# BEC, the $\tau$-model, and jets at the Z pole 

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#### Abstract

Bose-Einstein correlations of pairs of identical charged pions produced in hadronic Z decays are analyzed in terms of various parametrizations. A good description is achieved using a Lévy stable distribution in conjunction with a model where a particle's momentum is highly correlated with its space-time point of production, the $\tau$-model. However, a small but significant elongation of the particle emission region is observed in the Longitudinal Center of Mass frame, which is not accommodated in the $\tau$-model. Further, for three-jet events the region is found to be larger in the event plane than out of the plane.


## §1. Introduction

We have recently published ${ }^{1)}$ a study of Bose-Einstein correlations (BEC) in hadronic Z decay where we found good agreement with parametrizations arising in
 results are presented in Section 3.

The data were collected by the L 3 detector at an $\mathrm{e}^{+} \mathrm{e}^{-}$center-of-mass energy of $\sqrt{s} \simeq 91.2 \mathrm{GeV}$. Approximately 36 million like-sign pairs of well-measured charged tracks from about 0.8 million hadronic Z decays are used. ${ }^{4)}$ Events are classified as two- or three-jet events using calorimeter clusters with the Durham jet algorithm. To determine the event (thrust) axis we also use calorimeter clusters.

Two-particle BEC are measured by the BEC correlation function $R_{2}\left(p_{1}, p_{2}\right)=$ $\rho_{2}\left(p_{1}, p_{2}\right) / \rho_{0}\left(p_{1}, p_{2}\right)$, the ratio of the two-particle number density to that which would occur in the absence of BEC. An event mixing technique is used to construct $\rho_{0}$.

## §2. Summary of Previous Results ${ }^{1)}$

With a few assumptions, $R_{2}$ is related to the Fourier transform, $\tilde{f}(Q)$, of the (configuration space) density distribution of the source, $f(x)$ :

$$
R_{2}(Q)=\gamma\left[1+\lambda|\tilde{f}(Q)|^{2}\right](1+\delta Q),
$$

where $Q=\sqrt{-\left(p_{1}-p_{2}\right)^{2}}$. The parameter $\gamma$ and the $(1+\delta Q)$ term are introduced to parametrize possible long-range correlations inadequately accounted for in $\rho_{0}$, and $\lambda$ to measure the strength of the BEC. However, $(2 \cdot 1)$ is ruled out by the data, which show that $R_{2}$ has a significant dip below unity in the region $0.6-1.5 \mathrm{GeV}$, indicative of an anti-correlation.

This anti-correlation region is predicted in the $\tau$-model. ${ }^{2), 3)}$ and the data are in good agreement ${ }^{1)}$ with its predictions, both in its full and its simplified form, the latter being given by

$$
R_{2}(Q)=\gamma\left[1+\lambda \cos \left(\left(R_{\mathrm{a}} Q\right)^{2 \alpha}\right) \exp \left(-(R Q)^{2 \alpha}\right)\right](1+\epsilon Q),
$$

where $R$ is an effective radius, $R_{\mathrm{a}}$ is related to $R$ by

$$
R_{\mathrm{a}}^{2 \alpha}=\tan \left(\frac{\alpha \pi}{2}\right) R^{2 \alpha}
$$

and $\alpha$ is the index of stability of the Lévy distribution assumed to describe the proper-time distribution of particle emission. The strong coupling constant $\alpha_{\mathrm{s}}$ is related to $\alpha .{ }^{5), 6)}$

The $\tau$-model predicts that the two-particle BEC correlation function $R_{2}$ depends on the two-particle momentum difference only through $Q$, not through components of $Q$ separately. However, $R_{2}$ has been found to depend on components of $Q,{ }^{7 /-11)}$ the shape of the region of homogeneity being elongated along the event (thrust) axis. The question is whether this is an artifact of the Edgeworth or Gaussian parametrizations used in these studies or shows a defect of the $\tau$-model.

This is investigated in the Longitudinal Center of Mass System*) (LCMS), where

$$
\begin{align*}
Q^{2} & =Q_{\mathrm{L}}^{2}+Q_{\text {side }}^{2}+Q_{\text {out }}^{2}-(\Delta E)^{2} \\
& =Q_{\mathrm{L}}^{2}+Q_{\text {side }}^{2}+Q_{\text {out }}^{2}\left(1-\beta^{2}\right), \quad \beta=\frac{p_{1 \mathrm{out}}+p_{2 \mathrm{out}}}{E_{1}+E_{2}}
\end{align*}
$$

Assuming azimuthal symmetry about the event axis suggests that the region of homogeneity have an ellipsoidal shape with the longitudinal axis along the event axis. In $(2 \cdot 2) R^{2} Q^{2}$ is then replaced by

$$
R^{2} Q^{2} \Longrightarrow A^{2}=R_{\mathrm{L}}^{2} Q_{\mathrm{L}}^{2}+R_{\text {side }}^{2} Q_{\mathrm{side}}^{2}+\rho_{\mathrm{out}}^{2} Q_{\mathrm{out}}^{2}
$$

which results in

$$
\begin{align*}
R_{2}(Q)= & \gamma\left[1+\lambda \cos \left(\tan \left(\frac{\alpha \pi}{2}\right) A^{2 \alpha}\right) \exp \left(-A^{2 \alpha}\right)\right] \\
& \cdot\left(1+\epsilon_{\mathrm{L}} Q_{\mathrm{L}}+\epsilon_{\text {side }} Q_{\text {side }}+\epsilon_{\text {out }} Q_{\text {out }}\right)
\end{align*}
$$

The longitudinal and transverse size of the source are measured by $R_{\mathrm{L}}$ and $R_{\text {side }}$, respectively, whereas $\rho_{\text {out }}$ reflects both the transverse and temporal sizes. ${ }^{* *)}$ We also investigate two other decompositions of $Q$ :

$$
\begin{array}{ll}
Q^{2}=Q_{\mathrm{LE}}^{2}+Q_{\text {side }}^{2}+Q_{\mathrm{out}}^{2}, & Q_{\mathrm{LE}}^{2}=Q_{\mathrm{L}}^{2}-(\Delta E)^{2} \\
A^{2}=R_{\mathrm{LE}}^{2} Q_{\mathrm{LE}}^{2}+R_{\text {side }}^{2} Q_{\text {side }}^{2}+R_{\text {out }}^{2} Q_{\mathrm{out}}^{2} ; & \\
Q^{2}=Q_{\mathrm{L}}^{2}+Q_{\text {side }}^{2}+q_{\text {out }}^{2}, & q_{\mathrm{out}}^{2}=Q_{\mathrm{out}}^{2}-(\Delta E)^{2}, \\
A^{2}=R_{\mathrm{L}}^{2} Q_{\mathrm{L}}^{2}+R_{\text {side }}^{2} Q_{\text {side }}^{2}+r_{\text {out }}^{2} q_{\mathrm{out}}^{2} . &
\end{array}
$$

The first, $(2 \cdot 8 \mathrm{a})$, corresponds to the LCMS frame where the longitudinal and energy terms are combined; its three components of $Q$ are invariant with respect to Lorentz boosts along the event axis. The second, $(2 \cdot 8 \mathrm{c})$, corresponds to the LCMS frame

[^0]boosted to the rest frame of the pair; its three components are invariant under Lorentz boosts along the out direction.

Fits of $(2 \cdot 7)$ with $(2 \cdot 6),(2 \cdot 8 \mathrm{~b})$, and $(2 \cdot 8 \mathrm{~d})$ show that $R_{2}$ depends differently on the components of $Q$. Also, the values of $R_{\text {side }} / R_{\mathrm{L}}$ found are consistent with values found previously using Gaussian or Edgeworth parametrizations. ${ }^{74-11)}$

## §3. New (Preliminary) Results

Recent work investigates the dependence of the BEC radius on the 'jettiness' of the event, using the simplified $\tau$-model parametrization, $(2 \cdot 2)$, and its extension $(2 \cdot 7)$ to dependence on $\vec{Q}$ rather than $Q$.

Using the Durham algorithm, events can be classified according to the number of jets. The number of jets in a particular event depends on $y_{\text {cut }}$. We define $y_{23}$ as that value of $y_{\text {cut }}$ at which the number of jets changes from two to three. The event sample is then split


Fig. 1. The radius $R$ from fits of (2•2) for various $y_{23}$ subsamples. into subsamples according to the value of $y_{23}$. The subsample with the smallest value of $y_{23}$ corresponds to narrow two-jet events, whereas that with the largest $y_{23}$ consists of three or more very well separated jets. Fits of $(2 \cdot 2)$ are performed for each subsample. The estimates of $\alpha$ and $R$ are very highly correlated in the fits. Therefore, to stabilize the fits we fix the value of $\alpha$ to the value found in a fit of the entire sample: $\alpha=0.443$. We see in Fig. 1 that $R$ increases with $y_{23}$. This is consistent with an earlier observation of OPAL. ${ }^{13)}$

The dependence on $y_{23}$ of the radii for components of $Q,(2 \cdot 6)$ and $(2 \cdot 8 \mathrm{~d})$, is shown in Fig. 2. While the values of $R_{\mathrm{L}}$ found in the LCMS-rest frame fits are systematically lower than in the LCMS frame, the values of $R_{\text {side }} / R_{\mathrm{L}}$ agree extremely well. Note that at all values of $y_{23} R_{\text {side }}<R_{\mathrm{L}}$ while $r_{\text {out }}>R_{\mathrm{L}}$. Thus there is no azimuthal symmetry about the thrust axis, not even for the narrowest two-jet sample. Further, we observe that $R_{\mathrm{L}}$ and $R_{\text {out }}$ are approximately independent of $y_{23}$, whereas both $R_{\text {side }}$ and $r_{\text {out }}$ increase with $y_{23}$.

We find (cf. Fig. 3) that the out direction tends to be in the direction of the major axis, i.e., that the out direction tends to be in the event plane, or equivalently, that the side direction tends to be out of the event plane.

To further investigate the dependence on the event plane, each $y_{23}$ subsample is divided into 'in-plane' and 'out-of-plane' samples which use, respectively, only particles having azimuthal angle less than or greater than $45^{\circ}$ of the major axis.. The values of $R$ from fits of $(2 \cdot 2)$ are shown in Fig. 4. We see that for small $y_{23}$ there is little dependence of $R$ on whether the tracks are in or out of the event plane, but for large $y_{23} R$ is larger for the in-plane sample.

## References

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Fig. 2. The radii from fits in the LCMS and LCMS-rest frames for various $y_{23}$ subsamples.


Fig. 3. The angle between out and major.


Fig. 4. The radius $R$ from fits of (2-2) for various $y_{23}$ subsamples, which are split into 'in-plane' and 'out-of-plane' samples.
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[^0]:    ${ }^{*)}$ Also known as the Longitudinal Co-Moving System; for definition of variables, see Ref. 1).
    ${ }^{* *)}$ In the literature ${ }^{7)-12)}$ the coefficient of $Q_{\text {out }}^{2}$ in (2•6) is usually denoted $R_{\text {out }}^{2}$. We prefer to use $\rho_{\text {out }}^{2}$ to emphasize that, unlike $R_{\mathrm{L}}$ and $R_{\text {side }}, \rho_{\text {out }}$ contains a dependence on $\beta$.

