Bose-Einstein Correlations in e^+e^- Annihilation and $e^+e^- \rightarrow W^+W^-$ [1]

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Abstract. Results on Bose-Einstein correlations in $e^+e^- \rightarrow$ hadrons are reviewed.

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INTRODUCTION

To study Bose-Einstein correlations (BEC) among q identical particles, one usually examines the ratio, R_q, of the q-particle inclusive density, $\rho_q(p_1,\dots,p_q)$, to that expected when BEC are absent, ρ_{0q}: $R_q(p_1,\dots,p_q) = \rho_q(p_1,\dots,p_q)/\rho_{0q}(p_1,\dots,p_q)$. This ratio is usually regarded as a function of Q, where $Q^2 = M_q^2 - (qm)^2$ with M_q the mass of the q particles and m the mass of each particle. If the particles have identical 4-momenta, $Q = 0$. For 2 particles, Q^2 is simply the 4-momentum difference. Thus, *e.g.*, 2-particle BEC are studied using $R_2(Q) = \rho(Q)/\rho_0(Q)$, where the subscript q has been suppressed.

It can be shown in a variety of ways that R_q is related to the spatial distribution of the particle production [2, 3]. For example, assuming incoherent particle production and a spatial source density of pion emitters, $S(x)$, leads to $R_2(Q) = 1 + |G(Q)|^2$, where $G(Q) = \int \mathrm{d}x\, e^{iQx} S(x) = |G|\, e^{i\phi}$ is the Fourier transform of $S(x)$. Assuming $S(x)$ to be a Gaussian with radius r results in

$$R_2(Q) = 1 + \lambda e^{-Q^2 r^2} , \tag{1}$$

where we have inserted, as is customary, an additional parameter, λ, which is meant to account for several effects such as partial coherence (completely coherent particle production would imply $\lambda = 0$), multiple sources and particle purity.

The lack of time dependence in S is certainly wrong. The assumption of a spherical Gaussian distribution of particle emitters may seem unlikely in e^+e^- annihilation, where there is a definite jet structure. However, we must keep in mind that BEC only occur among particles produced close to each other in phase space. Thus, success of (1) in describing the data does not imply that the hadronization volume is a sphere of radius r.

Other parametrizations have been considered in the literature. Nevertheless, in spite of the above-noted limitations, this Gaussian parametrization (1) is most frequently used by

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[1] Given the space limitations of these proceedings, this write-up is very condensed. Almost all figures and tables, as well as much of the discussion, have been eliminated. They can be found in the referenced publications or in the long version of this write-up [1].

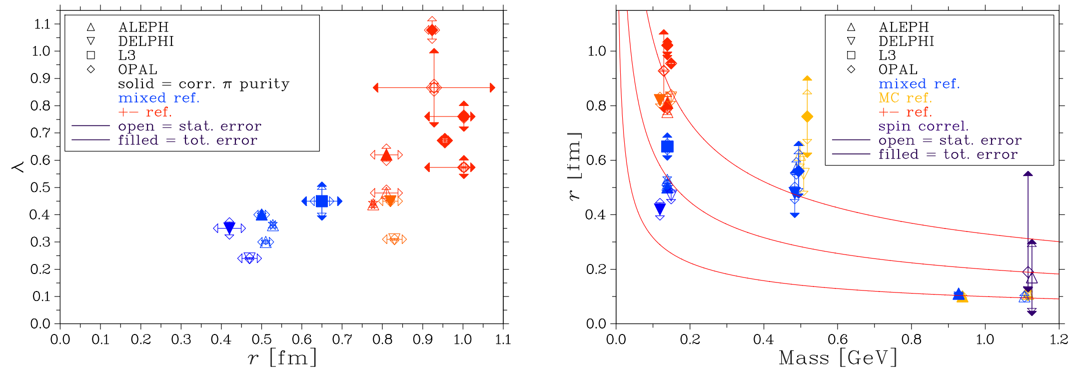

FIGURE 1. (left) λ and r at $\sqrt{s} = M_Z$ found in the LEP experiments [5, 6, 7, 8, 9, 10, 11]. (right) Dependence of r on the mass of the particle as determined at $\sqrt{s} = M_Z$ from 2-particle BEC for charged pions [5, 6, 7, 8, 9, 10, 11], charged kaons [12, 13] and neutral kaons [14, 12, 15] and from Fermi-Dirac correlations for protons [14] and lambdas [16, 17]. The curves illustrate a $1/\sqrt{m}$ dependence.

experimentalists. When it does not fit well, an expansion about the Gaussian (Edgeworth expansion [4]) can be used instead. In the interest of comparison of as much e^+e^- data as possible, I only consider results using the Gaussian or Edgeworth parametrizations.

EXPERIMENTAL DIFFICULTIES

There are several experimental problems affecting BEC results and their interpretation. Particle purity, resonances and weak decays all affect the measured values of λ and r. Other problems are the effect of final-state interactions, both Coulomb and strong, and the choice of the "reference sample," the sample for which ρ_0 is the density. For a discussion of these problems, see [1]. Finally, there is the effect of long-range correlations not adequately taken into account by the reference sample. R_2 is not usually found to be constant at large Q. To account for this the right hand side of (1) is multiplied by an appropriate factor, usually a linear dependence on Q: $\gamma(1+\delta Q)$.

EXPERIMENTAL RESULTS

Dependence on the reference sample. The values of λ and r found for charged-pion pairs from hadronic Z decays by ALEPH [5, 6], DELPHI [7], L3 [8] and OPAL [9, 10, 11] are displayed in Fig. 1. Solid points are corrected for pion purity; open points are not. This correction increases the value of λ but has little effect on the value of r. All of the results with $r > 0.7$ fm were obtained using an unlike-sign reference sample, while those with smaller r were obtained with a mixed reference sample. The choice of reference sample clearly has a large effect on the observed values of λ and r. In comparing results we must therefore be sure that the reference samples used are comparable.

Dependence on the center-of-mass energy. Comparison of values of r obtained using the same reference sample for $\sqrt{s} = 29$–91 GeV shows no evidence of dependence [1].

Dependence on the particle mass. It has been suggested, on several grounds [18], that r should depend on the particle mass as $r \propto 1/\sqrt{m}$. Values of r found at LEP for various types of particle are shown in Fig. 1. Comparing only results using the same type of reference sample (in this case mixed), we see no evidence for a $1/\sqrt{m}$ dependence. Rather, the data suggest one value of r for mesons and a smaller value for baryons. The value for baryons, about 0.1 fm, seems very small; if true it is telling us something unexpected about the mechanism of baryon production.

Dependence on particle multiplicity. The values of λ and r from charged-pion 2-particle BEC depend on the charged particle multiplicity, n_{ch}, of the events: λ decreases with n_{ch} while r increases. However, such a dependence is also seen on the number of jets [10]. When 2-jet events are selected, little, if any, dependence on n_{ch} remains. For 3-jet events, r seems independent of n_{ch}, although λ does decrease with n_{ch}. Thus, the dependence of λ and r on n_{ch} seems largely due to a dependence on the number of jets.

Elongation of the source. The Gaussian parametrization (1) assumes a spherical source. Given the jet structure of $\mathrm{e^+e^-}$ events, one might expect a more ellipsoidal shape. To investigate this, the parametrization is generalized to allow different radii along and perpendicular to the jet axis. The analysis uses the longitudinal center-of-mass system (LCMS) with the longitudinal axis along the jet-axis, *e.g.*, the thrust axis. The advantage of the LCMS is that the energy difference, and therefore the difference in emission time of the pions, couples only to the out-component, Q_{out}. Thus Q_{L} and Q_{side} reflect only spatial dimensions of the source, while Q_{out} reflects a mixture of spatial and temporal dimensions. Three-dimensional analyses parametrizing R_2 as a function of Q_{L}, Q_{side}, and Q_{out} have been performed by L3 [19] and OPAL [11], and two-dimensional analyses by ALEPH [6] and DELPHI [20], in which the out and side components are replaced by a transverse one, $Q_{\mathrm{t}}^2 = Q_{\mathrm{out}}^2 + Q_{\mathrm{side}}^2$. However, the interpretation of the corresponding parameter, r_{t}, as a transverse radius is not unambiguous, since it includes the effect of the difference in emission time. The longitudal radius is found to be about 20% larger than the side (transverse) radius. Further, the amount of the elongation increases when narrower 2-jet events are selected [11].

It should also be mentioned that ZEUS [21] performed a similar 2-dimensional analysis in deep inelastic ep interactions. The ratio $r_{\mathrm{t}}/r_{\mathrm{L}}$ found, similar to that found by DELPHI and ALEPH, is independent of the virtuality of the exchanged photon.

$\pi^0\pi^0$. In hadronization models with local charge conservation, *e.g.*, string models, neutral pions can be produced closer together than identical charged pions. One could expect this to be reflected in a smaller BEC radius for $\pi^0\pi^0$ than for $\pi^\pm\pi^\pm$. Only two $\mathrm{e^+e^-}$ experiments have attempted a BEC analysis of $\pi^0\pi^0$, L3 [22] and OPAL [23]. The experimental selections used are quite different. While L3 requires the energy of the pions to be less than 6 GeV, OPAL demands π^0 momenta greater than 1 GeV. Further, OPAL uses only 2-jet events, defined as having thrust larger than 0.9.

In order to make a comparison with charged pions, L3 also analyzes $\pi^\pm\pi^\pm$ with a selection similar to its π^0 selection. Results are listed in Table 1. Comparison of the L3 values for $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$ with the same selection indicates that both r and λ are smaller in the $\pi^0\pi^0$ case, but the significance is only about 1.5 standard deviations.

TABLE 1. Results of fits to R_2 for $\pi^0\pi^0$ [22, 23] and $\pi^\pm\pi^\pm$ [22, 8, 11]. The indicated uncertainties combine statistical and systematic uncertainties.

	Experiment	Selection	Ref. sample	r (fm)	λ
$\pi^0\pi^0$	L3	$E_\pi < 6\,\mathrm{GeV}$	MC	0.31 ± 0.10	0.16 ± 0.09
	OPAL	$p_\pi > 1\,\mathrm{GeV}$, 2-jet	mix	0.59 ± 0.09	0.55 ± 0.14
$\pi^\pm\pi^\pm$	L3	$E_\pi < 6\,\mathrm{GeV}$	MC	0.46 ± 0.01	0.29 ± 0.03
	L3		mix	0.65 ± 0.04	0.45 ± 0.07
	OPAL		+−	$1.00^{+0.03}_{-0.10}$	0.57 ± 0.05

For OPAL, the comparison is more difficult to make, since OPAL's charged-pi results use a different reference sample and different selection. Other experiments have found the ratio of r using a mixed reference sample to that using unlike-sign to be about 0.68 (ALEPH [6]) or 0.56 (DELPHI [7]). Applying such a factor would lower the OPAL value to about 0.62 fm, which agrees well with their result for $\pi^0\pi^0$. However, it is not clear what the effect of the 2-jet and π^0-momentum cuts is. In the L3 case, requiring the pions to have $E < 6\,\mathrm{GeV}$ and using a MC reference sample rather than a mixed one decreased r by about 30%. It is therefore conceivable that the OPAL requirement of $p > 1\,\mathrm{GeV}$ would increase r, in which case r for $\pi^0\pi^0$ would be smaller than for $\pi^\pm\pi^\pm$.

3-particle BEC. Correlations among more than two particles are classified as trivial, a consequence of lower-order correlations, and genuine. We study 3-particle BEC using $R_3(Q_3)$. Note that $Q_3^2 = Q_{12}^2 + Q_{23}^2 + Q_{31}^2$. The same assumptions that lead to (1) for R_2, lead to [24, 25] R_3^{gen}, the R_3 that would occur if there were no 2-particle BEC:

$$R_3^{\mathrm{gen}}(Q_{12}, Q_{23}, Q_{31}) = 1 + 2\lambda^{1.5}\,\Re\{G(Q_{12})G(Q_{23})G(Q_{31})\}\ , \qquad (2)$$

where $\Re$ denotes the real part. Thus R_3^{gen} depends on the phase of G.

Both R_2 and R_3^{gen} are measured by L3 [8] and fit with both a Gaussian and an Edgeworth parametrization, the latter providing a better fit.

By combining these measurements, one can extract $\omega = \cos(\phi_{12} + \phi_{23} + \phi_{13})$. If the particle production is completely incoherent, the phase ϕ_{ij} is expected to be zero, and $\omega = 1$. Incoherence also implies $\lambda = 1$, but λ is affected by many other factors, which should not affect ω. The measurements find ω consistent with unity; we conclude that particle production is completely incoherent.

BEC among three pions have previously been observed at lower energies [26, 27]. At LEP, DELPHI [28] and OPAL [29] have also observed genuine 3-pion BEC. Unfortunately, none of these experiments measured ω.

$\mathrm{W} \to \mathrm{q\bar{q}}$. Having found no evidence for a $\sqrt{s}$ dependence of BEC, and noting that in any case the masses of the Z and W are not much different, there is only one reason to expect BEC to be different in W decays than in Z decays. Whereas about 20% of Z decays are to $\mathrm{b\bar{b}}$, almost no W decays involve a b-quark. The long lifetime of the b results in diminished BEC. Thus, we should expect BEC in W-decay to be like that in the decay of the Z to light (udsc) quarks. This is indeed found to be the case [30, 31].

Inter-string BEC. A more interesting case is $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$. The W-bosons are produced not far above threshold and consequently travel only about 0.1 fm before decaying, less than the distance over which hadronization occurs. There can therefore be a significant degree of overlap of the two hadronizing systems, resulting in BEC among particles from different W-bosons. On the other hand, in the string picture no BEC is expected between particles arising from different strings. The existence of inter-string BEC is thus a fundamental question for the string picture.

The LEP experiments have tested the expectation [32] of no inter-string BEC using various test quantities. All four LEP experiments have reported final results [33, 34, 31, 35]. Only DELPHI [34] claims to see significant inter-W BEC. Their distribution of $\delta_I(Q)$, the correlation function of genuine inter-W BEC [36], clearly shows an enhancement at small Q, while no enhancement is seen in the MC where BEC (BE_{32} model [37]) is included only between pions from the same W. When BE_{32} is also applied to pions from different W-bosons, a larger enhancement is seen than that of the data. However, the interpretation of the enhancement in the data as inter-W BEC is somewhat clouded by their observance of an enhancement, albeit smaller, in the δ_I distribution of unlike-sign pions. This enhancement in the data is larger than that for MC with inter-W BEC.

The final results of the four LEP experiments, expressed as the ratio of the effect seen to that expected in the BE_{32} model are combined to give the preliminary LEP result [38] of 0.17 ± 0.13. The data are thus compatible with no inter-W BEC.

BEC IN THE LUND STRING MODEL

In the Lund string model, the longitudinal break-up of the color string is governed by the area law. The matrix element to get a final state depends on the area A spanned by the string. Transverse momentum arises via a tunneling mechanism, which is also related to b. To incorporate BEC, the probability of a final state is given by the square of the sum of the matrix elements corresponding to the areas of all the permutations of identical bosons [39, 40, 41]. This model results in 2- and genuine 3-particle BEC. It predicts elongation along the jet axis and a radius for $\pi^0\pi^0$ smaller than for $\pi^\pm\pi^\pm$. There is no mechanism to create correlations between particles from different strings.

The model has been incorporated in MC for a $q\bar{q}$ string. While the formalism to do so for the more realistic case of a string with multiple gluons has been worked out [42], a successful MC implementation has, unfortunately, so far proved elusive.

CONCLUSIONS

The study of BEC in e^+e^- presents a number of problems, both experimental and theoretical. Values obtained for parameters vary considerably among experiments, even when the same parametrization is used. Nevertheless, certain features are clear: BEC, both 2-particle and genuine 3-particle, exist; they seem independent of cms energy; the source shape is somewhat elongated in the jet direction; and the (Fermi-Dirac) radius for baryons is smaller than the radius for mesons. Experimentally, it is not clear whether the radius for neutral pions is smaller than that for charged pions. BEC in W decay is the same as in light-quark Z decay. The data are compatible with no inter-W BEC.

The implementation of BEC in the Lund string model appears consistent with these experimental findings. However, the experimental evidence that pion production is completely incoherent seems at odds with the coherent addition of amplitudes in the model.

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