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Parametrization of Bose–Einstein correlations and reconstruction of the space–time evolution of pion production in e^+e^- annihilation

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ABSTRACT

A parametrization of the Bose–Einstein correlation function of pairs of identical pions produced in hadronic e^+e^- annihilation is proposed within the framework of a model (the τ-model) in which space–time and momentum space are very strongly correlated. Using information from the Bose–Einstein correlations as well as from single-pion spectra, it is then possible to reconstruct the space–time evolution of pion production.

1. Introduction

In particle and nuclear physics, intensity interferometry provides a direct experimental method for the determination of sizes, shapes and lifetimes of particle-emitting sources (for reviews see, e.g., [1–5]). In particular, boson interferometry provides a powerful tool for the investigation of the space–time structure of particle production processes, since Bose–Einstein correlations (BEC) of two identical bosons reflect both geometrical and dynamical properties of the particle radiating source. Given the point-like nature of the underlying interaction, e^+e^- annihilation provides an ideal environment to study these properties in multiparticle production by quark fragmentation.

2. Bose–Einstein correlation function

The two-particle correlation function of two particles with four-momenta p_1 and p_2 is given by the ratio of the two-particle number density, $\rho_2(p_1, p_2)$, to the product of the two single-particle number densities, $\rho_1(p_1)\rho_1(p_2)$. Being only interested in the correlation R_2 due to Bose–Einstein interference, the product of single-particle densities is replaced by $\rho_0(p_1, p_2)$, the two-particle density that would occur in the absence of Bose–Einstein correlations:

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)}. \tag{1}$$

Since the mass of the two identical particles of the pair is fixed to the pion mass, the correlation function is defined in six-dimensional momentum space. Since Bose–Einstein correlations can be large only at small four-momentum difference $Q = \sqrt{-(p_1 - p_2)^2}$, they are often parametrized in terms of this one-dimensional distance measure. There is no reason, however, to expect the hadron source for jet fragmentation to be spherically symmetric. Recent investigations, using the Bertsch–Pratt parametrization [6,7], have, in fact, found an elongation of the source along the jet axis [8–12] in the longitudinal center-of-mass (LCMS) frame [13]. While this effect is well established, the elongation is actually only about 20%, which suggests that a parametrization in terms of the single variable Q, may be a good approximation.

There have been indications that the size of the source, as measured using BEC, depends on the transverse mass, $m_t = \sqrt{m^2 + p_t^2} = \sqrt{E^2 - p_z^2}$, of the pions [12,14,15]. It has been shown [16,17] that such a dependence can be understood if the produced pions satisfy, approximately, the (generalized) Bjorken–Gottfried condition [18–23], whereby the four-momentum of a produced particle and the space–time position at which it is produced are linearly related: $x = dp$. Such a correlation between space–time and momentum–energy is also a feature of the Lund string model as incorporated in JETSET [24], which is very successful in describing detailed features of the hadronic final states of e^+e^- annihilation. Recently, experimental support for this strong correlation has been found [12].

A model which predicts both a Q- and an m_t-dependence while incorporating the Bjorken–Gottfried condition is the so-called τ-model [25]. In this article we develop this model further and apply it to the reconstruction of the space–time evolution of pion production in e^+e^- annihilation.

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3. BEC in the τ model

In the τ-model, it is assumed that the average production point in the overall center-of-mass system, $\bar{x} = (\bar{t}, \bar{r}_x, \bar{r}_y, \bar{r}_z)$, of particles with a given four-momentum p is given by

$$\bar{x}(p) = a\tau p. \tag{2}$$

In the case of two-jet events, $a = 1/m_t$ where m_t is the transverse mass and $\tau = \sqrt{\bar{t}^2 - \bar{r}_z^2}$ is the longitudinal proper-time.[2] For isotropically distributed particle production, the transverse mass is replaced by the mass in the definition of a and τ is the proper-time. In the case of three-jet events the relation is more complicated.

The correlation between coordinate space and momentum space variables is described by the distribution of $x(p)$ about its average by $\delta_\Delta(x(p) - \bar{x}(p)) = \delta_\Delta(x - a\tau p)$. The emission function of the τ-model is then given by [25]

$$S(x, p) = \int_0^\infty \mathrm{d}\tau\, H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p), \tag{3}$$

where $H(\tau)$ is the (longitudinal) proper-time distribution and $\rho_1(p)$ is the experimentally measurable single-particle momentum spectrum, both $H(\tau)$ and $\rho_1(p)$ being normalized to unity.

The two-pion distribution, $\rho_2(p_1, p_2)$, is related to $S(x, p)$, in the plane-wave approximation, by the Yano–Koonin formula [26]:

$$\rho_2(p_1, p_2) = \int \mathrm{d}^4x_1\, \mathrm{d}^4x_2\, S(x_1, p_1)S(x_2, p_2) \times \bigl\{1 + \cos\bigl[(p_1 - p_2)(x_1 - x_2)\bigr]\bigr\}. \tag{4}$$

Assuming that the distribution of $x(p)$ about its average is much narrower than the proper-time distribution, Eq. (4) can be evaluated in a saddle-point approximation. Approximating the function δ_Δ by a Dirac delta function yields the same result. Thus the integral of Eq. (3) becomes

$$\int_0^\infty \mathrm{d}\tau\, H(\tau)\rho_1\left(\frac{x}{a\tau}\right), \tag{5}$$

and the argument of the cosine in Eq. (4) becomes

$$(p_1 - p_2)(\bar{x}_1 - \bar{x}_2) = -0.5(a_1\tau_1 + a_2\tau_2)Q^2. \tag{6}$$

Substituting Eqs. (5) and (6) in Eq. (4) leads to the following approximation of the two-particle Bose–Einstein correlation function:

$$R_2(Q, a_1, a_2) = 1 + \mathrm{Re}\,\tilde{H}\left(\frac{a_1 Q^2}{2}\right)\tilde{H}\left(\frac{a_2 Q^2}{2}\right), \tag{7}$$

where $\tilde{H}(\omega) = \int \mathrm{d}\tau\, H(\tau)\exp(i\omega\tau)$ is the Fourier transform of $H(\tau)$.

This formula simplifies further if R_2 is measured with the restriction

$$a_1 \approx a_2 \approx \bar{a}. \tag{8}$$

In that case, R_2 becomes

$$R_2(Q, \bar{a}) = 1 + \mathrm{Re}\,\tilde{H}^2\left(\frac{\bar{a}Q^2}{2}\right). \tag{9}$$

Thus for a given average of a of the two particles, R_2 is found to depend only on the invariant relative momentum Q. Further, the model predicts a specific dependence on $\bar{a}$, which for two-jet events is a specific dependence on $\bar{m}_t$.[3]

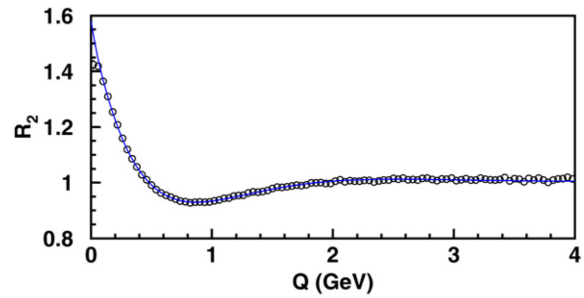

Fig. 1. The Bose–Einstein correlation function R_2 for events generated by PYTHIA. The curve corresponds to a fit of the one-sided Lévy parametrization, Eq. (13).

Since there is no particle production before the onset of the collision, $H(\tau)$ should be a one-sided distribution. We choose a one-sided Lévy distribution, which has the characteristic function (Fourier transform) [27] (for $\alpha \neq 1$)[4]

$$\tilde{H}(\omega) = \exp\left\{-\frac{1}{2}\left(\Delta\tau|\omega|\right)^\alpha \times \left[1 - i\,\mathrm{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right] + i\omega\tau_0\right\}, \tag{10}$$

where the parameter τ_0 is the proper-time of the onset of particle production and $\Delta\tau$ is a measure of the width of the proper-time distribution. Using this characteristic function in Eq. (9) yields

$$R_2(Q, \bar{a}) = 1 + \cos\left[\bar{a}\tau_0 Q^2 + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\bar{a}\Delta\tau Q^2}{2}\right)^\alpha\right] \times \exp\left[-\left(\frac{\bar{a}\Delta\tau Q^2}{2}\right)^\alpha\right], \tag{11}$$

which for two-jet events is

$$R_2(Q, \bar{m}_t) = 1 + \cos\left[\frac{\tau_0 Q^2}{\bar{m}_t} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2\bar{m}_t}\right)^\alpha\right] \times \exp\left[-\left(\frac{\Delta\tau Q^2}{2\bar{m}_t}\right)^\alpha\right]. \tag{12}$$

We now consider a simplification of the equation obtained by assuming (a) that particle production starts immediately, i.e., $\tau_0 = 0$, and (b) an average a-dependence, which is implemented in an approximate way by defining an effective radius, $R = \sqrt{\bar{a}\Delta\tau/2}$, which for 2-jet events becomes $R = \sqrt{\Delta\tau/(2\bar{m}_t)}$. This results in:

$$R_2(Q) = 1 + \cos\bigl[(R_a Q)^{2\alpha}\bigr]\exp\bigl[-(RQ)^{2\alpha}\bigr], \tag{13}$$

where R_a is related to R by

$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right)R^{2\alpha}. \tag{14}$$

To illustrate that Eq. (13) can provide a reasonable parametrization, we show in Fig. 1 a fit of Eq. (13) with R_a a free parameter to Z-boson decays generated by PYTHIA [29] with BEC simulated by the BE_{32} algorithm [30] as tuned to L3 data [31]. In particular, it describes well the dip in R_2 below unity in the Q-region 0.5–1.5 GeV, unlike the usual Gaussian or exponential parametrizations. While generalizations [32] of the Gaussian by an Edgeworth expansion and of the exponential by a Laguerre expansion can

[2] The terminology 'longitudinal' proper-time and 'transverse' mass seems customary in the literature even though their definitions are analogous $\tau = \sqrt{\bar{t}^2 - \bar{r}_z^2}$ and $m_t = \sqrt{E^2 - p_z^2}$.

[3] In the initial formulation of the τ-model this dependence was averaged over [25] due to the lack of m_t dependent data at that time.

[4] For the special case $\alpha = 1$, see, e.g., Ref. [28].

describe the dip, they require more additional parameters than Eq. (13). Recently the L3 Collaboration has presented preliminary results showing that Eq. (13) describes their data on hadronic Z decay [33].

4. The emission function of two-jet events

Within the framework of the τ-model, we now show how to reconstruct the space–time picture of pion emission. We restrict ourselves to two-jet events where we know what a is, namely $a = 1/m_t$. The emission function in configuration space, $S_x(x)$, is the proper-time derivative of the integral over p of $S(x,p)$, which in the τ-model is given by Eq. (3). Approximating δ_Δ by a Dirac delta function, we find

$$S_x(x) = \frac{1}{\bar{n}}\frac{d^4 n}{d\tau\, d^3 x} = \left(\frac{m_t}{\tau}\right)^3 H(\tau)\rho_1\left(p = \frac{m_t x}{\tau}\right), \tag{15}$$

where n and $\bar{n}$ are the number and average number of pions produced, respectively.

Given the symmetry of two-jet events, S_x does not depend on the azimuthal angle, and we can write it in cylindrical coordinates as

$$S_x(r,z,t) = P(r,\eta)H(\tau), \tag{16}$$

where η is the space–time rapidity. With the strongly correlated phase-space of the τ-model, $\eta = y$ and $r = p_t\tau/m_t$. Consequently,

$$P(r,\eta) = \left(\frac{m_t}{\tau}\right)^3 \rho_{p_t,y}(rm_t/\tau,\eta), \tag{17}$$

where $\rho_{p_t,y}$ is the joint single-particle distribution of p_t and y.

The reconstruction of S_x is simplified if $\rho_{p_t,y}$ can be factorized in the product of the single-particle p_t and rapidity distributions, i.e., $\rho_{p_t,y} = \rho_{p_t}(p_t)\rho_y(y)$. Then Eq. (17) becomes

$$P(r,\eta) = \left(\frac{m_t}{\tau}\right)^3 \rho_{p_t}(rm_t/\tau)\rho_y(\eta). \tag{18}$$

The transverse part of the emission function is obtained by integrating over z as well as azimuthal angle. Pictures of this function evaluated at successive times would together form a movie revealing the time evolution of particle production in 2-jet events in e^+e^- annihilation.

To summarize: Within the τ-model, $H(\tau)$ is obtained from a fit of Eq. (12) to the Bose–Einstein correlation function. From $H(\tau)$ together with the inclusive distribution of rapidity and p_t, the full emission function in configuration space, S_x, can then be reconstructed.

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