Parametrization of Bose-Einstein Correlations and Reconstruction of the Source Function in Hadronic Z-boson Decays using the L3 Detector

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Bose-Einstein correlations of pairs of identical charged pions produced in hadronic Z decays are analyzed in terms of various parametrizations. A good description is achieved using a Lévy stable distribution in conjunction with a hadronization model having highly correlated configuration and momentum space, the τ-model. Using these results, the source function is reconstructed.

Keywords: Bose-Einstein Correlations

I. INTRODUCTION

In particle and nuclear physics, intensity interferometry provides a direct experimental method for the determination of sizes, shapes and lifetimes of particle-emitting sources [1–5]. In particular, boson interferometry provides a tool for the investigation of the space-time structure of particle production processes, since Bose-Einstein correlations (BEC) of two identical bosons reflect both geometrical and dynamical properties of the particle radiating source.

Here we study BEC in hadronic Z decay. We investigate various static parametrizations of the correlation function in terms of the four-momentum difference, $Q = \sqrt{-(p_1 - p_2)^2}$ and find that none give an adequate description. However, within the framework of models assuming strongly correlated coordinate and momentum space a good description is achieved. We then reconstruct the complete space-time picture of the particle emitting source.

The data were collected by the L3 detector at an e^+e^- center-of-mass energy of $\sqrt{s} \simeq 91.2$ GeV. Approximately 36 million like-sign pairs of well-measured charged tracks from about 0.8 million hadronic Z decays are used [6].

We perform analyses on the complete sample as well as on two- and three-jet samples. The latter are found using calorimeter clusters with the Durham jet algorithm with jet resolution parameter $y_{\rm cut} = 0.006$. To determine the thrust axis of the event we also use calorimeter clusters.

II. BOSE-EINSTEIN CORRELATION FUNCTION

The two-particle correlation function of two particles with four-momenta p_1 and p_2 is given by the ratio of the two-particle number density, $\rho_2(p_1, p_2)$, to the product of the two single-particle densities, $\rho_1(p_1)\rho_1(p_2)$. Since we are interested only in the correlation, R_2, due to Bose-Einstein interference, the product of single-particle densities is replaced by $\rho_0(p_1, p_2)$, the two-particle density that would occur in the absence of BEC:

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)} \,. \tag{1}$$

This ρ_2 is corrected for detector acceptance and efficiency using Monte Carlo (MC) events, to which a full detector simulation has been applied, on a bin-by-bin basis. An event mixing technique is used to construct ρ_0. Since all correlations are removed, not just BEC, ρ_0 is corrected for this using MC [6].

Since the mass of the identical particles of the pair is fixed, R_2 is defined in six-dimensional momentum space. Since BEC can be large only at small Q, they are often parametrized in this one-dimensional distance measure. But there is no reason to expect the hadron source to be spherically symmetric in jet fragmentation. Recent investigations have, in fact, found an elongation of the source along the jet axis [7–10]. While this effect is well established, the elongation is actually only about 20%, which suggests that a parametrization in terms of the single variable Q, may be a good approximation.

On the other hand, in heavy-ion and hadron-hadron interactions BEC are found not to depend simply on Q, but on components of the momentum difference separately [5, 11–14]. However, in e^+e^- annihilation at lower energy [15] it has been observed that Q is the appropriate variable. We confirm that this is indeed the case: We observe [6], both for 2-jet and 3-jet events, that R_2 does not decrease when both $q^2 = (\vec{p}_1 - \vec{p}_2)^2$ and $q_0^2 = (E_1 - E_2)^2$ are large while $Q^2 = q^2 - q_0^2$ is small, but is maximal for $Q^2 = q^2 - q_0^2 = 0$, independent of the individual values of q and q_0. The same is observed in a different decomposition: $Q^2 = Q_{\rm t}^2 + Q_{L,B}^2$, where $Q_{\rm t}^2 = (\vec{p}_{\rm t1} - \vec{p}_{\rm t2})^2$ is the component transverse to the thrust axis and $Q_{L,B}^2 = (p_{\rm L1} - p_{\rm L2})^2 - (E_1 - E_2)^2$ combines the longitudinal momentum and energy differences. Again, R_2 is maximal along the line $Q = 0$. Hence, a parametrization in terms of Q can be considered adequate for the purposes of this article.

III. PARAMETRIZATIONS OF BEC

With a few assumptions [2, 5, 16], R_2 is related to the Fourier transformed source distribution:

$$R_2(p_1, p_2) = \gamma\left[1 + \lambda|\tilde{f}(Q)|^2\right](1 + \delta Q) \,, \tag{2}$$

*Visitor from Budapest, Hungary, sponsored by the Scientific Exchange between Hungary (OTKA) and The Netherlands (NWO), project B64-27/N25186.

where $f(x)$ is the (configuration space) density distribution of the source, and $\tilde{f}(Q)$ is the Fourier transform (characteristic function) of $f(x)$. The parameter γ and the $(1+\delta Q)$ term are introduced to parametrize possible long-range correlations not adequately accounted for in the reference sample, and λ to account for several factors, such as lack of complete incoherence of particle production and presence of long-lived resonance decays resulting in a small, resolvable core and a halo with unresolvable large length scales [17, 18].

A. Static parametrizations

The simplest assumption is that the source has a symmetric Gaussian distribution, in which case

$$R_2(Q) = \gamma\left[1+\lambda\exp\left(-(RQ)^2\right)\right](1+\delta Q)\ . \qquad (3)$$

Fits of Eq. (3) to the data result in an unacceptably low confidence level, both for two-jet and for three-jet events. The fits are particularly bad at low Q values, where the parametrization underestimates the height of the peak. We conclude that the shape of the source deviates from a Gaussian.

A model-independent way to study deviations from Eq. (3) is to use [5, 19, 20] the Edgeworth expansion about a Gaussian. Keeping only the first non-Gaussian term, we have

$$R_2(Q) = \gamma\left(1+\lambda\exp\left(-(RQ)^2\right)\left[1+\frac{\kappa}{3!}H_3(RQ)\right]\right)(1+\delta Q)\ , \qquad (4)$$

where κ is the third-order cumulant moment and $H_3(RQ) \equiv (\sqrt{2}RQ)^3 - 3\sqrt{2}RQ$ is the third-order Hermite polynomial. The second-order cumulant corresponds to the radius R.

Fits of Eq. (4) to the two-jet and three-jet data are indeed much better than the purely Gaussian fits. However, the confidence levels are still marginal, and close inspection shows that the fit curves are systematically above the data in the region 0.6–1.2 GeV and that the data for $Q \geq 1.5$ GeV appear flatter than the fit. This is also the case for the purely Gaussian fit.

The symmetric Lévy stable distribution has three parameters, x_0, R, and α, and its characteristic function can be written as

$$\tilde{f}(Q) = \exp\left(iQx_0 - \frac{|RQ|^\alpha}{2}\right)\ . \qquad (5)$$

The index of stability, α, satisfies the inequality $0 < \alpha \leq 2$. The case $\alpha = 2$ corresponds to a Gaussian source distribution with mean x_0 and standard deviation R. For more details, see, *e.g.*, [21]. R_2 has the following form [22]:

$$R_2(Q) = \gamma[1+\lambda\exp(-(RQ)^\alpha)](1+\delta Q)\ . \qquad (6)$$

From the fit of Eq. (6) to the two-jet data, shown in Fig. 1, it is clear that the correlation function is far from Gaussian: $\alpha = 1.34 \pm 0.04$. The confidence levels, although improved compared to the fit of Eq. (3), are still unacceptably low, in fact worse than that for the Edgeworth parametrization. The same is true for 3-jet events.

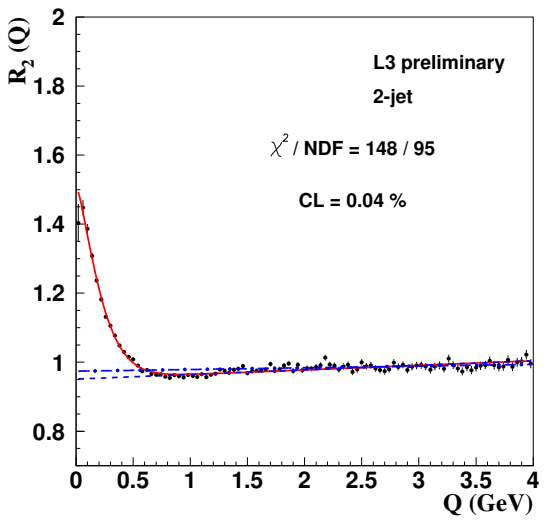

FIG. 1: The Bose-Einstein correlation function R_2 for two-jet events. The curve corresponds to the fit of the symmetric Lévy parametrization, Eq. (6). The dashed line represents the long-range part of the fit, *i.e.*, $\gamma(1+\delta Q)$. The dot-dashed line represents a linear fit in the region $Q > 1.5$ GeV.

Both the symmetric Lévy parametrization and the Edgeworth parametrizations do a fair job of describing the region $Q < 0.6$ GeV, but fail at higher Q. R_2 in the region $Q \geq 1.5$ GeV is nearly constant (≈ 1). However, in the region 0.6–1.5 GeV R_2 has a smaller value, dipping below unity, indicative of an anti-correlation. This is clearly seen in Fig. 1 by comparing the data in this region to an extrapolation of a linear fit, Eq. (6) with $\lambda = 0$, in the region $Q \geq 1.5$ GeV. The inability to describe this dip in R_2 is the primary reason for the failure of both parametrizations.

B. Time dependence of the source

The parametrizations discussed so far all assume a static source. The parameter R, representing the size of the source as seen in the rest frame of the pion pair, is a constant. It has, however, been observed that R depends on the transverse mass, $m_t = \sqrt{m^2+p_t^2} = \sqrt{E^2-p_z^2}$, of the pions [23, 24]. It has been shown [25, 26] that this dependence can be understood if the produced pions satisfy, approximately, the (generalized) Bjorken-Gottfried condition, whereby the four-momentum of a particle and the space-time position at which it is produced are linearly related: $x^\mu = dk^\mu$. Such a correlation is also a feature of the Lund string model as incorporated in JETSET, which is very successful in describing detailed features of the hadronic final states of e^+e^- annihilation.

In the previous section we have seen that BEC depend, at least approximately, only on Q and not on its components separately. Further, we have seen that R_2 in the region 0.6–1.5 GeV dips below its values at higher Q. A model which predicts such Q-dependence while incorporating the Bjorken-Gottfried condition is the τ-model.

1. The τ model

A model of strongly correlated phase-space, known as the τ-model [27], explains the experimentally found invariant relative momentum dependence of BEC in e^+e^- reactions. This model also predicts a specific transverse mass dependence of R_2, that we subject to an experimental test here.

In this model, it is assumed that the average production point in the overall center-of-mass system, $\overline{x} = (\overline{t}, \overline{r}_x, \overline{r}_y, \overline{r}_z)$, of particles with a given four-momentum k is given by

$$\overline{x}^\mu(k^\mu) = dk^\mu \; . \tag{7}$$

In the case of two-jet events,

$$d = \tau / m_t \; , \tag{8}$$

where m_t is the transverse mass and $\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the longitudinal proper time [41]. For isotropically distributed particle production, the transverse mass is replaced by the mass in Eq. (8), while for the case of three-jet events the relation is more complicated. The second assumption is that the distribution of $x^\mu(k^\mu)$ about its average, $\delta_\Delta(x^\mu(k^\mu) - \overline{x}^\mu(k^\mu))$, is narrower than the proper-time distribution. Then

$$S(x,k) = \int_0^\infty d\tau H(\tau) \delta_\Delta(x - dk) \rho_1(k) \; , \tag{9}$$

where $H(\tau)$ is the longitudinal proper-time distribution, the factor $\delta_\Delta(x-dk)$ describes the strength of the correlations between coordinate- and momentum-space variables and $\rho_1(k)$ is the experimentally measurable single-particle spectrum.

$S(x,k)$ and $\rho_2(k_1,k_2)$ are related in the plane-wave approximation, by the Yano-Koonin formula [28]:

$$\begin{aligned} \rho_2(k_1,k_2) \; &= \; \int d^4x_1 d^4x_2 S(x_1,k_1) S(x_2,k_2) \\ &\quad \cdot \left(1 + \cos\left([k_1 - k_2][x_1 - x_2] \right) \right) \; . \end{aligned} \tag{10}$$

Approximating δ_Δ by a Dirac δ-function, the argument of the cosine becomes $(k_1 - k_2)(\bar{x}_1 - \bar{x}_2) = -0.5(d_1 + d_2)Q^2$. Then R_2 is approximated by

$$R_2(k_1,k_2) = 1 + \lambda \mathrm{Re} \widetilde{H}^2 \left(\frac{Q^2}{2\overline{m}_t} \right) \; , \tag{11}$$

where $\widetilde{H}(\omega) = \int d\tau H(\tau) \exp(i\omega\tau)$ is the Fourier transform of $H(\tau)$. Thus R_2 depends on Q, not its components. R_2 also depends on the average transverse mass of the two pions, $\overline{m}_t$.

Since there is no particle production before the onset of the collision, $H(\tau)$ should be a one-sided distribution. We choose a one-sided Lévy distribution, which has characteristic function [22] (for $\alpha \neq 1$—see, *e.g.*, [21] for the special case $\alpha = 1$)

$$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2} \left(\Delta\tau |\omega| \right)^\alpha \left(1 - i \, \mathrm{sign}(\omega) \tan\left(\frac{\alpha\pi}{2} \right) \right) + i\omega\tau_0 \right] \tag{12}$$

where τ_0 is the proper time of the onset of particle production and $\Delta\tau$ is a measure of the width of $H(\tau)$. Then Eq. (11) becomes

$$\begin{aligned} R_2(Q,\overline{m}_t) \; &= \; \gamma \left[1 + \lambda \cos\left(\frac{\tau_0 Q^2}{\overline{m}_t} + \tan\left(\frac{\alpha\pi}{2} \right) \left(\frac{\Delta\tau Q^2}{2\overline{m}_t} \right)^\alpha \right) \right. \\ &\quad \left. \cdot \exp\left(- \left(\frac{\Delta\tau Q^2}{2\overline{m}_t} \right)^\alpha \right) \right] (1 + \delta Q) \; . \end{aligned} \tag{13}$$

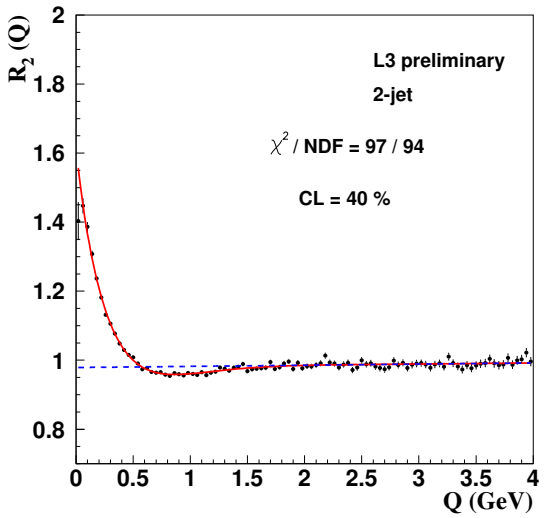

FIG. 2: The Bose-Einstein correlation function R_2 for two-jet events. The curve corresponds to the fit of the one-sided Lévy parametrization, Eq. (14). The dashed line represents the long-range part of the fit, *i.e.*, $\gamma(1 + \delta Q)$.

2. The τ model for average m_t

Before proceeding to fits of Eq. (13), we first consider a simplification obtained by assuming (a) that particle production starts immediately, *i.e.*, $\tau_0 = 0$, and (b) an average m_t-dependence, which is implemented in an approximate way by defining an effective radius, $R = \sqrt{\Delta\tau / (2\overline{m}_t)}$. This results in

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left[(R_a Q)^{2\alpha} \right] \exp\left(-(RQ)^{2\alpha} \right) \right] (1 + \delta Q) \; , \tag{14}$$

where R_a is related to R by

$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2} \right) R^{2\alpha} \; . \tag{15}$$

Fits of Eq. (14) are first performed with R_a as a free parameter. The fit results obtained, for two-jet, three-jet, and all events are listed in Table I and shown in Fig. 2 for two-jet events. They have acceptable confidence levels, describing well the dip in the 0.6–1.5 GeV region, as well as the low-Q peak.

The fit parameters for the two-jet events satisfy Eq. (15). However, those for three-jet and all events do not. We note that the values of the parameters α and R do not differ greatly between 2- and 3-jet samples, the most significant difference appearing to be nearly 3σ for α. However, these parameters are rather highly correlated which makes the simple calculation of the statistical significance of differences in the parameters unreliable.

TABLE I: Results of fits of Eq. (14) for two-jet, three-jet, and all events. The uncertainties are only statistical.

parameter	2-jet	3-jet	all
α	0.42 ± 0.02	0.35 ± 0.01	0.38 ± 0.01
λ	0.67 ± 0.03	0.84 ± 0.04	0.73 ± 0.02
R (fm)	0.79 ± 0.04	0.89 ± 0.03	0.81 ± 0.03
R_a (fm)	0.59 ± 0.03	0.88 ± 0.04	0.81 ± 0.02
δ	0.003 ± 0.002	-0.003 ± 0.002	0.003 ± 0.001
γ	0.979 ± 0.005	1.001 ± 0.005	0.997 ± 0.003
χ^2/DoF	97/94	102/94	98/94
confidence level	40%	27%	37%

Fit results imposing Eq. (15) result, for two-jet events, in values of the parameters which are the same as in the fit with R_a free—only the uncertainties have changed. For three-jet and all events, the imposition of Eq. (15) results in values of α and R closer to those for two-jet events, but the confidence levels are very bad, a consequence of incompatibility with Eq. (15), an incompatibility that is not surprizing given that Eq. (8) is only valid for two-jet events. Therefore, we only consider two-jet events in the rest of this article.

3. *The τ model with m_t dependence*

Fits of Eq. (13) to the two-jet data are performed in several m_t intervals. The quality of the fits is acceptable and the fitted values of the parameters, α, τ_0 and $\Delta\tau$, are stable and within errors independent of m_t, as expected in the τ-model. Their values, $\tau_0 \approx 0$ fm, $\alpha \approx 0.38 \pm 0.05$ and $\Delta\tau \approx 3.5 \pm 0.6$ fm, are consistent with the fit of Eq. (14) in the previous section, including the value of R, which, combined with the average value of m_t (0.563 GeV), corresponds to $\Delta\tau = 3.5$ fm.

IV. THE EMISSION FUNCTION OF TWO-JET EVENTS

Using the τ-model, we now reconstruct the space-time picture of the emitting process for two-jet events. The emission function in configuration space, $S(x)$, is the proper time derivative of the integral over k of $S(x,k)$, which is given by Eq. (9). Approximating δ_Δ by a Dirac δ-function yields

$$S(x) = \frac{d^4n}{d\tau d^3r} = \left(\frac{m_t}{\tau}\right)^3 H(\tau)\rho_1\left(k = \frac{m_t r}{\tau}\right) . \qquad (16)$$

To simplify the reconstruction of $S(x)$ we assume that it can be factorized: $S(r,z,t) = I(r)G(\eta)H(\tau)$, where $I(r)$ is the single-particle transverse distribution, $G(\eta)$ the space-time rapidity distribution, and $H(\tau)$ the proper-time distribution. With the τ-model's strongly correlated phase-space, $\eta = y$ and $r = p_t\tau/m_t$. Hence, $G(\eta) = N_y(\eta)$ and $I(r) = \left(\frac{m_t}{\tau}\right)^3 N_{p_t}(rm_t/\tau)$, where N_y and N_{p_t} are the single-particle inclusive rapidity and p_t distributions, respectively. The factorization of transverse and longitudinal distributions has been checked. The distribution of p_t is, to a good approximation, independent of y [6].

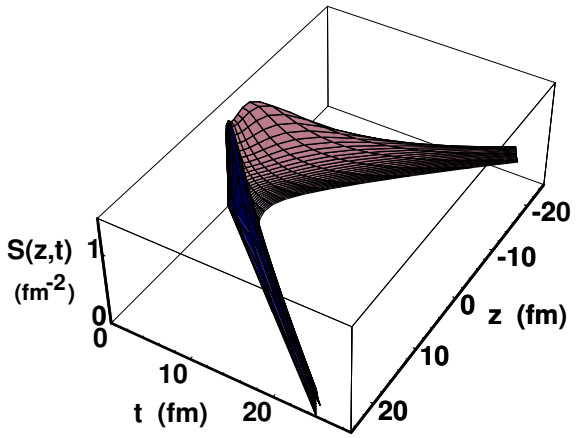

FIG. 3: The temporal-longitudinal part of the source function normalized to the average number of pions per event.

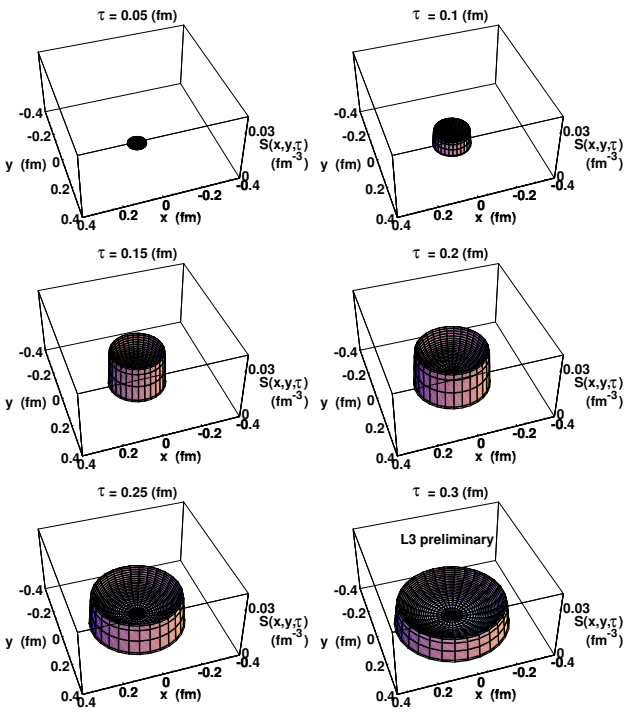

FIG. 4: The transverse source function normalized to the average number of pions per event for various proper times.

With these assumptions and using $H(\tau)$ as obtained from the fit of Eq. (13) together with the inclusive rapidity and p_t distributions [6], the full emission function is reconstructed. Its integral over the transverse distribution is plotted in Fig. 3. It exhibits a "boomerang" shape with a maximum at low t and z but with tails reaching out to very large values of t and z, a feature also observed in hadron-hadron [29] and heavy ion collisions [30]. The transverse part of the emission function, obtained by integrating over z and azimuthal angle, is shown in Fig. 4 for various proper times. Particle production starts immediately, increases rapidly and decreases slowly forming an expanding ring-like structure.

V. DISCUSSION

BEC of all events as well as two- and three-jet events are observed to be well-described by a Lévy parametrization incorporating strong correlations between configuration- and momentum-space. A Lévy distribution arises naturally from a fractal, or from a random walk or anomalous diffusion [31], and the parton shower of the leading log approximation of QCD is a fractal [32–34]. In this case, the Lévy index of stability, α, is related to the strong coupling constant, α_s [35, 36]:

$$3\alpha_s = 2\pi\alpha^2 \,. \tag{17}$$

Assuming (generalized) local parton hadron duality [37–39], one can expect that the distribution of hadrons retains the features of the gluon distribution. Using the value of α found in fits of Eq. (14) for two-jet events we find $\alpha_s = 0.37 \pm 0.04$. This is a reasonable value for a scale of 1–2 GeV, which is where the production of hadrons takes place. For comparison, from τ decay, $\alpha_s(m_\tau \approx 1.8\ \text{GeV}) = 0.35 \pm 0.03$ [40].

It is of particular interest to point out the m_t dependence of the "width" of the source. In Eq. (13) the parameter associated with the width is $\Delta\tau$. Note that it enters Eq. (13) as $\Delta\tau Q^2/\overline{m}_t$. In a Gaussian parametrization the radius R enters the parametrization as R^2Q^2. Our observance that $\Delta\tau$ is independent of $\overline{m}_t$ thus corresponds to $R \propto 1/\sqrt{\overline{m}_t}$ and can be interpreted as confirmation of the observance [23, 24] of such a dependence of the Gaussian radii in 2- and 3-dimensional analyses of Z decays. The lack of dependence of the parameters of Eq. (13) on m_t is in accordance with the τ-model.

Using the BEC fit results and the τ-model, the emission function of two-jet events is reconstructed. Particle production begins immediately after collision, increases rapidly and then decreases slowly, occuring predominantly close to the light cone. In the transverse plane a ring-like structure expands outwards, which is similar to the picture in hadron-hadron interactions but unlike that of heavy ion collisions.

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