{p_\mathrm{t}} = M_\varphi$.

Perturbative QCD predicts[7] that* $\Phi_q = D - \frac{3\phi_q + D}{q}$, where D (=3) is the number of dimensions in which the analysis is done, increases with q reaching a maximum at the same value of q for which the H_q have their first minimum, *i.e.*, at $q \approx 5$. The results for ϕ_q and Φ_q are shown in Fig. 5. It is observed that Φ_q has a maximum, but at a lower value of q. We also find that the ϕ_q from JETSET agree well with the data.

3. Conclusions

Although the minimum of H_q at $q \approx 5$ predicted by MLLA and NNLLA appears to be present in the data, the oscillatory behavior of NNLLA can not be confirmed because of uncertainty on the statistical bias of the H_q. The maximum in Φ_q occurs at $q \approx 2$ rather than the predicted 5. Both H_q and ϕ_q are reasonably well-described by JETSET.

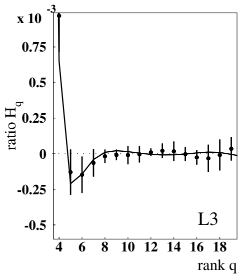

Figure 4. Preliminary untruncated L3 data (points) compared to JETSET.

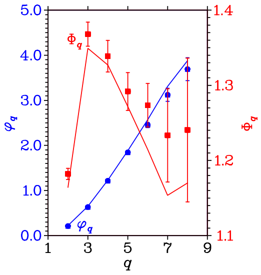

Figure 5. Preliminary L3 results for ϕ_q and Φ_q (points) compared to JETSET.

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*Our expression contains $3\phi_q$ rather than ϕ_q because our M in (3) is the number of bins in 3 dimensions rather than the number of bins in each of the variables.