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MULTIPLICITY FLUCTUATIONS AND MULTIPARTICLE CORRELATIONS*

W. J. METZGER

University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands
E-mail: W.Metzger@cern.ch

A short review of contributions 149, 297, 541, 542, 543 and 802 on multiplicity fluctuations and multiparticle correlations is given.

1 Introduction

The contributions discussed[1–6] in this mini-review are concerned with fluctuations in the number of particles and correlations between particles. Data are compared to the predictions of Monte Carlo (MC) models and/or to analytic perturbative QCD calculations. In the latter case, the predictions are actually for partons, and to be able to make a comparison one assumes the validity of Local Parton-Hadron Duality (LPHD), *i.e.*, that an observable for particles is proportional to the same observable for partons.

2 Local Multiplicity Fluctuations

Fluctuations are often studied using the normalised factorial moments (NFMs), F_q, and cumulants, K_q, of the local multiplicity distribution, $P_n(\delta)$, which is the probability to find n particles in a phase space bin of size δ:

$$F_q = \frac{\sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n}{\left(\sum_{n=1}^{\infty} n\, P_n\right)^q} \tag{1}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \binom{q-1}{m} K_{q-m}\, F_m \tag{2}$$

The F_q measure q-particle fluctuations. The K_q, which are obtained from the F_q by subtracting the contributions of fewer than q particles, measure the genuine q-particle fluctuations. Sometimes the ratio, $H_q = K_q/F_q$, is also used. Other normalizations of the F_q and various corrections will not concern us here.

From the definition (1) it is clear that the F_q suffer from a statistical bias. The value of F_q depends on the high multiplicity tail of the multiplicity distribution, the measurement of which is limited by the statistics of the experiment, resulting in an underestimate of the value. The size of this bias is greater for experiments having smaller statistics. Furthermore, the experiment must be capable of resolving the maximum number of particles.

An alternative to NFMs which overcomes these problems is the bunching parameters (BPs):[7]

$$\eta_q = \frac{q}{q-1}\,\frac{P_q P_{q-2}}{P_{q-1}^2} \tag{3}$$

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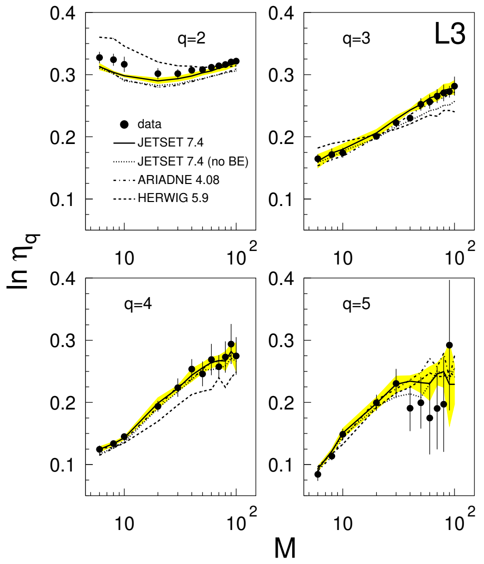

Figure 1: Bunching parameters as a function of the number of bins, M, which is inversely proportional to the bin size, δ.

All NFMs and BPs are equal to one if P_n is a Poisson distribution. In the case of dynamical fluctuations giving rise to scale-invariant correlations (intermittency) the NFMs have a power-law dependence on the bin size: $F_q(\delta) \propto \delta^{-\phi_q}$, where the ϕ_q are related to the anomalous fractal dimensions. The η_q for $q > 2$ are independent of δ in the case of monofractal fluctuations but not for multifractal fluctuations.

L3 has studied[4] both NFMs and BPs in bins of rapidity with respect to the thrust axis (Fig. 1). The η_q for $q > 2$ establish unequivocally that the fluctuations are multifractal, as is expected from QCD. There is good agreement with the MC models with the exception of η_2 where HERWIG overestimates the data for large bins (small M), which indicates that HERWIG's multiplicity distributions for such bins are too broad.

OPAL has studied[2] the NFMs and the cumulants in bins of rapidity, transverse momentum, and azimuthal angle with respect to the sphericity axis using one-, two- and three-dimensional bins. Figs. 2 and 3 show examples of their results.

Monte Carlo models show generally good agreement with the NFMs and cumulants of both L3 and OPAL,

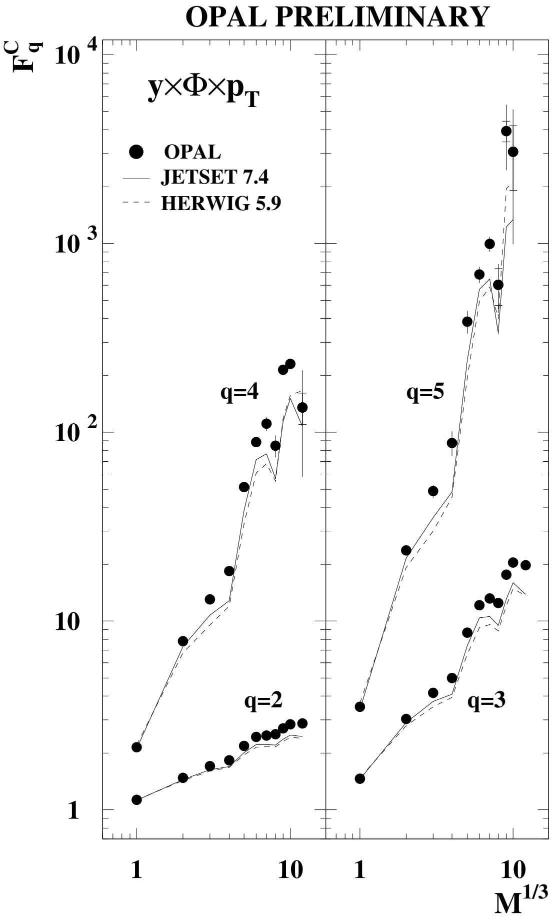

Figure 2: NFMs for 3-dimensional bins.

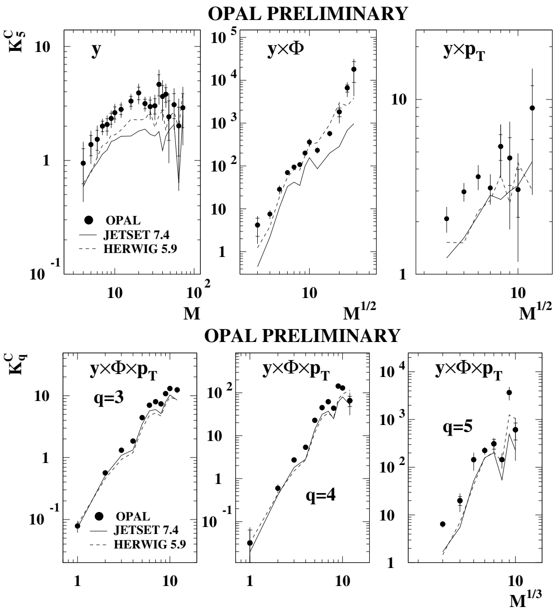

Figure 3: Cumulants for 1-, 2- and 3-dimensional bins.

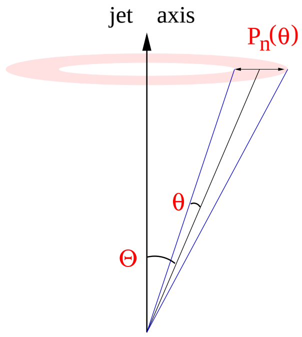

Figure 4: Definition of angular bins.

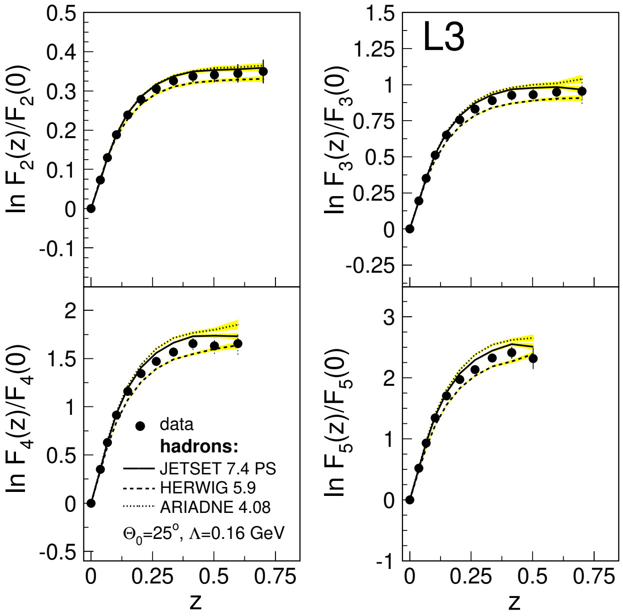

Figure 5: NFMs for angular bins compared to MC models.

although in some cases the MC slightly underestimates the data, particularly for small bin sizes (large M).

L3[5] and ZEUS[6] have studied fluctuations in angular bins (rings) of half-width θ about angle Θ with respect to the jet axis[†], as illustrated in Fig. 4. Rather than θ, both experiments use[‡]

$$z = \frac{\ln(\Theta/\theta)}{\ln(E_{\rm jet}\Theta/\Lambda)} \tag{4}$$

L3 finds good agreement between MC and data for the $F_q(z)/F_q(0)$ (Fig. 5). (The normalization to $F_q(0)$ is done to reduce hadronization effects.) However, various perturbative QCD calculations at DLLA disagree (Fig. 6) with this data. ZEUS applies a QCD-inspired normalization to the factorial moments, which has the effect of lessening their energy dependence. They then compare (Fig. 7) to asymptotic perturbative QCD predictions, also finding disagreement. ZEUS too finds good agreement with MC (ARIADNE) predictions.

[†]The ZEUS analysis is performed in the Breit frame.
[‡]ZEUS calls this variable ϵ.

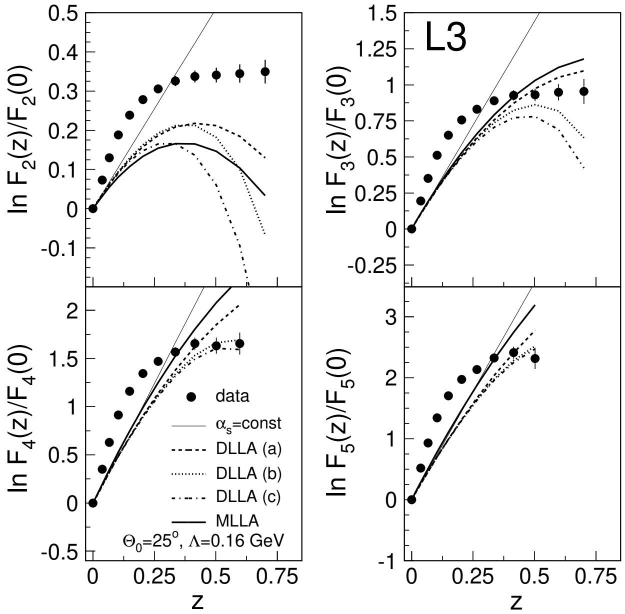

Figure 6: NFMs for angular bins compared to QCD predictions.

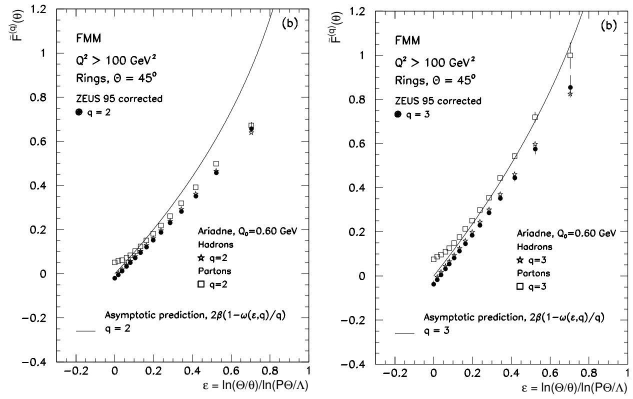

Figure 7: Modified NFMs for angular bins compared to an asymptotic QCD prediction and to ARIADNE MC.

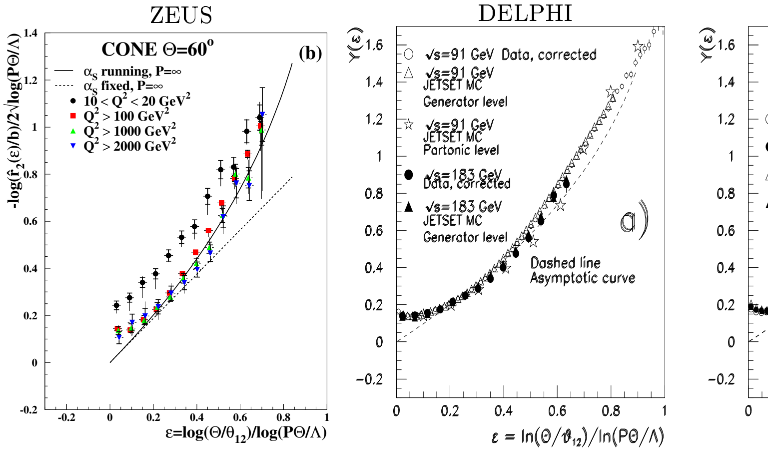

Figure 8: "Trivial" angular correlatins, $\hat{r}_2$.

3 Angular Correlations

ZEUS[6] and DELPHI[1] have studied correlations between two particles within the cone defined by Θ in Fig. 4 as a function of the difference in polar angles of the two particles, $\theta_{12} = \mid \theta_1 - \theta_2 \mid$, using two normalizations of the two-particle correlation function ρ_2:

$$\hat{r}_2(\theta_{12}) = \frac{\rho_2(\theta_{12})}{\langle N(\Theta)\rangle^2} \tag{5}$$

$$r_2(\theta_{12}) = \frac{\rho_2(\theta_{12})}{\rho_1 \otimes \rho_1(\theta_{12})} \tag{6}$$

where N is the number of particle within the cone and

$$\rho_2(\theta_{12}) = \int_\Theta \mathrm{d}^3k_1 \mathrm{d}^3k_2\, \rho_2(k_1,k_2)\, \delta\left(\theta_{12} - \theta(k_1,k_2)\right)$$

$$\rho_1 \otimes \rho_1(\theta_{12}) = \int_\Theta \mathrm{d}^3k_1 \mathrm{d}^3k_2\, \rho_1(k_1)\rho_1(k_2)\, \delta\left(\theta_{12} - \theta(k_1,k_2)\right)$$

One can write the correlation function ρ_2 as $\rho_2 = \kappa + \rho_1 \otimes \rho_1$, *i.e.*, as a sum of genuine correlations, κ, plus the "trivial" correlations, $\rho_1 \otimes \rho_1$, arising solely as a consequence of the single-particle distributions. It then turns out that $\hat{r}_2$ is dominated by the "trivial" correlations, whereas r_2 measures the genuine correlations. Experimentally, ρ_2 is measured using events and $\rho_1 \otimes \rho_1$ using particles from different events. Both experiments use the scaling variable

$$\epsilon = \frac{\log\left(\frac{\Theta}{\theta_{12}}\right)}{\log\left(\frac{P\Theta}{\Lambda}\right)} \tag{7}$$

where $P = E_{\rm jet}$, which is half the CMS energy for DELPHI and $Q/2$ for ZEUS. Λ is taken as 150 MeV.

There are some inconsistencies between the two experiments. For $\hat{r}_2$ (Fig. 8) ZEUS finds agreement with asymptotic QCD predictions for their highest energy jets ($\sim$31 GeV), whereas DELPHI with 45-GeV jets does not, although DELPHI can get better agreement by using $\Lambda = 300$ MeV.

Neither experiment finds agreement with the asymptotic QCD predictions for r_2 (Fig. 9), although DELPHI's data for $\sqrt{s} = 183$ GeV come much closer than does the 91 GeV data. Here too there is a discrepancy between the data of the two experiments: The values of r_2 for ZEUS are approximately equal to those for DELPHI's highest energy jets (91 GeV), but the ZEUS jets are of much lower energy.

While there is little agreement with asymptotic QCD predictions, DELPHI does find very good agreement with the predictions of the JETSET MC.

4 Oscillations in H_q

The ratio of cumulant to factorial moments, H_q, of the soft gluon multiplicity distribution is calculable[8] in perturbative QCD. Assuming the validity of LPHD, the calculation should also be valid for the charged particle multiplicity distribution. Its behavior, plotted against the rank of the moments, q, depends greatly on the approximations used (Fig. 10). SLD has observed[9] the behavior expected in NNLLA. L3 has confirmed[3] this observation. Further, L3 finds (Fig. 11) that the minimum plus oscillations seen in the data are well reproduced by the MC models JETSET and ARIADNE. HERWIG produces a similar behavior, but the amplitudes of its oscillations are much larger. This good agreement of the MCs is rather suspicious, since these MC models are not NNLLA, but more in the spirit of MLLA, for which no oscillations are expected.

To investigate whether the oscillations are indeed due to a perturbative process (which is what is calculated in NNLLA), L3 also studied H_q calculated from the jet multiplicity distribution using both Durham and Cambridge jet algorithms at various energy scales, $\sqrt{sy_{\mathrm{cut}}}$. The results for Durham are shown in Fig. 12. The results for Cambridge are quite similar. At a small energy scale (220 MeV corresponding to $y_{\mathrm{cut}} = 6\times10^{-6}$) the minimum and oscillations are present. But as one proceeds to larger energy scales, the minimum shifts to lower values of q and the oscillations disappear. It thus appears that the oscillatory behavior arises in the hadronization phase and that its similarity to NNLLA predictions is fortuitous.

This conclusion is reinforced by the comparison with MC. The MC programs have three different implementations of the perturbative parton shower. The two programs which use the same hadronization model (JETSET and ARIADNE) give nearly identical predictions, whereas the other (HERWIG) gives quite different predictions, the difference decreasing as one progresses from particles to perturbative-scale jets.

5 Conclusions

Fluctuations and correlations are well described by Monte Carlo models such as JETSET, ARIADNE, and HERWIG. From these studies there is little need for additional physics in the MCs.

Perturbative QCD with the assumption of LPHD is less successful. Comparisons with data are close enough that there is hope, but it is clear that either the present calculations are not accurate enough or that LPHD is invalid.

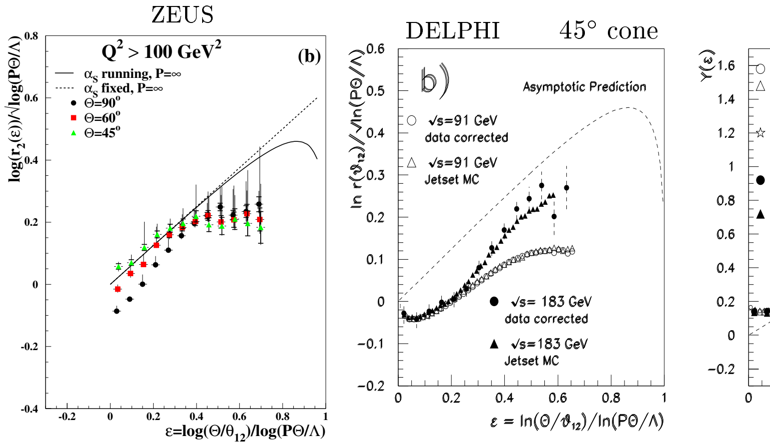

Figure 9: "Genuine" angular correlatins, r_2.

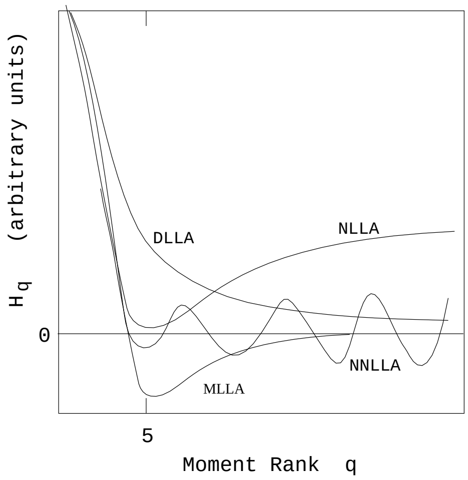

Figure 10: Behavior of H_q expected from QCD.

The success of the MC models suggests that calculations need to go beyond DLA, to be done for finite rather than infinite energy, and need to include energy and momentum conservation and quark mass effects.

Comparisons of parton level and hadron level in MC models, as done in several of the contributions gives no clear picture of the importance of hadronization. If the two levels give nearly equal values for an observable, one is tempted to conclude that hadronization effects are small and that LPHD is valid. Such a conclusion is, however, confounded by the tuning of the MC models where parameters affecting the perturbative parton shower are correlated with hadronization parameters. Moreover, different MC models often lead to contradictory conclusions.[1,5]

The L3 study of H_q oscillations suggests that this behavior is produced in the non-perturbative hadronization rather than in the perturbative parton shower.

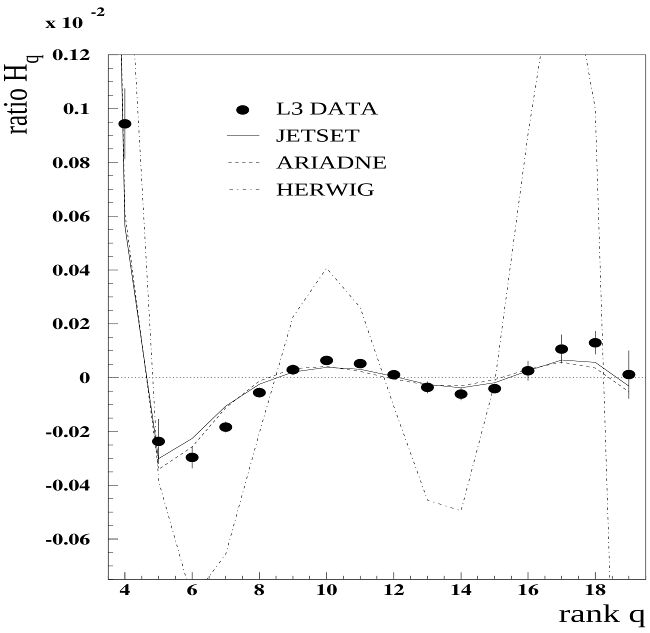

Figure 11: H_q of the charged particle multiplicity distribution.

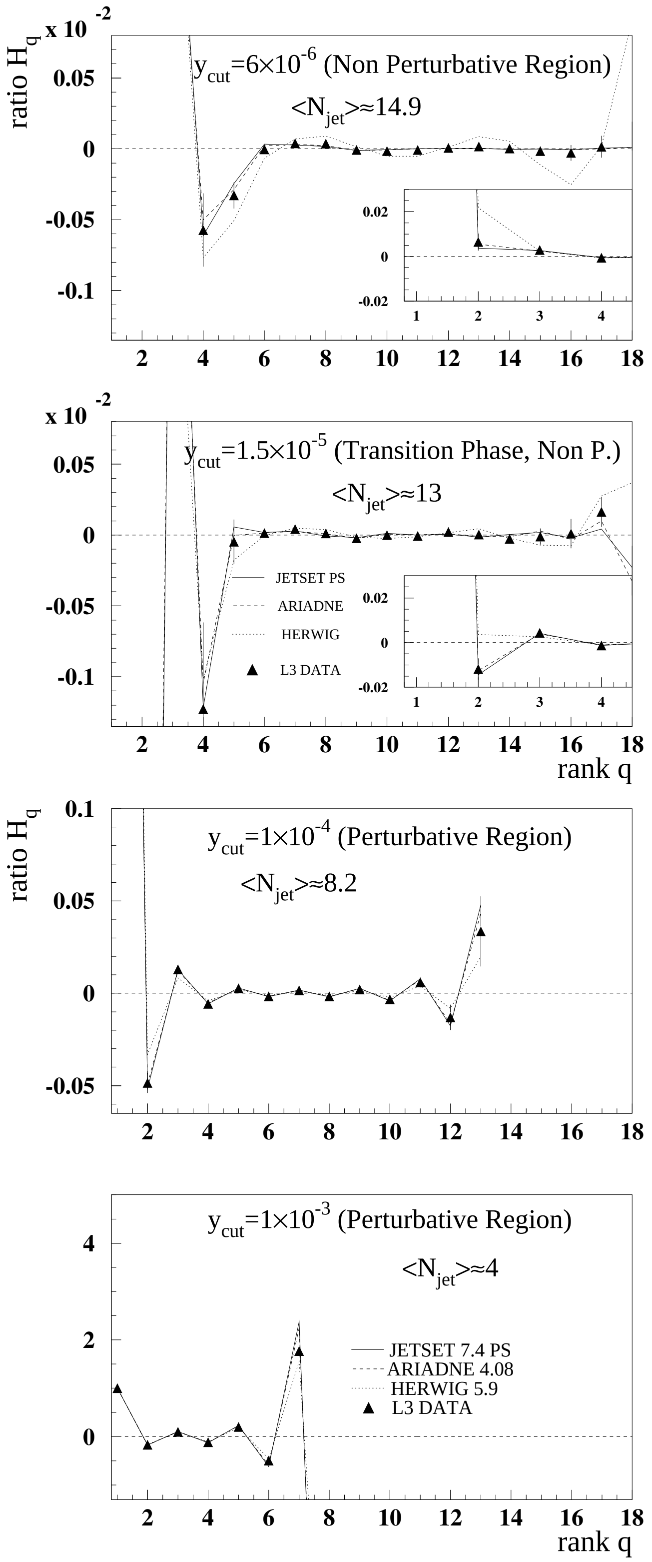

Figure 12: H_q of the jet multiplicity distribution for the Durham jet algorithm at various values of $y_{\rm cut}$.

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