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Oscillation of H_q Moments as NOT a Test of QCD*

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The ratio of cumulant factorial to factorial moments of the charged particle multiplicity distribution shows oscillatory behaviour similar to that predicted by NNLLA. However, an analysis of the jet multiplicity distribution finds these oscillations only at non-perturbative scales. We investigate also how it is possible to reproduce the oscillations within the framework of Monte Carlo models.

1 Introduction

The normalised factorial moments, F_q, and their cumulants, K_q, of order q are defined in terms of the multiplicity distribution, P_n, by

$$F_q = \frac{\sum_{n=q}^{\infty} n(n-1)....(n-q+1)P_n}{(\sum_{n=1}^{\infty} nP_n)^q}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m} F_m$$

Their ratio, $H_q = K_q/F_q$, has been calculated[1] for the soft gluon multiplicity distribution in different approximations of perturbative QCD (pQCD). The expected behaviour of H_q with respect to q depends strongly on the approximation: In the Double Leading Logarithm Approximation (DLLA), H_q decreases to 0 as q^{-2}. In the Modified Leading Logarithm Approximation (MLLA), H_q decreases to a negative minimum at $q \approx 5$ and then rises to approach 0 asymptotically. In the Next to Leading Logarithm Approximation (NLLA), H_q decreases to a positive minimum at $q \approx 5$ and then increases to a positive constant value for large q. In the Next to Next to Leading Logarithm Approximation (NNLLA), H_q decreases to a negative first minimum at $q \approx 5$, and then shows quasi-oscillations about 0.

Validity of the Local Parton Hadron Duality hypothesis (LPHD) would imply similar behaviour for the charged particle multiplicity distribution.

Previous measurements of the H_q for the charged particle multiplicity distribution in e^+e^- collisions[2,3] at 91.2 GeV and also in hadron collisions[4] have shown, qualitatively, the behaviour predicted in NNLLA.

While it is tempting to conclude that the observed behaviour is a confirmation of the NNLLA prediction, it must be realized that such a conclusion rests strongly on the validity of LPHD. Although the LPHD hypothesis works well for single-particle distributions and mean multiplicity, difficulties have been detected when resonances are involved.[5,6] Further, heavy quark decays will cause a modification of the multiplicity distribution which is unlikely to be

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properly accounted for by LPHD.[7] For these reasons, we consider the assumption of LPHD as highly questionable.

Therefore, in this paper we present preliminary measurements[8] of the ratio of cumulant to factorial moments not only for the charged particle multiplicity distribution but also for the jet multiplicity distributions. The jet multiplicity distributions should enable us to extend the moment-ratio analysis to the energy scales of partons, in particular to scales where perturbative QCD should be reliable ($1 - 2$ GeV), thus reducing the dependence on the assumption of LPHD.

2 Experimental procedures

This analysis is based on data collected by the L3 detector[9] in 1994 at the energy of the Z and uses 900000 selected hadronic events.[8]

The factorial moments calculated from the raw data, $F_q^{\rm raw}$, were corrected for detector effects by the following method: Events are generated by the Monte Carlo (MC) programs JETSET 7.4 PS,[10] HERWIG 5.9,[11] and ARIADNE 4.08,[12] which have been tuned using L3 data.[13] For each sample the factorial moments, $F_q^{\rm gen}$, are calculated. The events are passed through the simulation of the L3 detector[18] and reconstructed in the same way as the data. After the selection procedure factorial moments, $F_q^{\rm sim}$, are calculated. The $F_q^{\rm raw}$ are then corrected by $F_q = F_q^{\rm raw} \times (F_q^{\rm gen}/F_q^{\rm sim})$. From these F_q, the H_q are calculated. The corrections turn out to be rather small, and there is quite satisfactory agreement among the corrections obtained using different MC models, as is shown in fig. 1.

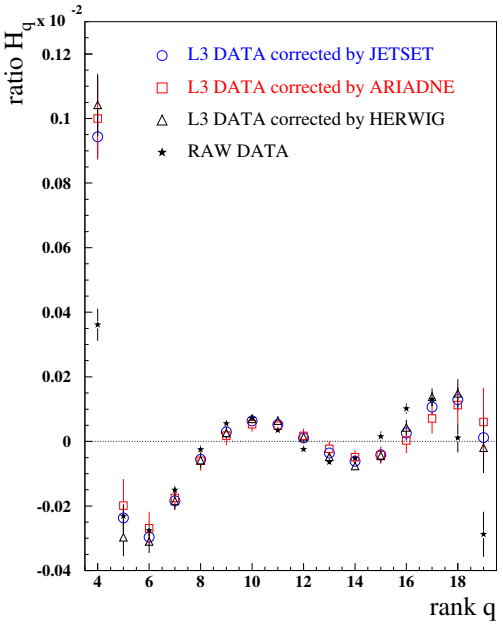

Figure 1: H_q of the charged particle multiplicity distribution for raw data and after correction by various Monte Carlo models.

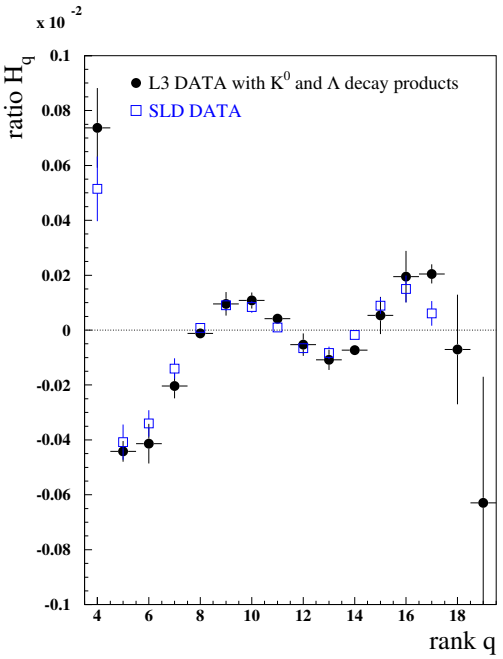

Figure 2: H_q calculated from the corrected P_n compared with SLD.

To calculate the statistical errors of the H_q, 10000 multiplicity distributions are generated from the experimental particle multiplicity distribution according to a Poissonian fluctuation

of each P_n. This is also done for the generated and simulated multiplicity distributions. The statistical errors on the H_q are taken as the standard deviation of the 10000 values of H_q obtained from the 10000 corrected F_q. The standard deviation of the values of the H_q corrected using different MC models is taken as a systematic error. Statistical and systematic errors were added in quadrature.

An alternative method to obtain corrected H_q is to obtain first the corrected charged particle multiplicity distribution and then to calculate the moments from it. However, the unfolding procedure is complicated and the systematic errors are not yet fully understood. Nevertheless, very preliminary results agree well with the SLD data[2] when the decay products of K^0 and Λ are included in the multiplicity, as SLD has done. This is shown in fig. 2. This procedure is even more complicated for the jet multiplicity distribution, since the jet multiplicity need not be even. Consequently we have chosen to present all results using the previously described method of correction of the F_q rather than the P_n. Further, the decay products of K^0 and Λ are not included in the moments of the charged particle multiplicity distribution.

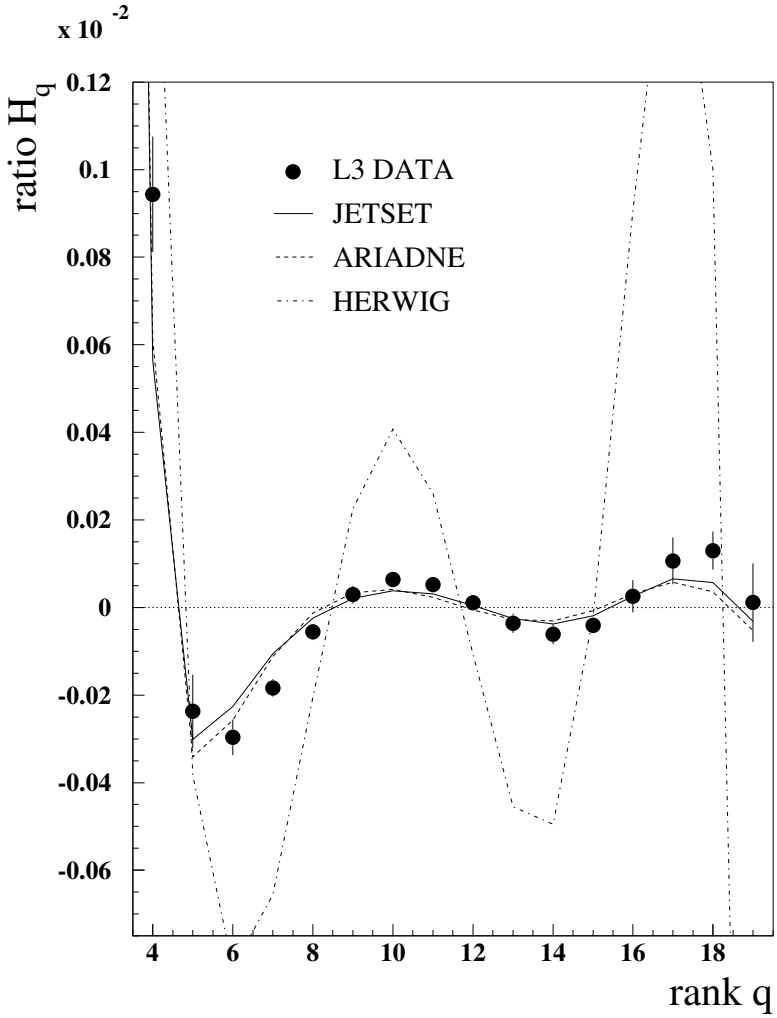

Figure 3: H_q of the charged particle multiplicity distribution.

3 Charged particle multiplicity analysis

The H_q are very sensitive to statistical fluctuations, which are large for high multiplicities due to the limited statistics of the sample. Therefore, the data and the simulated multiplicity distributions are truncated at 45 (events with $N_{\rm ch} \geq 45$ are rejected) and the generated multiplicity distributions at 50. The value 45 corresponds to the first high-multiplicity bin with fewer than 35 events. The use of truncated distributions decreases both systematic and statistical errors.

The corrected H_q, shown in fig. 3, have a negative first minimum at $q = 6$ and quasi-oscillations around 0 for greater q. These results are in qualitative agreement with those previously measured. The amplitudes of the oscillations and $|H_6|$ are found to increase if the multiplicity distribution is further truncated (not shown). The H_q calculated from JETSET and ARIADNE agree quite well with the data when they are truncated in the same way as the data. The H_q calculated from HERWIG show the same features but the amplitudes of the oscillations are much larger.

Since the behaviour of the H_q agrees qualitatively with that predicted in NNLLA, it is natural to interpret this agreement as a confirmation of the NNLLA prediction, and SLD made this interpretation. However, JETSET and ARIADNE agree with the data even though their parton showers are closer to MLLA than to NNLLA. Hence their H_q should not be expected to oscillate. That they do, together with doubt about the LPHD assumption, suggests that we view the NNLLA interpretation with scepticism.

4 Jet multiplicity analysis

This analysis assumes that the jet multiplicity distribution is related to the parton multiplicity distribution at the energy scale corresponding to the jet resolution. By choosing a scale where perturbative QCD should be applicable ($\gtrsim 1$ GeV), we should be able to test the pQCD predictions for the behaviour of the H_q. We define jets using both the Durham[20] and the new Cambridge[21] jet algorithms. Previous analyses of hadronic Z decays have shown[14–17,22] that the mean jet multiplicity using the Durham algorithm agrees well with the NLLA prediction in the perturbative region.

Since this study starts with the charged track multiplicity distribution, we have also constructed jets using only charged tracks. The H_q measured for jet multiplicity distributions obtained from both the Durham and Cambridge jet algorithms have been corrected using the same procedure as for the charged particle multiplicity distribution, again truncating the distribution at the highest multiplicities. The resulting H_q are shown for the Durham algorithm (similar results are found[8] with the Cambridge algorithm) in figures 4 and 5, for four different values of the jet resolution parameter, $y_{\rm cut}$, representing both non-perturbative and pertubative energy scales (scale $= E_{\rm cms}\sqrt{y_{\rm cut}}$).

The H_q calculated from JETSET and ARIADNE agree well with the data for each $y_{\rm cut}$, while the H_q calculated from HERWIG disagree in magnitude with the data for low values of $y_{\rm cut}$, where the jet multiplicity distributions are closer to the charged particle multiplicity distribution.

A first negative minimum at $q = 4$ followed by oscillations about 0, is seen for $y_{\rm cut} = 6 \times 10^{-6}$ (energy scale ≈ 220 MeV) (figure 4a). This is qualitatively similar to that for the charged particle multiplicity distribution but the position of the first minimum is shifted to a lower value of q.

As $y_{\rm cut}$ is increased, the behaviour of the H_q changes further. The negative first minimum near 5 disappears. For $q \leqslant 5$ each H_q alternates between positive and negative values. For

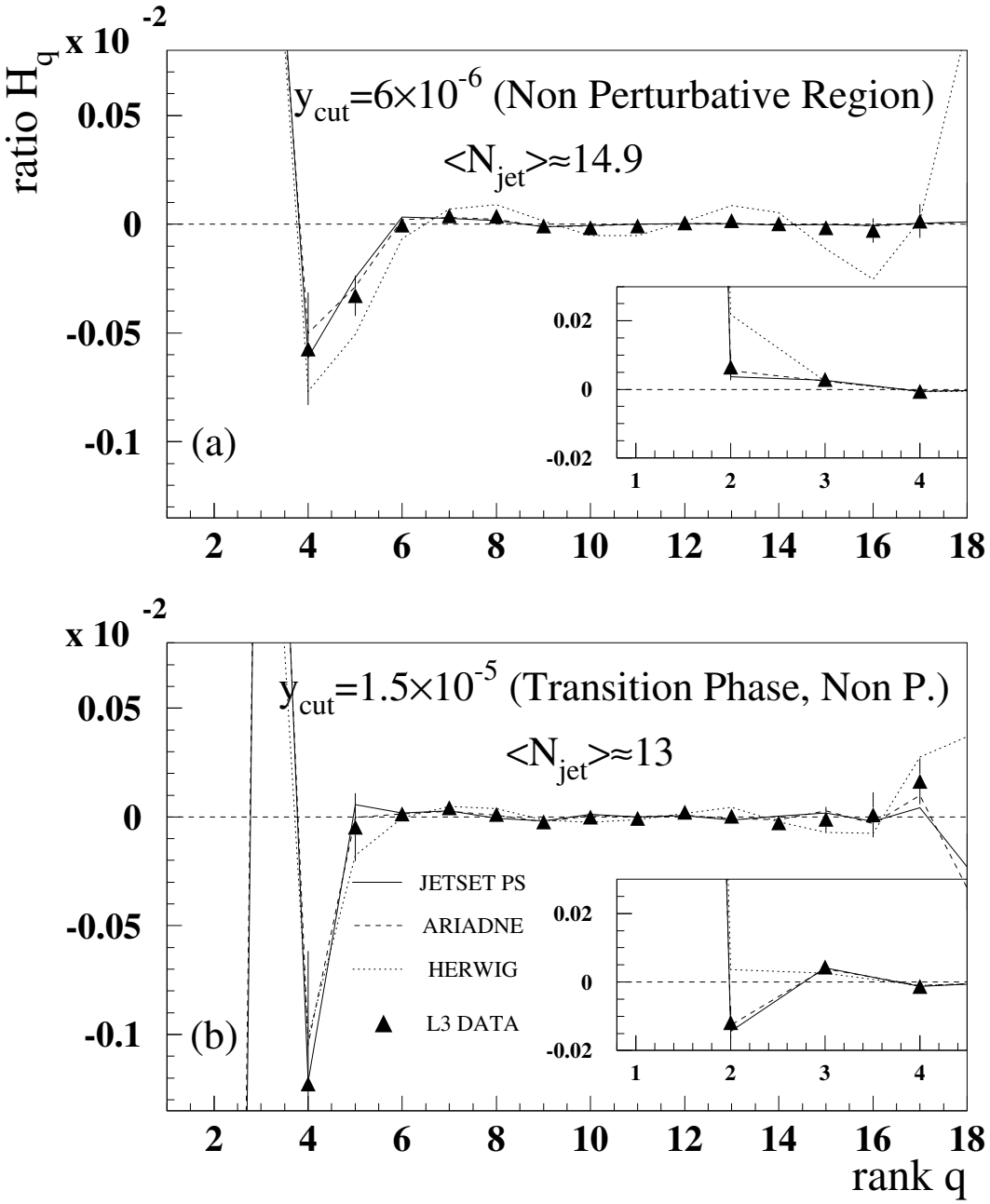

Figure 4: H_q of the jet multiplicity distributions at non-perturbative energy scales.

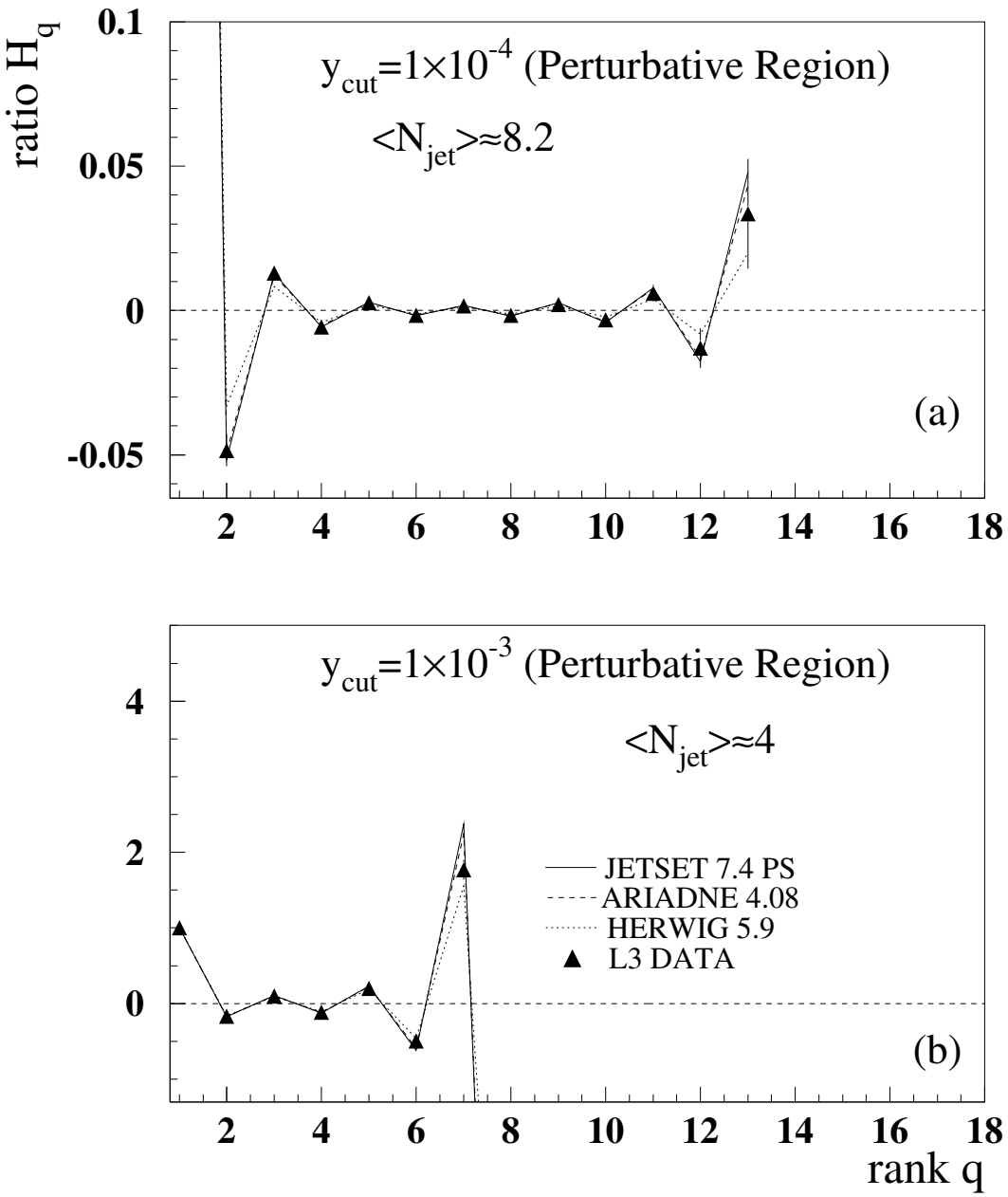

Figure 5: H_q of the jet multiplicity distributions at perturbative energy scales.

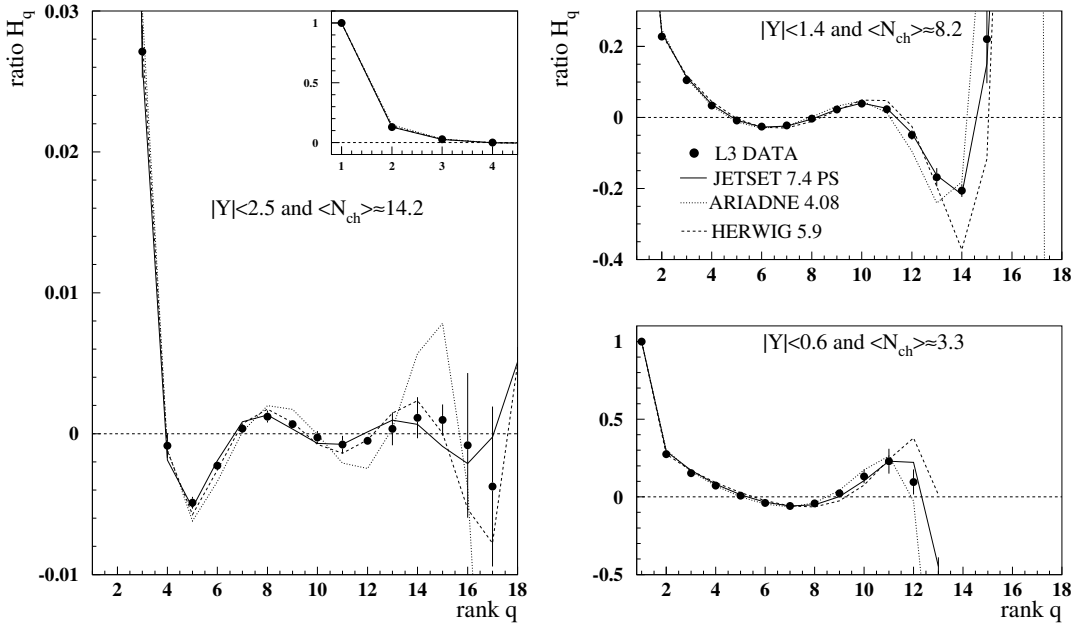

Figure 6: H_q for the charged particle multiplicity in restricted intervals of rapidity

greater q, the oscillations begin to disappear. For $y_{\rm cut} = 1.5 \times 10^{-5}$ (energy scale $\approx$ 350 MeV) these oscillations have almost disappeared, as seen in figure 4b.

For $y_{\rm cut} > 1.5 \times 10^{-5}$ and in particular in the perturbative region $y_{\rm cut} \geq 10^{-4}$ the oscillations and the negative first minimum near 5 have completely disappeared as seen in figure 5 for $y_{\rm cut} = 10^{-4}$ (energy scale ≈ 0.9 GeV) and $y_{\rm cut} = 10^{-3}$ (energy scale ≈ 2.8 GeV). Instead, H_q alternates between positive and negative values for each q.

None of the predictions made at different approximations of pQCD for the H_q is observed at perturbative scales. Even though pQCD describes the average jet multiplicity well, it seems to be unable to describe the shape of these distributions. Therefore, the minimum and oscillatory behaviour of the H_q seen for the charged particle multiplicity distribution and for very low values of $y_{\rm cut}$ appears unrelated to the behaviour of the H_q calculated in pQCD, and in particular in NNLLA.

This conclusion might be questioned, since the mean jet multiplicity decreases rapidly with increasing $y_{\rm cut}$. To investigate whether a small mean jet multiplicity necessarily leads to the behaviour seen at large $y_{\rm cut}$, we have examined the H_q of the charged particle multiplicity distribution in restricted intervals of rapidity along the thrust axis. The intervals chosen have mean charged multiplicities comparable to the mean jet multiplicities found for the three largest values of $y_{\rm cut}$. Fig. 6 shows the H_q for three rapidity intervals ($|Y| < 2.5, 1.4,$ and 0.6). The H_q calculated from MC models agree well with the data for each interval. In each case, the H_q decrease sharply to a negative minimum followed by oscillation. Contrary to the case of the jet multiplicity distributions, the first negative minimum shifts to higher q when the mean multiplicity decreases. In no case is the quasi-oscillatory behaviour degraded to a change of sign with each order. Thus, decreasing the mean value of the multiplicity does not necessarily lead to the behaviour observed for jets.

5 Monte Carlo study of the oscillatory behaviour of the H_q

Given the success of the MC generators in the previous sections, we attempted to find which aspect of the models is responsible for the agreement.

First we used different models of parton generation in JETSET, keeping in all cases the Lund string fragmentation model: no angular ordering in the parton shower; $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ matrix elements; and matrix element production of only $q\bar{q}$.

Next we considered the possibility that this behaviour could come from the fragmentation model, which could simulate some hidden higher-order aspects of pQCD. We used JETSET with the various choices of parton generation used previously but with independent fragmentation for the hadronization process. We also used HERWIG, which uses cluster fragmentation.

In all cases the H_q have a negative first minimum near $q = 5$ and quasi-oscillations for greater q. Some examples are shown in figure 7. It thus appears that we can reproduce the oscillatory behaviour of the H_q without the need of the NNLLA of pQCD.

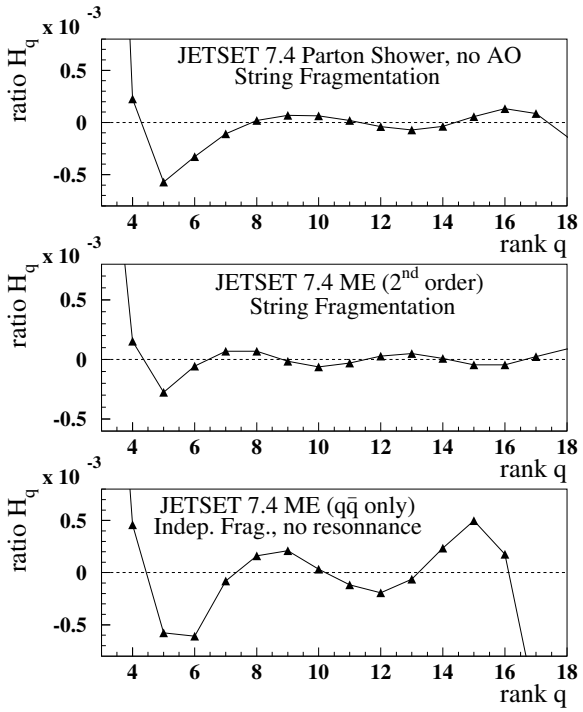

Figure 7: H_q for various Monte Carlo options

6 Conclusions

The H_q have been measured for both the charged particle and jet multiplicity distributions obtained with the Durham and Cambridge algorithms at various scales. The NNLLA prediction of a first minimum at $q \approx 5$ and quasi-oscillation around 0 for higher q is observed for the charged particle multiplicity distribution, confirming previous results. However, our analysis of the H_q of the jet multiplicity distributions reveals that this behaviour appears only for

very small $y_{\rm cut}$, corresponding to energy scales $\lesssim 200$ MeV, which is far from the perturbative region. Furthermore, a Monte Carlo investigation using different models of parton generation and different fragmentation models has found in each case oscillatory behaviour of H_q.

Our conclusion is that the similarity of the behaviour of H_q observed in the charged particle multiplicity distribution to that predicted by NNLLA perturbative QCD calculations is purely fortuitous.

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