QCD

(experimental)

(mostly e^+e^-)

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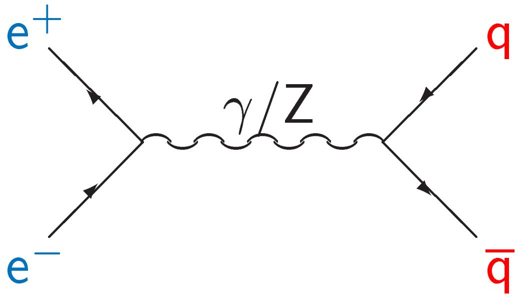
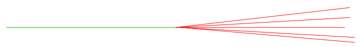
Why mostly e^+e^-?



- Well defined production process
  Hadronic activity only in final state

- At high energy,
  hard partons (q, $\bar{q}$, or g) $\Longrightarrow$ collimated jets

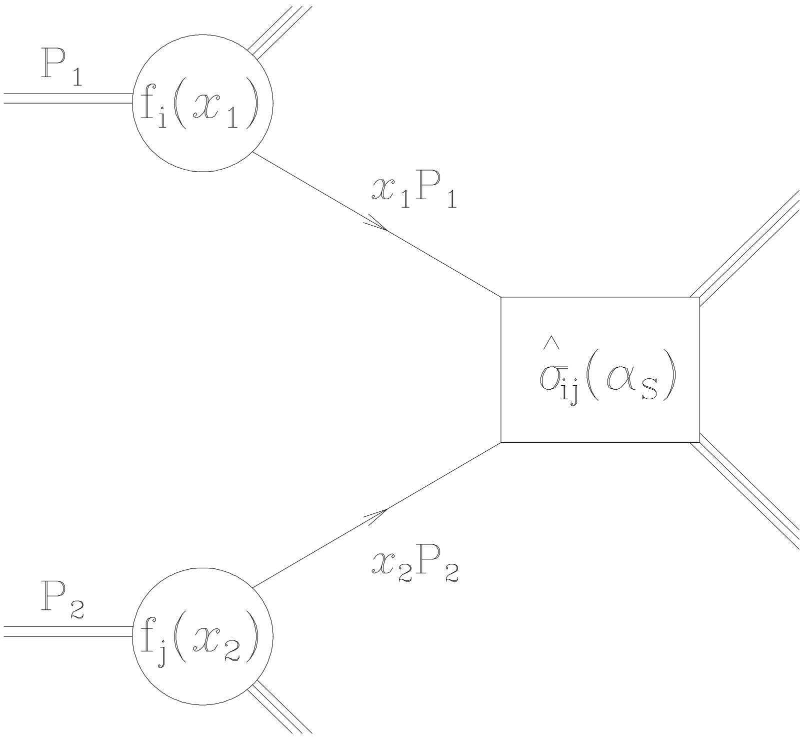
q,$\bar{q}$,g  jet of hadrons

- $\Longrightarrow$ clean tests of QCD
  Can extract physics with a minimum of assumptions

Why mostly e^+e^-?

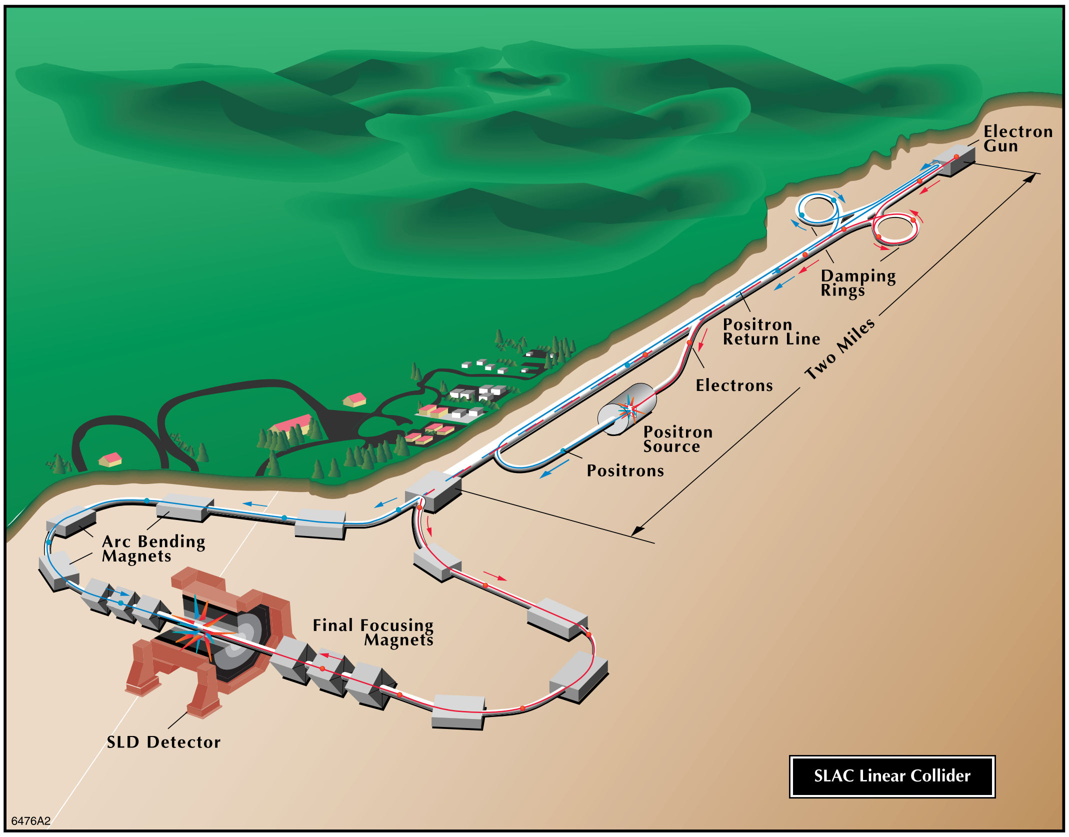
By contrast, ep or pp are messy because of proton structure

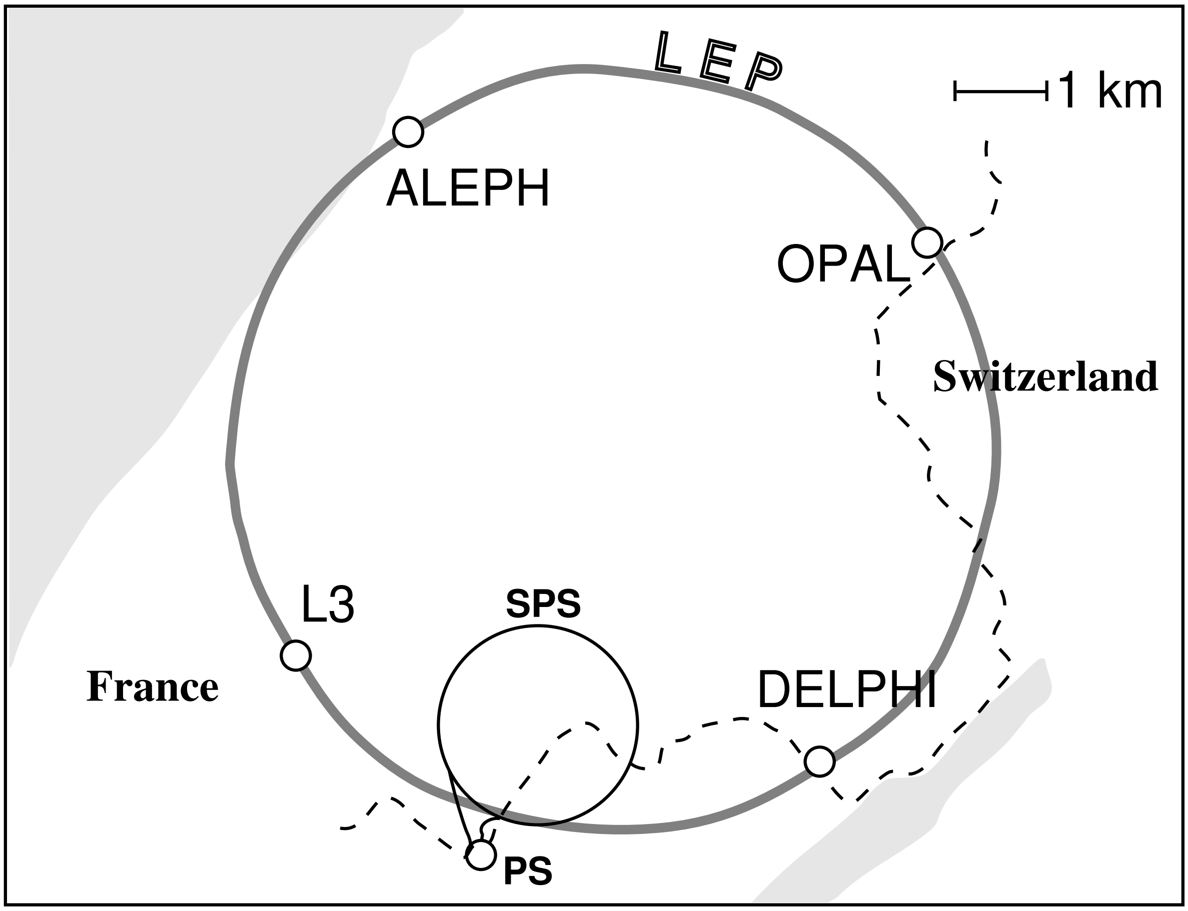
- initial state involves hadron(s)
  Need to know parton distribution functions



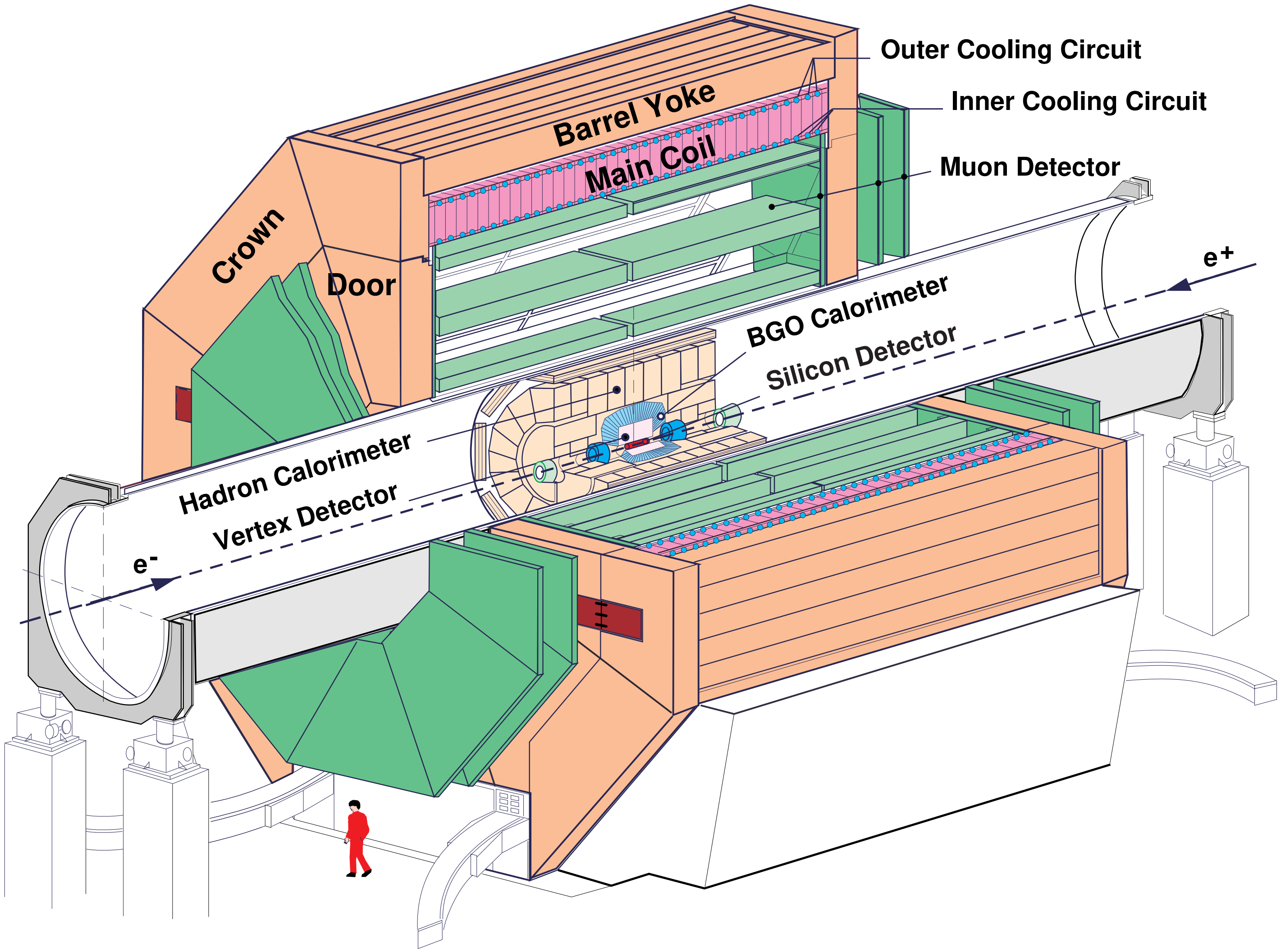
- hard jet and 'spectator' jet may overlap
  hard jets may be influenced by 'spectator' jets
  - colour field between hard jet and spectator jet
  - overlap of jets
    $\Longrightarrow$ ambiguous assignment of particles to jets

Today's High-Energy e^+e^- Machines





A "Typical" Detector

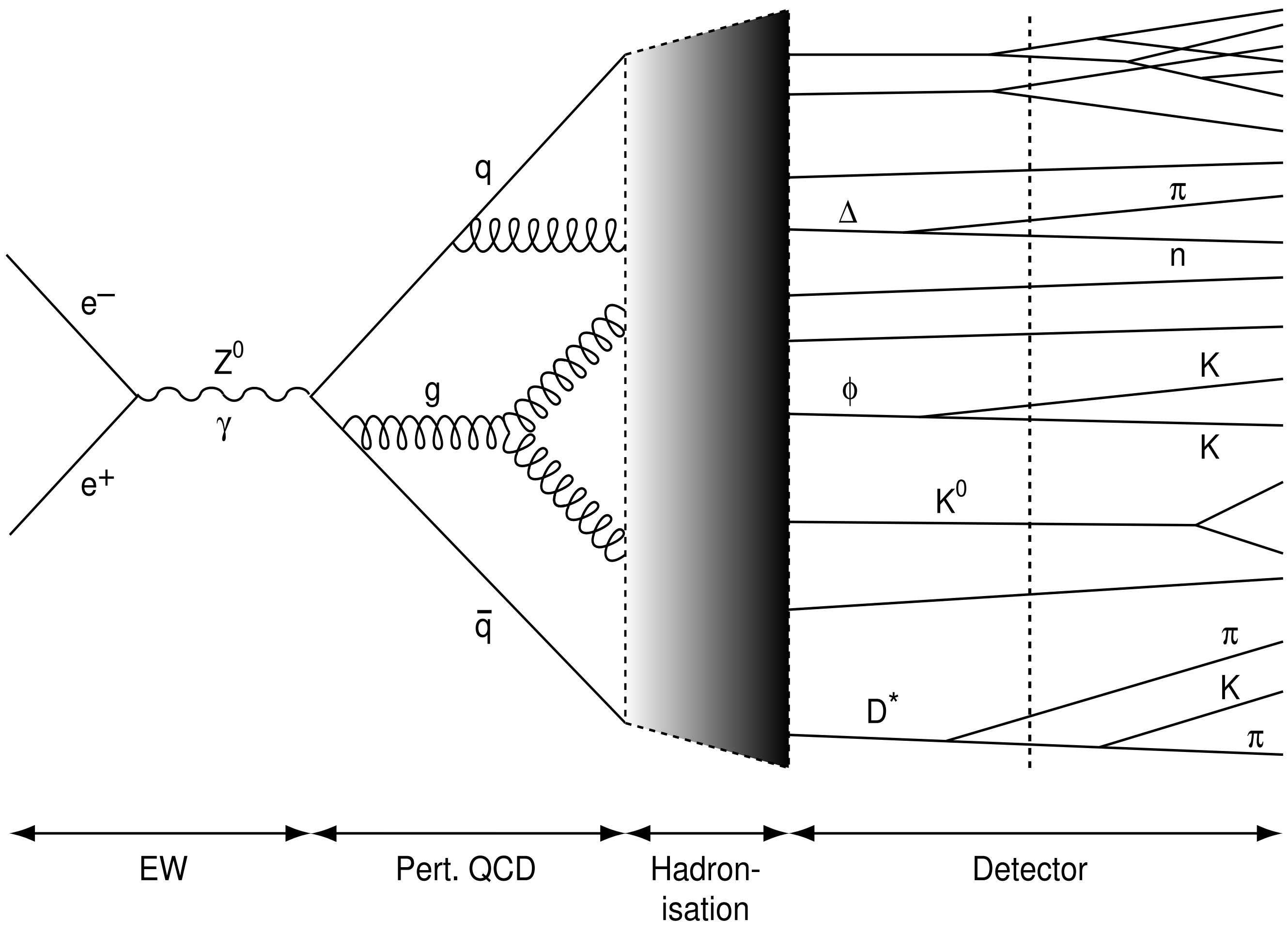


L3

Outline

- Introduction
- QCD – some basic tests
  - Does g have spin 1 ?
  - Is SU(3) the colour group ?
  - Does α_s run ?
    - Ways to measure α_s
- The Parton Shower and perturbative QCD results
- Softer aspects of QCD
  - quark/gluon fragmentation differences
  - multiplicities
- Conclusions

QCD in e^+e^- collisions



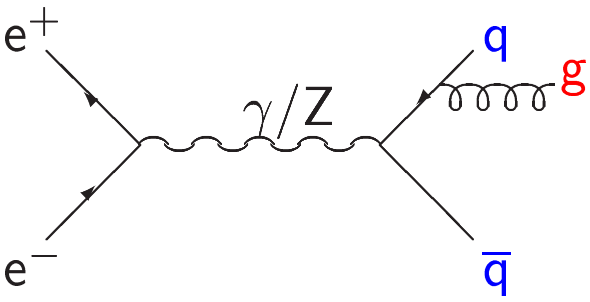
QCD

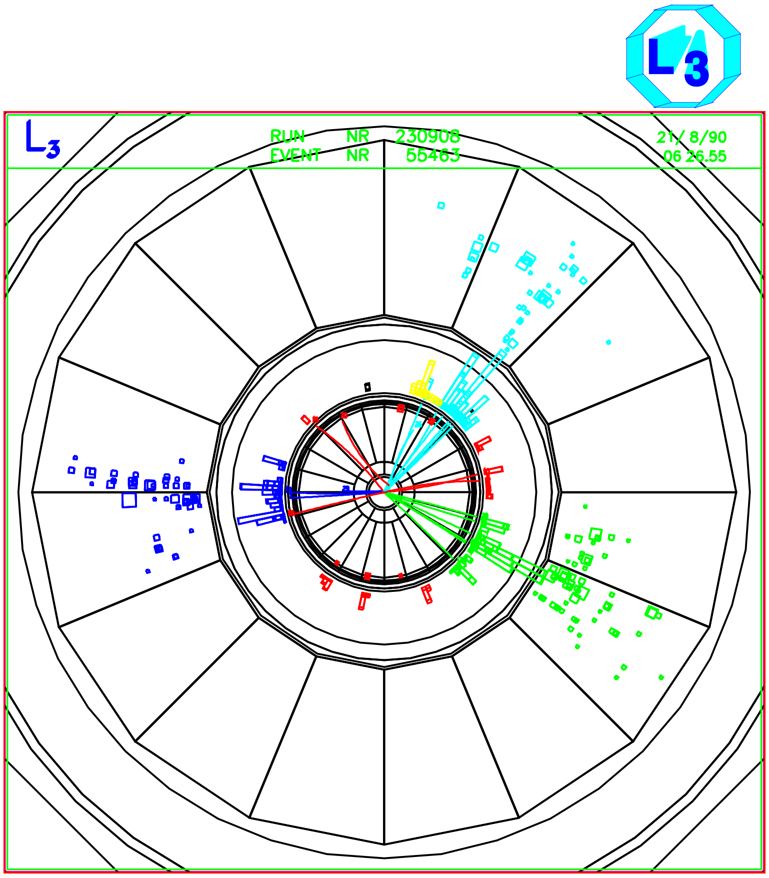
- $e^+e^- \to q\bar{q}$ (Electro-Weak)
  $\to$ 2 jets (∼kinematics)

  jet ≡ roughly collimated group of particles

- QCD: q $\to$ qg $\to$ 3-jet events
  (also 4-, 5-, ... jet events)

  seen long ago at PETRA (Germany not Jordan)
  (not that long ago)





Jet finding — ‘classical’ methods

When you’re a jet,
You’re a jet all the way.
From your first cigarette,
to your last dying day.

1. start with all particles
2. compute ‘distance’ y_{ij} between all pairs of particles i, j
3. for pair with smallest y_{ij},
   - if $y_{ij} < y_{\text{cut}}$
     - combine particles i, j into one “particle”
     - iterate
   - otherwise stop

Resulting “particles” called jets.

Need

1. def. of ‘distance’, y_{ij}
2. a value for y_{cut}
3. prescription for combining particles

Jet finding — 'classical' methods

- def. of 'distance' y_{ij}

algorithm	y_{ij}
mass	$\frac{(p_i+p_j)^2}{s}$
JADE	$\frac{2E_iE_j}{s}(1-\cos\theta_{ij})$
Geneva	$\frac{4}{9}\frac{2E_iE_j}{(E_i+E_j)^2}(1-\cos\theta_{ij})$
Durham (k_t)	$\frac{2\min(E_i^2,E_j^2)}{s}(1-\cos\theta_{ij})$
for massless particles, JADE = mass	
Durham	$sy_{ij} = p_\mathrm{t}^2$ of i w.r.t. j to 2nd order in θ_{ij} (if $E_i < E_j$)
Expt.: use E_vis^2 instead of s, $E_\mathrm{vis} = \sum E_i$	

- def. of combining i, $j \to k$

scheme	E	$E0$	P
$E_k =$ $\vec{p}_k =$	$E_i + E_j$ $\vec{p}_i + \vec{p}_j$	$E_i + E_j$ $E_{ij}\frac{\vec{p}_i+\vec{p}_j}{\lvert\vec{p}_i+\vec{p}_j\rvert}$	$\lvert\vec{p}_{ij}\rvert$ $\vec{p}_i + \vec{p}_j$
		produce massless "particles"	
$P0$ – like P, but E_vis recomputed at each step : $E_\mathrm{vis} = \sum E_k$			

- Most common are JADE (E or $E0$) and Durham (E)

Jet finding — 'classical' methods

algorithm	y_{ij}
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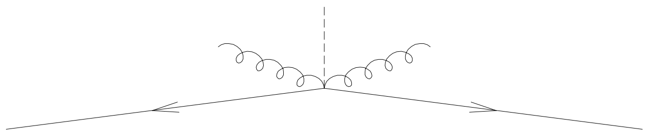
Durham (k_t)	$\frac{2\min(E_i^2,E_j^2)}{s}(1-\cos\theta_{ij})$

Problems:

- soft particles
  - JADE: 3 jets
  - Geneva, Durham: 2 jets



- hadronisation corrections to jet fractions R_2, R_3, ...
  (R_n = fraction of events that are n-jet)
  - JADE $E0$, Durham: small
  - Geneva: large
- Durham is most used

Hadronisation corrections to R_n

Bethke *et al.*, Nucl.Phys.B 370(1992)310

Jet finding — Cambridge

An improvement on Durham for soft particles:

Same def. of y: $$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{s}(1 - \cos\theta_{ij})$$

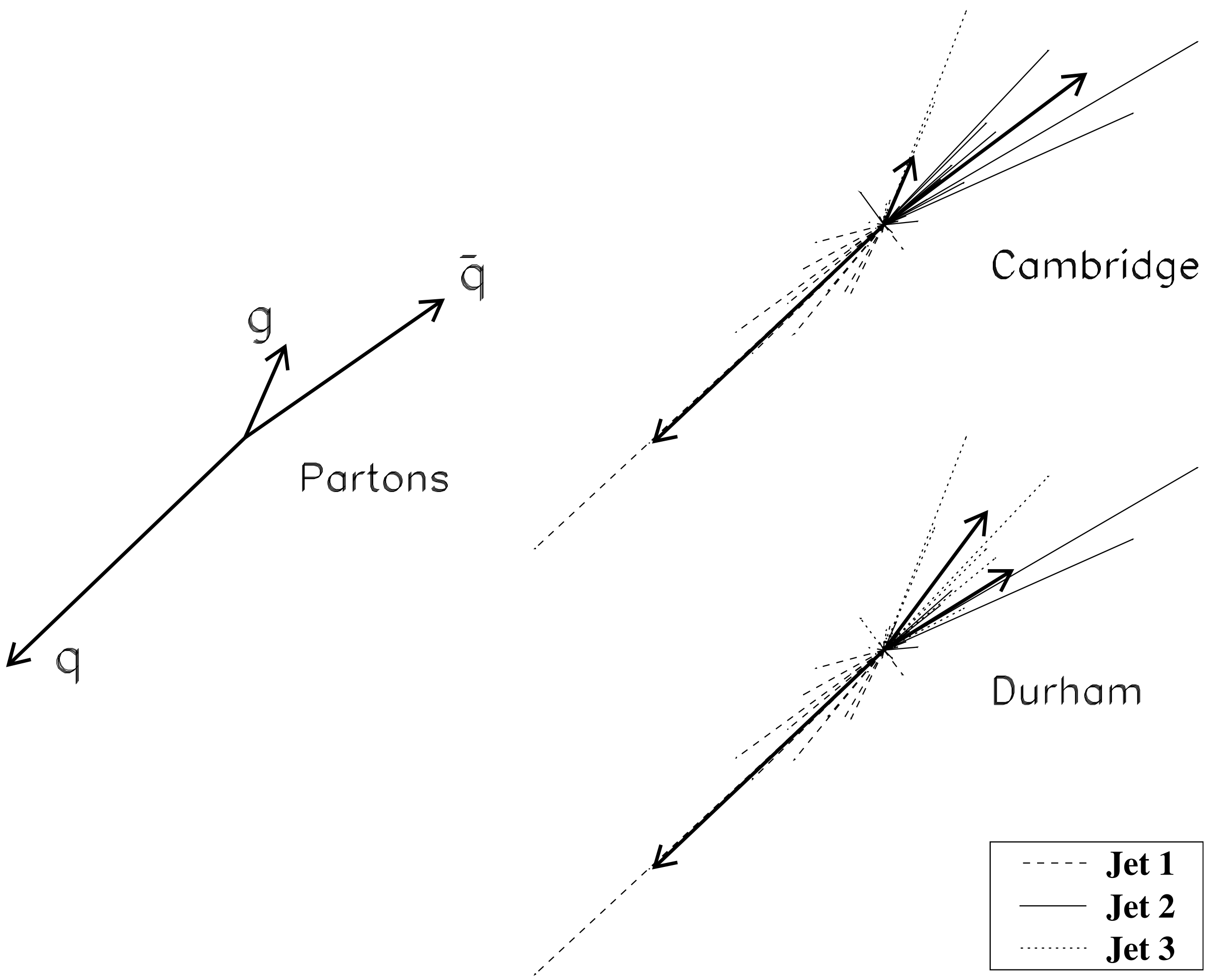
Define ordering variable: $$v_{ij} = 2(1 - \cos\theta_{ij})$$

1. start with all particles
2. compute v_{ij} between all pairs of particles i, j
3. for pair with smallest v_{ij},
   - if $y_{ij} < y_{\text{cut}}$:
     - combine particles i, j into one "particle" (E-scheme)
     - iterate
   - if $y_{ij} \geq y_{\text{cut}}$:
     - call particle (i or j) with smaller E a jet and remove it from list of particles ("soft freezing")
     - iterate

Note: Durham = Cambridge with $v_{ij} = y_{ij}$

Jet finding — Cambridge

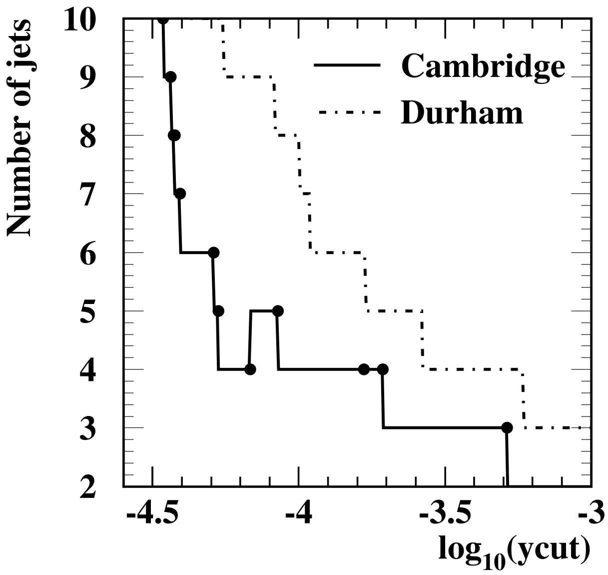
Soft freezing prevents extra particles being assigned to it.



Jet momenta are closer to parton momenta.

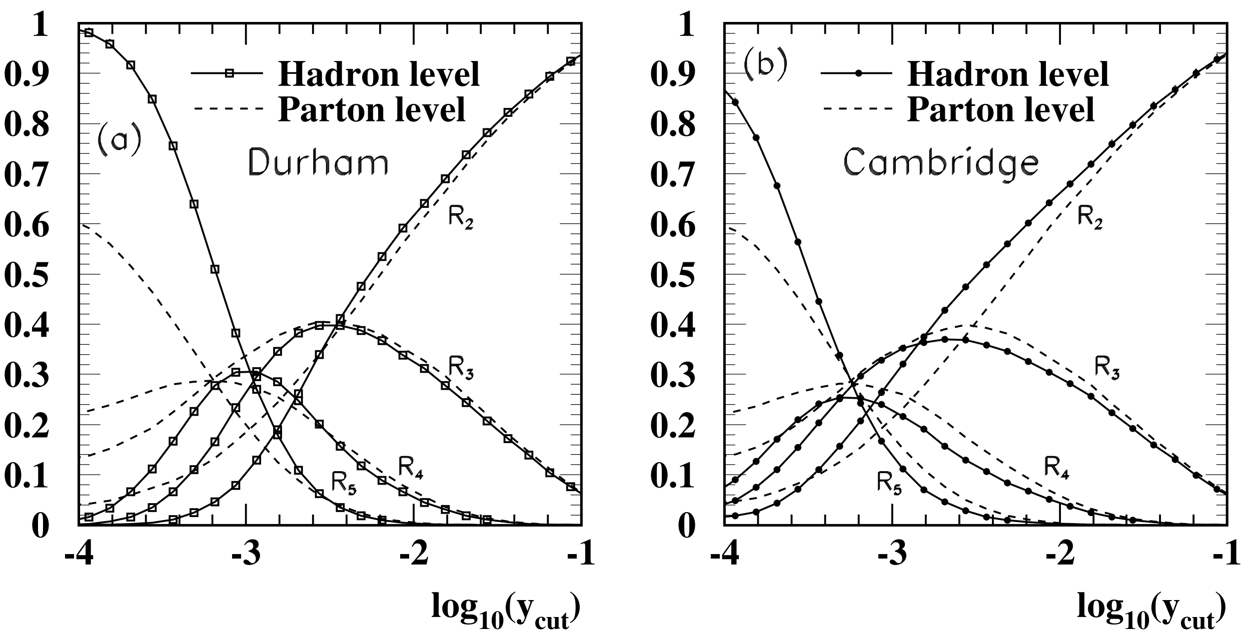
Jet finding — Cambridge

But number of jets $vs.$ $y_{\rm cut}$ is sometimes 'strange':



Disjoint intervals of $y_{\rm cut}$ can have same jet multiplicity and differing jet momenta.

And hadronisation corrections for jet fractions are larger for $y_{\rm cut} \gtrsim 0.003$ $(\log y_{\rm cut} \gtrsim -2.5)$



CERN-EP/98-043

Jet finding — other

LUCLUS

- Similar to Durham or Geneva except
  - distance,

$$d_{ij}^2 = \frac{4|\vec{p}_i|^2|\vec{p}_j|^2 \sin^2(\theta_{ij}/2)}{(|\vec{p}_i| + |\vec{p}_j|)^2} \approx \left(\frac{2E_iE_j \sin(\theta_{ij}/2)}{E_i + E_j}\right)^2$$

  - for small θ_{ij}, $d_{ij} \propto p_\mathrm{t}$ of i or j w.r.t. their sum
    Durham $\sqrt{y_{ij}} \propto p_\mathrm{t}$ of softer particle w.r.t. harder particle
  - reassignment of particles to other jets,
    if after combining, a particle in the combined jet is closer to a different jet

- Disadvantage: not easily calculable in pert. QCD

Jet finding — other

Cone – mainly used at TeVatron

- Jet $=$ all particles i such that
  - angle w.r.t. cone axis $< R$
  - cone axis is along $\sum \vec{p}_i$
  - $E_{\text{jet}} > \epsilon$

- Disadvantages:
  - cones may overlap (same particle in more than 1 jet)
  - some particles may not be in *any* jet
    (OK for TeVatron or HERA – spectator jets)

We can find JETS

jet:	ME	jet, jette, get
	OFr	get, giet, ject
	L	jactus, pp. of jacere

jet $\approx$ perturbative QCD parton

What do we learn from them?

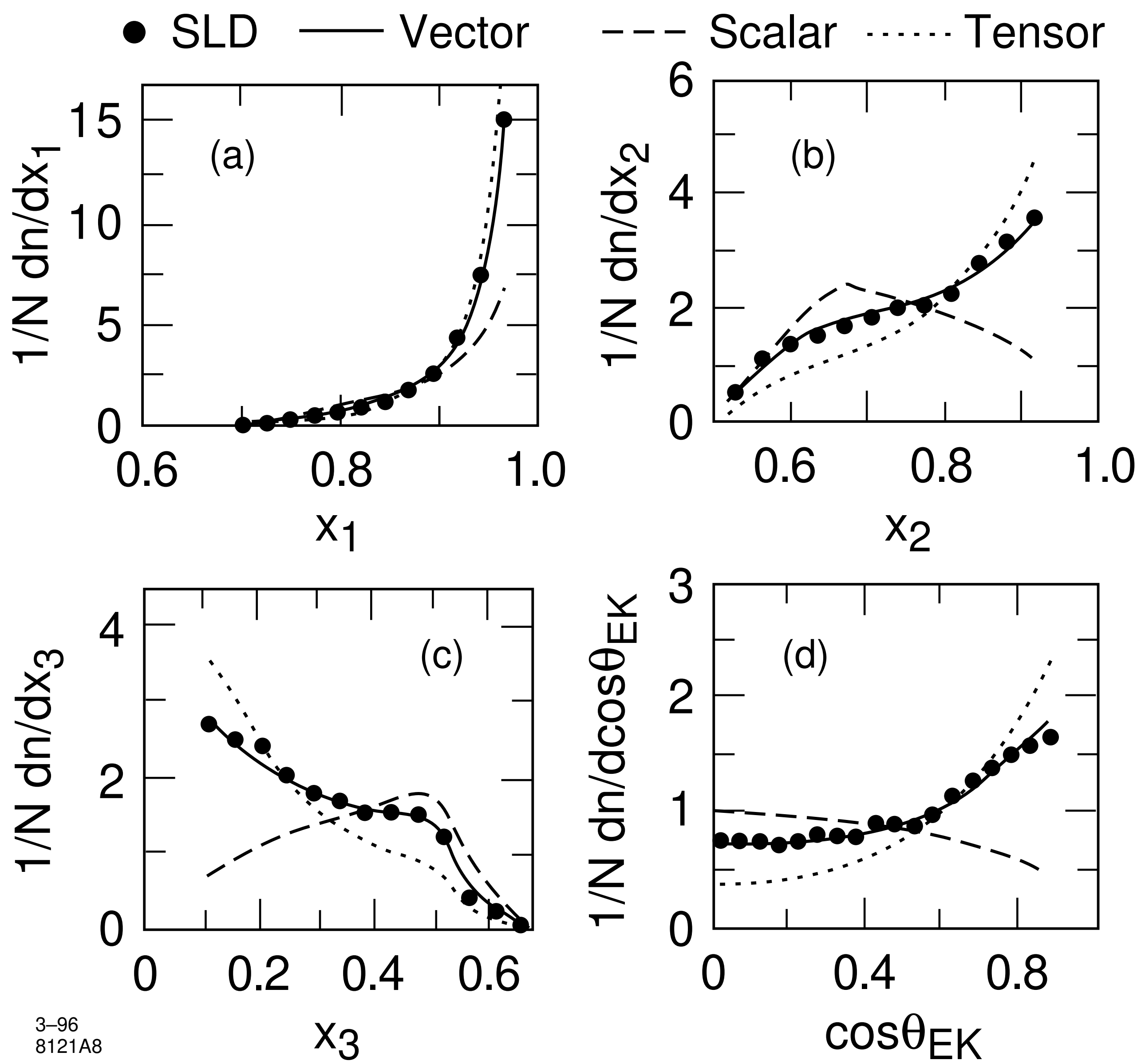
Basic tests of QCD

- Does g have spin 1 ?
- Is SU(3) the colour group ?
- Does α_s run ?

Does g have spin 1 ?

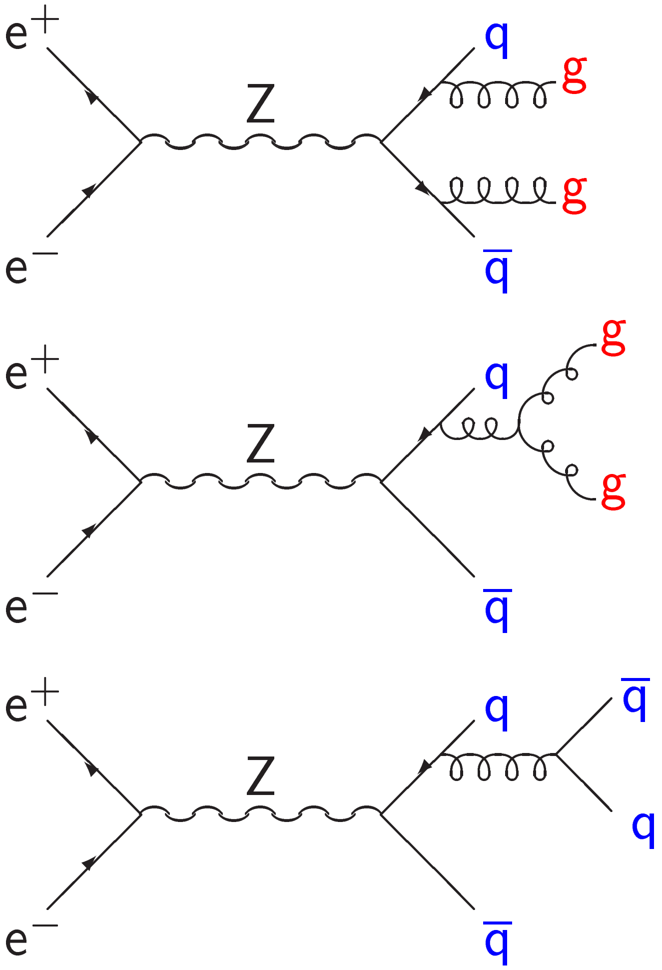
Energy and Ang. dists. of 3-jet events:
$x_i = \frac{2E_i}{E_{\text{cms}}}$, $E_1 > E_2 > E_3$
θ_{EK} = angle between jets 2,3 in rest frame of 1,2

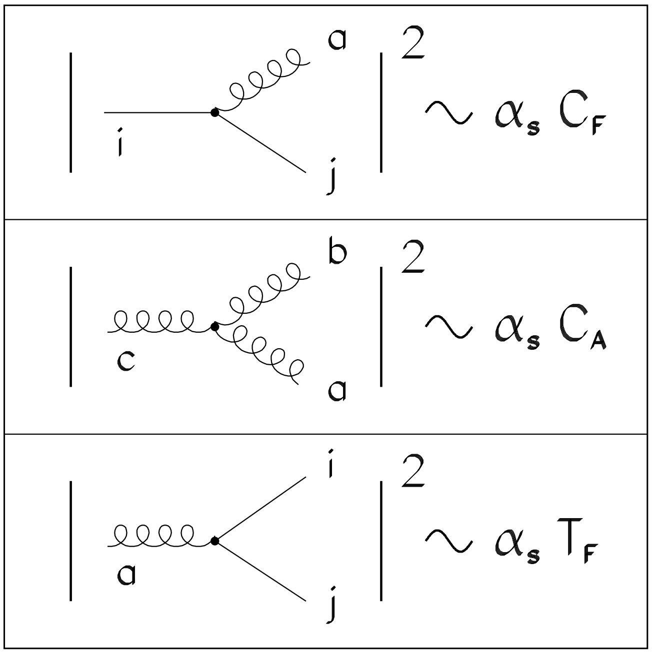
- Not spin 0, consistent with spin 1 – PETRA
- Not spin 2 – SLD: (PRD 55(1997)2533)



Is SU(3) the colour group ?

ALEPH (CERN-PPE/97-002): in 4-jet events,



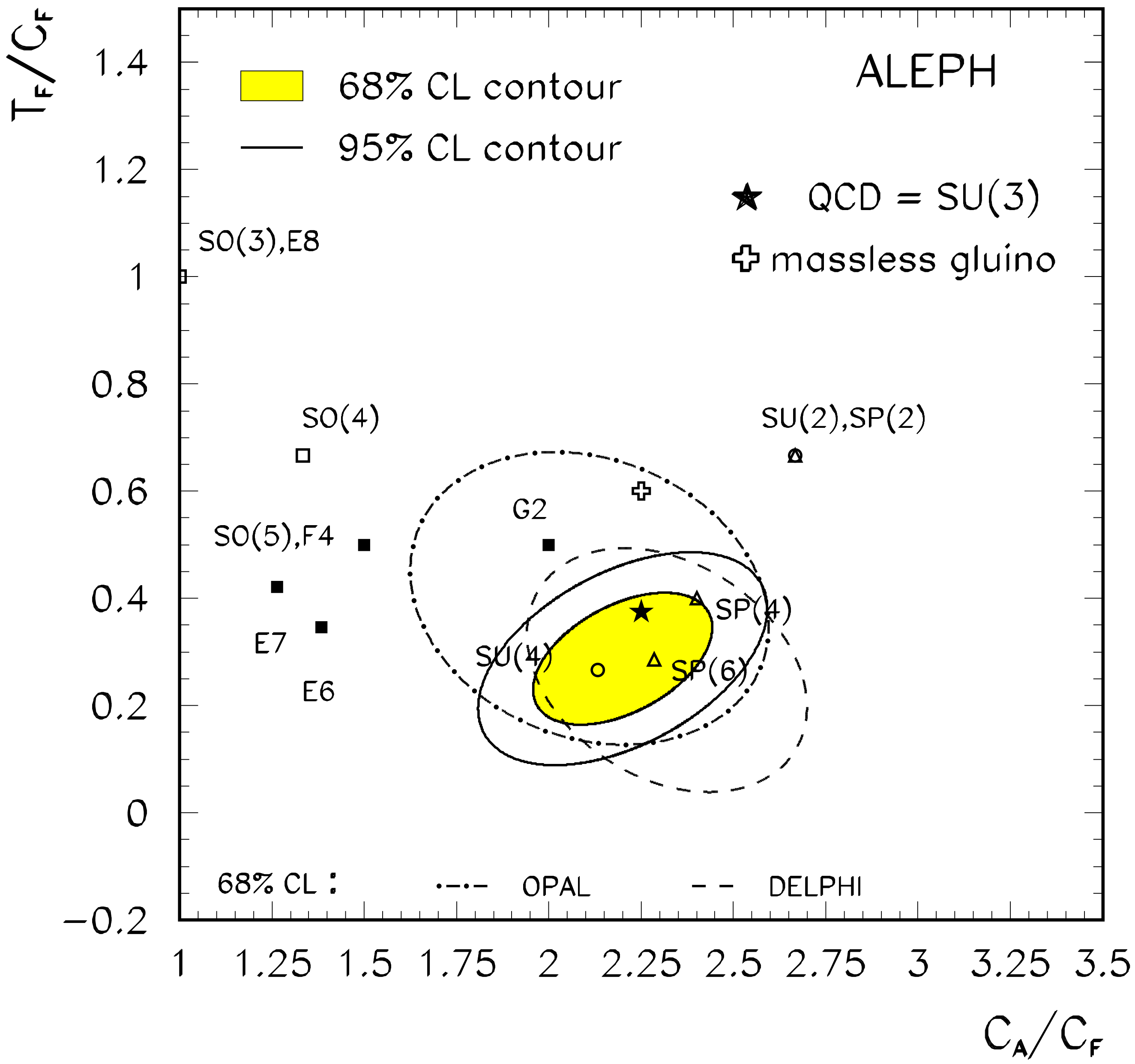


4-jet diagrams are related to the Casimir operators (colour factors) of colour group of QCD: C_{F}, C_{A}, T_{F}

From relative frequencies of 4-jet diagrams, can measure C_{F}, C_{A}, T_{F}

Jet E and Ang. dists. of diagrams differ

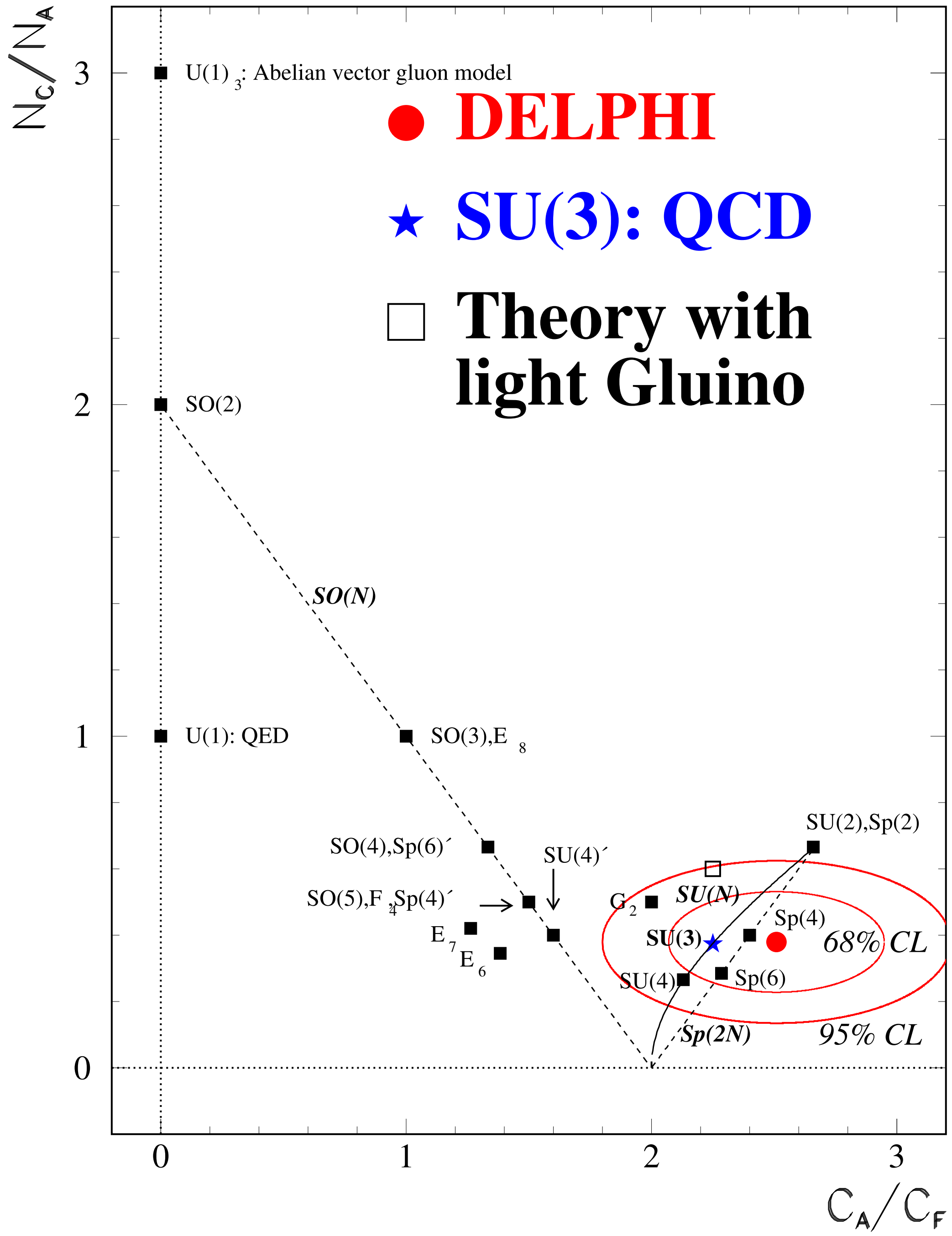
Colour Factors



Results agree with SU(3)

Colour Factors

DELPHI has done a similar analysis (EPS97 236):



Results agree with SU(3)
(and with ALEPH, but errors are larger)

Does α_s run ?

α_s is called the strong coupling constant
But it is *NOT* a constant!
α_s "runs", *i.e.*, depends on the scale Q^2
How α_s "runs", is predicted by QCD.

To test the running:

- Measure α_s at different scales Q^2
- Check if $\alpha_s(Q^2)$ depends on Q^2 as predicted by QCD

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left\{ 1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln(\ln(Q^2/\Lambda^2))}{\ln(Q^2/\Lambda^2)} + \ldots \right\}$$

$$\beta_0 = 11 - \frac{2}{3} n_f \qquad\qquad \beta_1 = 102 - \frac{38}{3} n_f$$

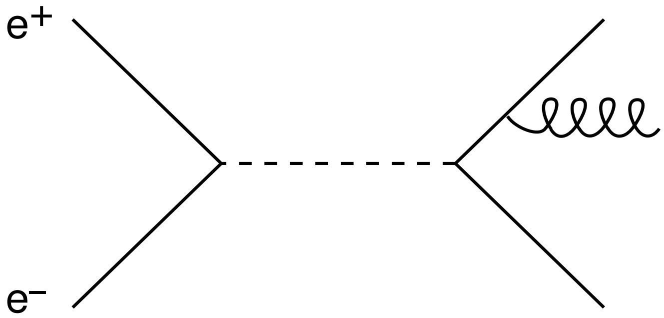
where n_f = the number of flavours

N.B. n_f depends on available E
– at low E, heavy quarks do not contribute

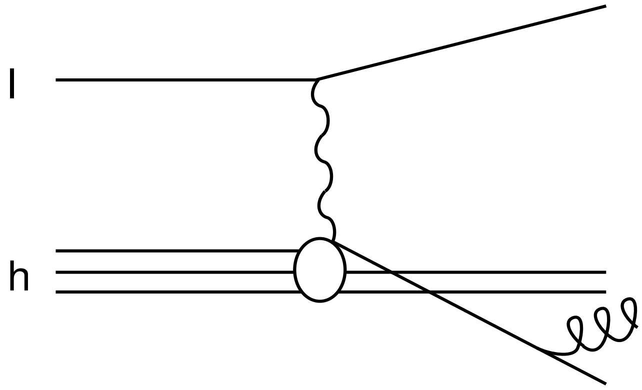
Measuring α_s

Can measure α_s practically anywhere you have hadrons

e^+e^-

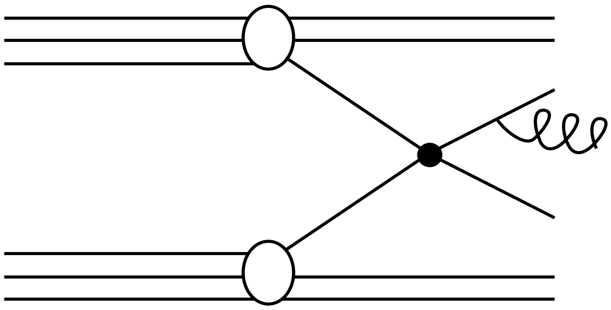


DIS

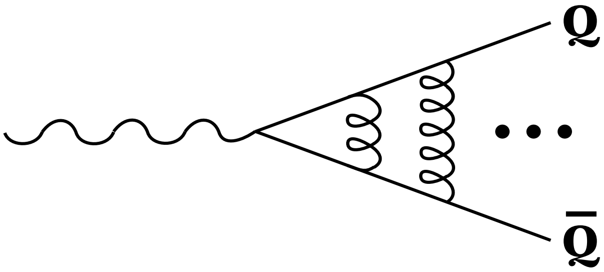


or structure of h

hh



Heavy quarkonia



Measuring α_s

- Choose an OBSERVABLE f
  - IR- and collinear-safe

- Choose RENORMALISATION SCHEME
  - generally $\overline{\text{MS}}$

- Choose RENORMALISATION SCALE (μ^2)
  - to minimize importance of higher-order terms
  - no consensus, much controversy
  - should be 'close' to 'natural' scale,

  $i.e.$, what you would naively expect,

  $e.g.$, $\mu^2 = s$ for $e^+e^- \rightarrow$ hadrons

- Calculate in PERTURBATION THEORY
  - at least to NLO:

  $\Longrightarrow f(\mu^2, \Lambda_{\overline{\text{MS}}})$

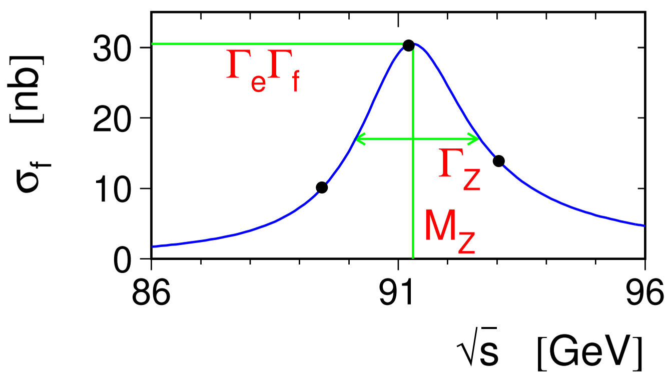
- Measure the OBSERVABLE, f, at scale μ^2

- measurement + calculation $\Longrightarrow \Lambda_{\overline{\text{MS}}}$

  With $\Lambda_{\overline{\text{MS}}}$ calculate $\alpha_s(Q^2)$ for any Q^2

Measuring α_s in e^+e^-

- $\sigma(e^+e^- \to \text{hadrons})$ or equivalently (at Z), Z lineshape



$$\Gamma_Z = \Gamma_{had} + \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + 3\Gamma_{\nu\nu}$$

$$\Gamma_{had} = 1.671\left(1 + \left(\frac{\alpha_s}{\pi}\right) + 0.75\left(\frac{\alpha_s}{\pi}\right)^2 - 15.3\left(\frac{\alpha_s}{\pi}\right)^3 + \ldots\right)$$

Concern about convergence of series

$\mathcal{O}(\alpha_s^4)$ calculation in progress

Measuring α_s in e^+e^-

- τ decays

$$
\begin{aligned}
R_\tau &= \frac{\mathrm{BR}(\tau \to \mathrm{had.} + \nu_\tau)}{\mathrm{BR}(\mathrm{e}\overline{\nu}_\mathrm{e}\nu_\tau)} \\
&= 3.0582\,(1 + \delta_{\mathrm{pert.}}(\alpha_s) + \delta_{\mathrm{non-pert.}}(\alpha_s)) \\
\delta_{\mathrm{pert.}}(\alpha_s) &= a + 5.2a^2 + 26.4a^3 + \mathcal{O}(a^4) \quad ,
\end{aligned}
$$

$$a = \alpha_s(M_\tau^2)/\pi$$

Convergence?

Large theoretical uncertainty since M_τ small,
especially on $\delta_{\mathrm{non-pert.}}$

Applicable at only 1 scale $(\alpha_s(M_\tau^2))$

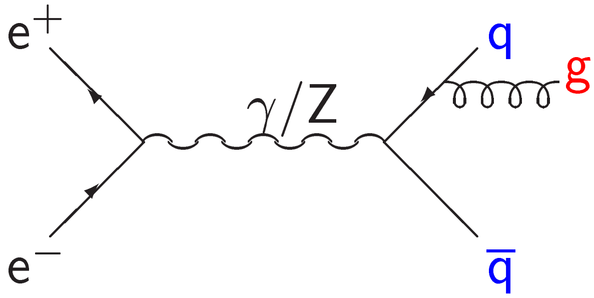
Measuring α_s in e^+e^-

- 3-jet fraction, R_3

$$R_3 = \frac{\alpha_s(\mu)}{2\pi} A_3(y_{cut}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \cdot \left[A_3(y_{cut}) \frac{\beta_0}{2} \ln\left(\frac{\mu^2}{s}\right) + B_3(y_{cut}) \right]$$

  A_3, B_3 calculable from M.E. for most jet algorithms

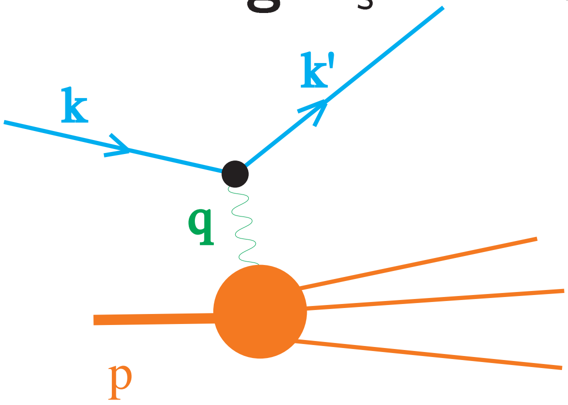
  dependence on jet algorithm
  (hadronisation corrections)
  scale uncertainties
  M.E. only to $\mathcal{O}(\alpha_s^2)$



- Hadronic event shapes (see later)
  - the more gluons radiated, the less 2-jet-like

- Scaling Violatons in fragmentation functions
  Choice of scale particularly difficult (see later)

Measuring α_s in DIS



- Structure Functions, $F(x, Q^2)$
  x = fract. of nucleon momentum
  carried by struck q or g
  $Q^2 = -q^2$ = 4-momentum transfer squared
  $F(x, Q^2)$ depends on probs. of branchings
  $\mathrm{q} \to \mathrm{qg}$, $\mathrm{g} \to \mathrm{gg}$, $\mathrm{g} \to \mathrm{q\bar{q}}$,
  $F(x, Q^2)$ described in QCD by DGLAP equation
  - Sum Rules, *e.g.*, Gross-Llewellyn Smith,

$$\int_0^1 \mathrm{d}x \left[F_3^{\nu\mathrm{p}}(x, Q^2) + F_3^{\bar{\nu}\mathrm{p}}(x, Q^2)\right]$$

$$= 6\left(1 - \Delta_{\mathrm{pert.}} - \Delta_{\mathrm{non-pert.}}\right)$$

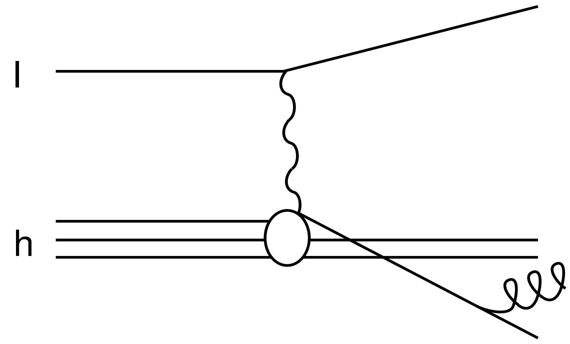
$$\Delta_{\mathrm{pert.}} = \frac{\alpha_s}{\pi} + 3.58\left(\frac{\alpha_s}{\pi}\right)^2 + 19.0\left(\frac{\alpha_s}{\pi}\right)^3$$

$$\Delta_{\mathrm{non-pert.}} = 0.032 \pm 0.016$$

  theoretical uncertainty dominated by $\Delta_{\mathrm{non-pert.}}$
  - Scaling violations
    In simple parton model, $F(x, Q^2)$ scales
    ($\to F(x)$ as energy, $Q^2 \to \infty$ with x=const.)
    But not in QCD (DGLAP)

Measuring α_s in DIS

- multi-jet production – additional jets from hard gluons analagous to jet fractions in e^+e^-



- event shapes in Breit frame
(very recent – Vancouver contribution from ZEUS, H1)

Breit frame for DIS jets

Breit frame: $$\begin{aligned} q &= (0,0,Q,0) \\ p &= \left(0,0,-\tfrac{q}{2},E\right)\ ,\ E \approx \tfrac{Q}{2} \\ p' &= \left(0,0,+\tfrac{q}{2},E\right) \end{aligned}$$

Compare to e^+e^- in CMS

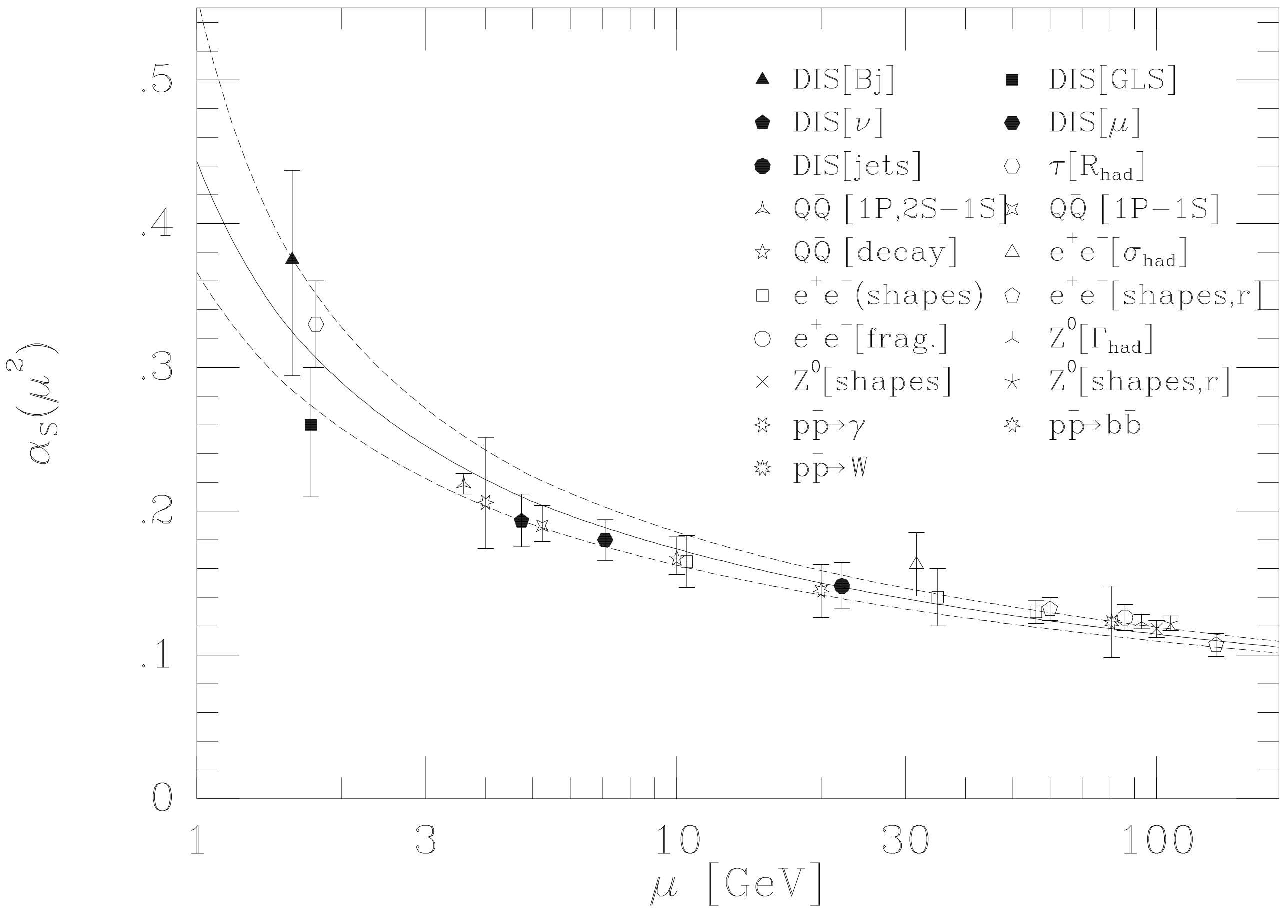
$$p' = \left(0,0,p'_z,\frac{E_{\text{cms}}}{2}\right)\ ,\ p'_z \approx \frac{E_{\text{cms}}}{2}$$

$$\begin{array}{ccc} p' \text{ in Breit frame} & \approx & \text{one hemisphere of } e^+e^- \\ Q & \Leftrightarrow & E_{\text{cms}} \end{array}$$

Most e^+e^- jet analyses can also be done in DIS

Does α_s run (walk, limp, crawl?)

Plot all α_s measurements and compare with QCD running



Ellis, Stirling and Webber, "QCD and Collider Physics", Camb.Univ.Press (1996)

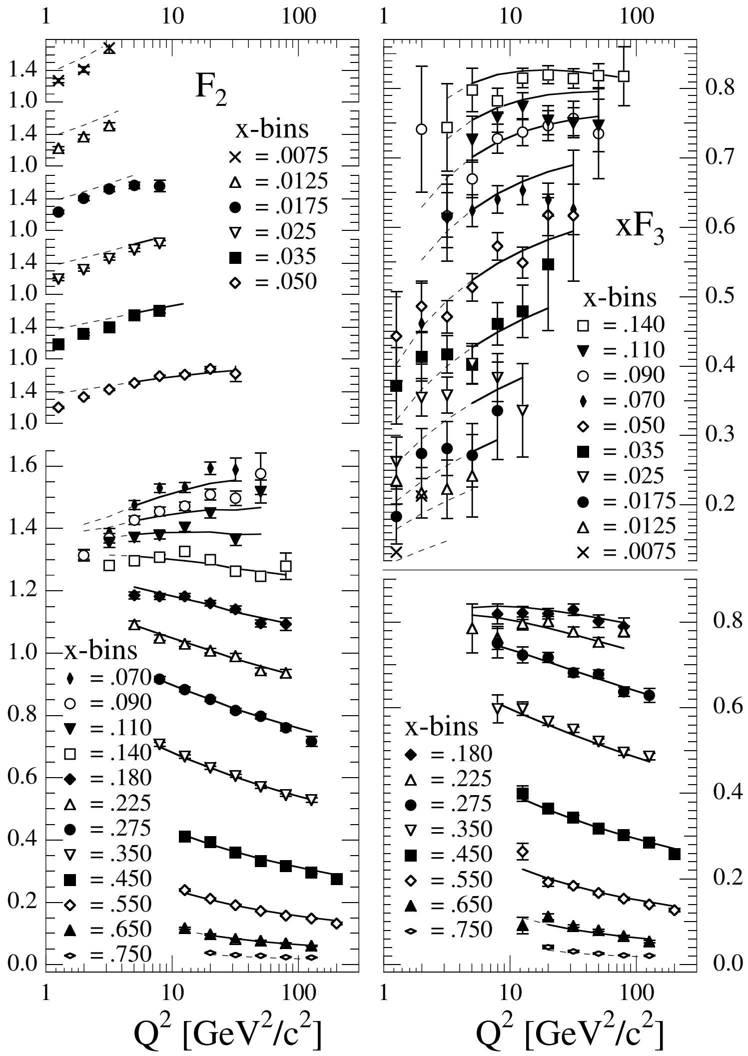
- Agrees with QCD

- But could also be something else

Does α_s run (walk, limp, crawl?)

To reduce systematic errors,
use same method at various scales
in same experiment

- DIS:
  - wide range of Q^2 in one expt.
  - fit F to all x, Q^2 simultaneously using DGLAP
  - It works. So consistent with QCD running:



νN
CCFR collab.,
hep-ex/9701017

  - Q^2 small compared to e$^+$e$^-$

- e$^+$e$^-$:
  - Use hadronic event shapes at various E_{cms}

Shape Variables – defs.

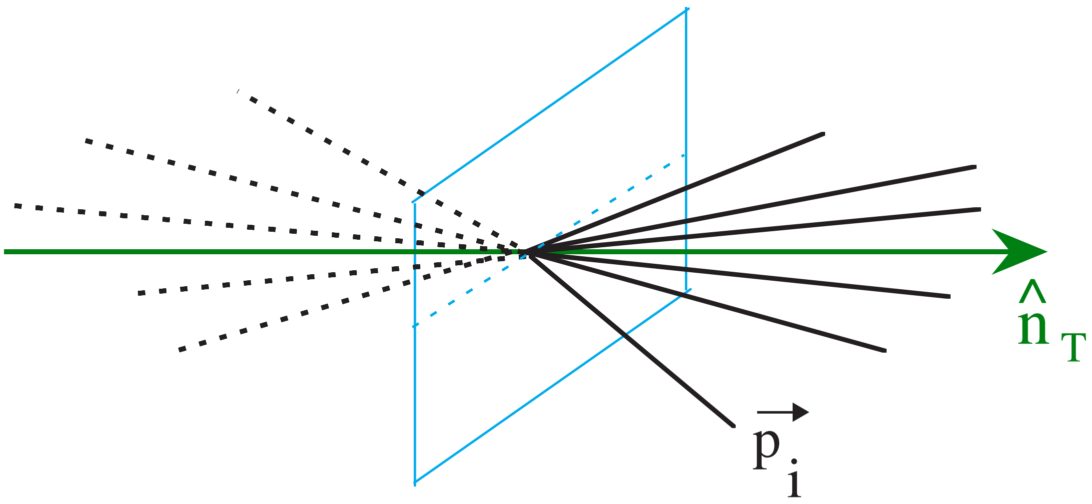
- Thrust

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i| \cdot \vec{n}}{\sum_i |\vec{p}_i|} \right)$$

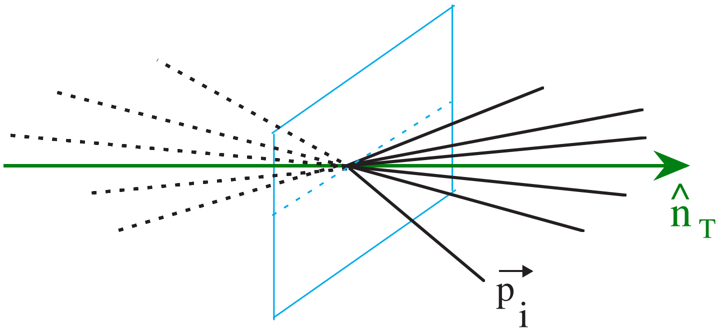
  Resulting $\vec{n}$ is called thrust direction, $\vec{n}_{\mathrm{T}}$
  Also define $\tau = 1 - T$

  – Thrust Major: T_{maj} found like T
    with $\vec{n}$ restricted to be $\perp \vec{n}_{\mathrm{T}}$
    Resulting $\vec{n}$ is called thrust-major direction, $\vec{n}_{\mathrm{maj}}$
  – Thrust Minor: T_{minor} found like T
    with $\vec{n} = \vec{n}_{\mathrm{minor}} \equiv \vec{n}_{\mathrm{maj}} \times \vec{n}_{\mathrm{T}}$ no max
  – Oblateness - $O \equiv T_{\mathrm{maj}} - T_{\mathrm{minor}}$

  Divide event into hemispheres
  by plane through origin $\perp \vec{n}_{\mathrm{T}}$



Shape Variables – defs.



- Jet Masses

  Let M_1,M_2 be inv. mass of particles in hemispheres 1,2.

  $$M_{\mathsf{H}} = \max\,(M_1, M_2) \quad , \qquad M_{\mathsf{L}} = \min\,(M_1, M_2)$$

  – Scaled Heavy Jet Mass $\rho = \ \rho_{\mathsf{H}} \equiv M_{\mathsf{H}}^2/s \approx M_{\mathsf{H}}^2/E_{\mathsf{vis}}^2$

  – Scaled Light Jet Mass $\rho_{\mathsf{L}} \equiv M_{\mathsf{L}}^2/s \approx M_{\mathsf{H}}^2/E_{\mathsf{vis}}^2$

  – Sum of Scaled Jet Masses $\rho_{\mathsf{S}} = \rho_{\mathsf{H}} + \rho_{\mathsf{L}}$

  – Diff. of Scaled Jet Masses $\rho_{\mathsf{D}} = \rho_{\mathsf{H}} - \rho_{\mathsf{L}}$

  – to 1^{st}-order in pert. QCD for massless partons,

  $$\rho = 1 - T \quad , \qquad \rho_{\mathsf{L}} = 0$$

- Jet Broadening For each hemisphere, $h = 1, 2$

  $$B_h = \frac{\sum_{i\in h} |\vec{p_i} \times \vec{n}_{\mathsf{T}}|}{2\sum_i |\vec{p_i}|}$$

  – Total Jet Broadening $B_{\mathsf{T}} \equiv B_1 + B_2$

  – Wide Jet Broadening $B_{\mathsf{W}} \equiv \max(B_1, B_2)$

  – Narrow Jet Broadening $B_{\mathsf{N}} \equiv \min(B_1, B_2)$

  – to 1^{st}-order in pert. QCD, $B_{\mathsf{T}} = B_{\mathsf{W}} = \frac{1}{2}O$, $B_{\mathsf{N}} = 0$

- ρ_{L}, B_{N} not useful for α_{s}

Shape Variables – defs.

- Sphericity S define sphericity tensor

$$S^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i |\vec{p}_i|^2} \quad , \qquad \alpha, \beta = 1, 2, 3\ (x, y, z)$$

order the eigenvalues of $S^{\alpha\beta}$: $Q_1 < Q_2 < Q_3$.

$$S \equiv \frac{3}{2}(Q_1 + Q_2)$$

– Aplanarity, $A \equiv \frac{3}{2} Q_1$

$S^{\alpha\beta}$ is quadratic in p $(p_i^\alpha p_i^\beta)$
$\Longrightarrow$ pert. QCD calculations diverge

- C-parameter linearise $S^{\alpha\beta}$:

$$\Theta^{\alpha\beta} = \frac{\sum_i \left(p_i^\alpha p_i^\beta\right) / |\vec{p}_i|}{\sum_i |\vec{p}_i|} \quad , \qquad \alpha, \beta = 1, 2, 3\ (x, y, z)$$

In terms of eigenvalues, λ_i of $\Theta^{\alpha\beta}$,

$$\begin{aligned} C &\equiv 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) \\ &= \frac{3}{2} \frac{\sum_{i,j} \left[\frac{1-(\vec{p}_i \cdot \vec{p}_j)^2}{|\vec{p}_i||\vec{p}_j|}\right]}{\left(\sum_i |\vec{p}_i|\right)^2} \end{aligned}$$

Shape Variables – defs.

- Spherocity

$$S = \left(\frac{4}{\pi}\right)^2 \min_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|} \right)^2$$

Thrust	maximizes	the	longitudinal	component
Spherocity	minimizes		transverse	

Related to shape variables are:

- y 'transition value', y_{nm}, *e.g.*,
  y_{23} = the value of y_{cut} for which
  event classification changes from 2-jet to 3-jet.

- Jet Rates, $R_n(y_{\text{cut}})$ fraction of events classified as n-jet

- Differential Jet Rates, $D_n(y_{\text{cut}})$, *e.g.*,

$$D_2(y_{\text{cut}}) \equiv \frac{R_2(y_{\text{cut}}) - R_2(y_{\text{cut}} - \Delta y_{\text{cut}})}{\Delta y_{\text{cut}}}$$

Shape Variables – defs.

Also related to shape variables:

- Energy-energy correlations (EEC)
  with χ the angle between a pair of particles,

$$\mathrm{EEC}(\chi) = \frac{1}{N_{\mathrm{ev}}}\frac{1}{\Delta\chi}\sum_{1}^{N_{\mathrm{ev}}}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{E_iE_j}{E_{\mathrm{vis}}^2}\delta_{\mathrm{bin}}(\chi-\chi_{ij})$$

where
$$\Delta\chi = \text{bin width}$$
$$\delta_{\mathrm{bin}}(\chi-\chi_{ij}) = 1 \text{ if } \chi_{ij} \text{ in same bin as } \chi$$
$$= 0 \text{ otherwise}$$

Energy-energy correlation asymmetry, EECA (or AEEC)

$$\mathrm{EECA}(\chi) = \mathrm{EEC}(\pi-\chi) - \mathrm{EEC}(\chi)$$

- Jet Cone Energy Fraction (JCEF)
  fraction of energy of jet in a conical shell (ring) of width $\Delta\chi$ about thrust axis at angle χ:

$$\mathrm{JCEF}(\chi) = \frac{1}{N_{\mathrm{ev}}\Delta\chi}\sum_{\mathrm{events}}\int_{\chi-\frac{\Delta\chi}{2}}^{\chi-\frac{\Delta\chi}{2}}\sum_{i}\frac{E_i}{E_{\mathrm{vis}}}\delta\left(\chi'-\chi_i\right)\,\mathrm{d}\chi'$$

where $\chi_i = \arccos\left(\frac{\vec{p}_i\cdot\vec{n}_T}{|\vec{p}_i|}\right)$

α_s from shape variables, I

Measure dist., $R(\alpha_s, Y) \equiv \frac{1}{\sigma}\frac{d\sigma}{dY}$, of shape variable, Y,
From $\mathcal{O}(\alpha_s^2)$ matrix elements, R is given by

$$R(\alpha_s, Y) = \frac{\alpha_s}{2\pi}A(Y) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[B(Y) + 2\pi\beta_0 \ln\frac{\mu^2}{s}A(Y)\right]$$

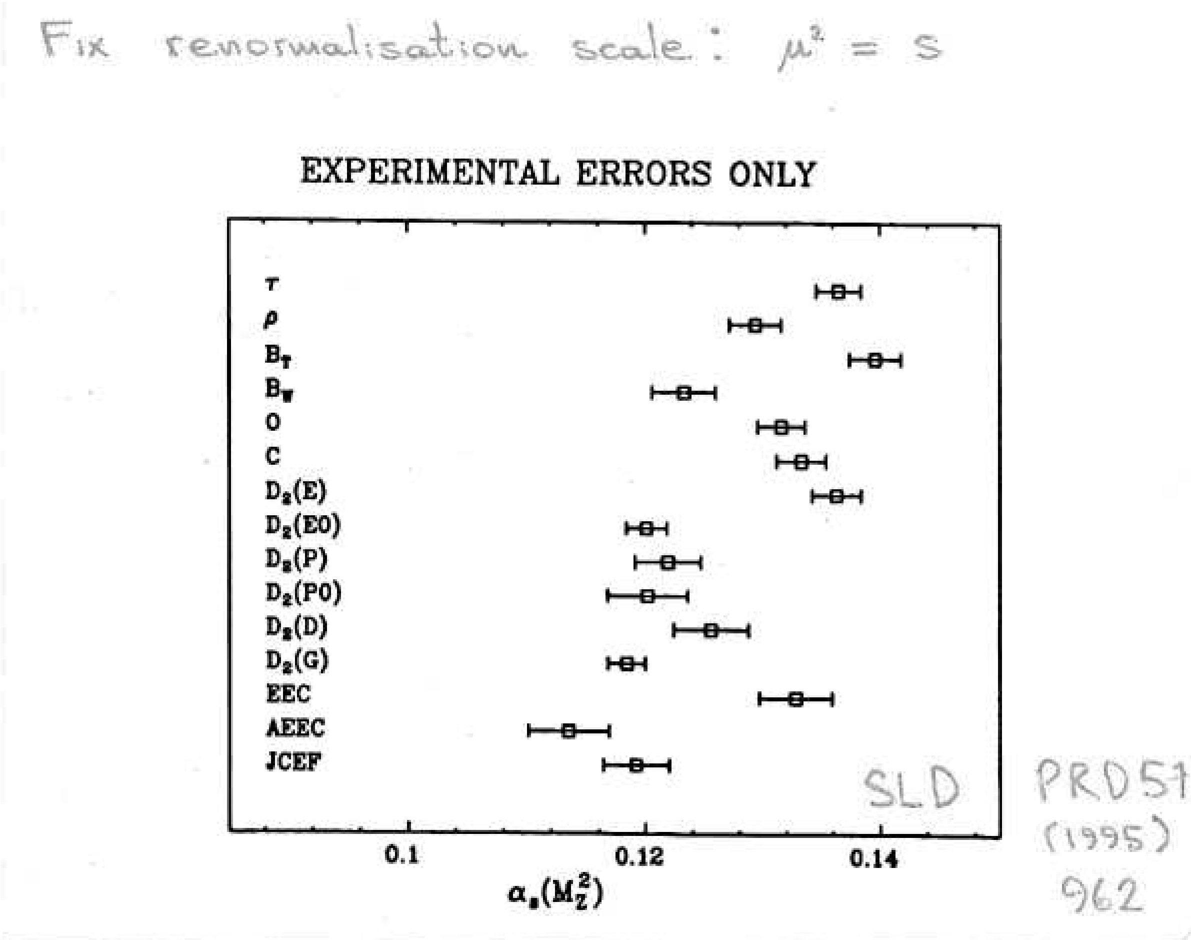
where $A(Y)$, $B(Y)$ are known functions
(different for each variable, Y)

So fit this to measured $R \Longrightarrow \alpha_s(\mu^2)$

But α_s from this expression is disastrous

α_s from shape variables, I

The disaster:



- 15 highly correlated observables
  measured using same data sample
  yield highly discrepant values of $\alpha_s(M_Z^2)$

- What's the problem?
  - Escape clause for QCD:
    higher-order terms must be important
  - Another source of worry: hadronisation corrections
    Perturbative calculations are for partons. So, must 'correct' measured shape variables to parton level

α_s from shape variables, II

- Relative importance of $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s)$ terms is nearly 1 for some shape variables.

- This suggests using different values of μ for different shape variables.

- But what?

  1. 'Optimized scales' - a 'best' scale for each observable
     But theorists disagree on best 'optimized scale'.
  2. Let the data decide: Use $\mu^2 = x_\mu s$,
     x_μ an additional fit parameter instead of $x_\mu = 1$

  DELPHI (ICHEP98 142) (1.5M Z events) tries both approaches, fitting to

$$R_\mu(\alpha_s, x_\mu, Y) \equiv \frac{1}{\sigma}\frac{\mathrm{d}^2\sigma}{\mathrm{d}Y\,\mathrm{d}\cos\theta_\mathrm{T}}$$

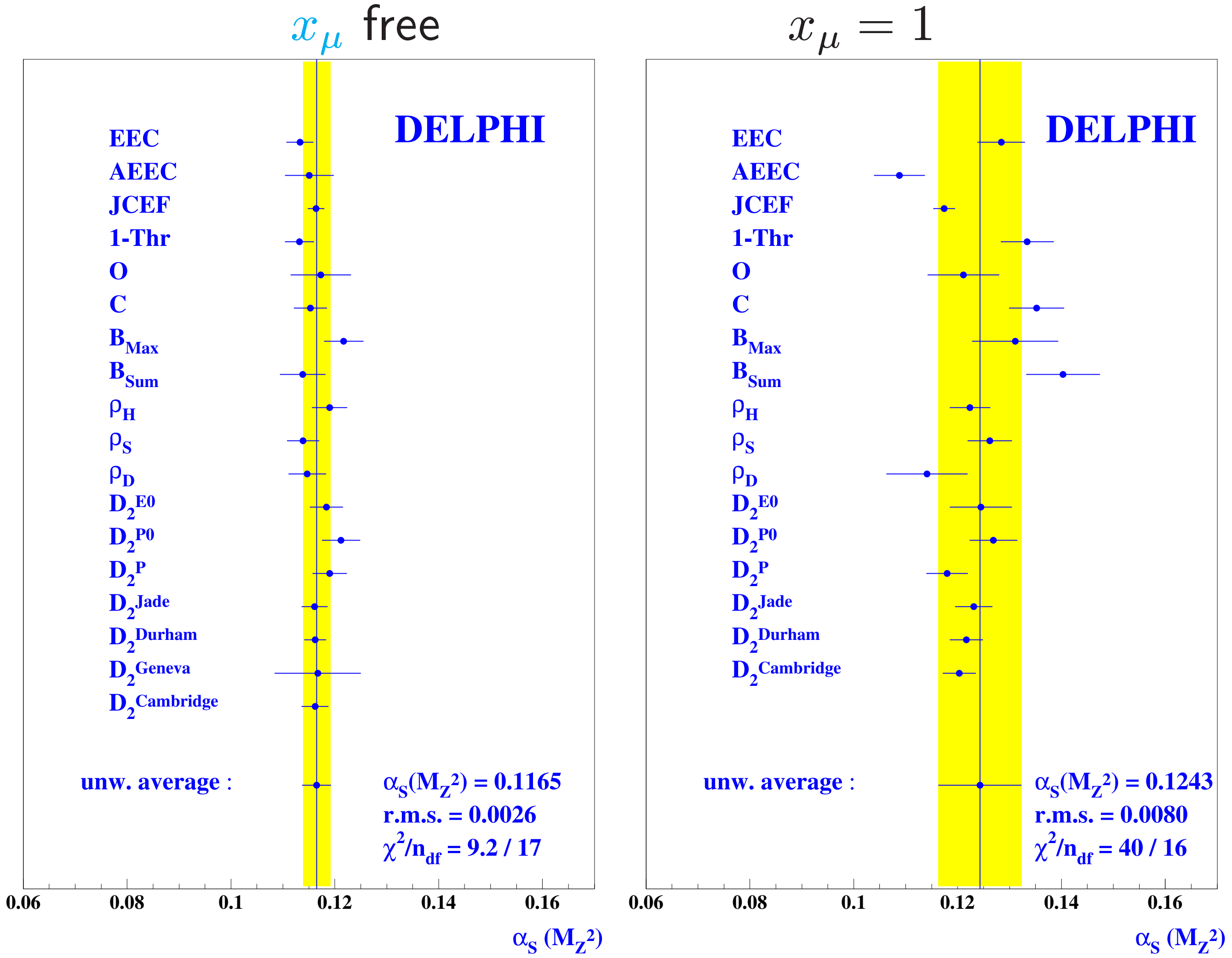
α_s from shape variables, x_μ free

Observable	Fit Range (Obs.)	x_μ	χ^2/n_{df}
EEC	$25.2^\circ - 158.4^\circ$	0.0112 ± 0.0007	0.85
AEEC	$21.6^\circ - 75.6^\circ$	0.0072 ± 0.0015	0.98
JCEF	$104.4^\circ - 172.8^\circ$	0.0991 ± 0.0050	1.13
$1 - T$	0.05 - 0.30	0.0033 ± 0.0002	1.24
O	0.24 - 0.44	2.08 ± 0.37	0.73
C	0.24 - 0.72	0.0068 ± 0.0006	1.02
B_{Max}	0.10 - 0.24	0.029 ± 0.014	0.96
B_{Sum}	0.12 - 0.24	0.0092 ± 0.0022	1.19
ρ_H	0.04 - 0.30	0.0459 ± 0.0026	0.90
ρ_S	0.05 - 0.25	0.0063 ± 0.0003	1.12
ρ_D	0.01 - 0.20	1.104 ± 0.019	1.18
D_2^{E0}	0.05 - 0.20	0.121 ± 0.022	0.82
D_2^{P0}	0.05 - 0.20	0.095 ± 0.041	0.88
D_2^{P}	0.10 - 0.25	0.0053 ± 0.0006	0.85
D_2^{Jade}	0.05 - 0.25	0.083 ± 0.015	0.80
D_2^{Durham}	0.015 - 0.16	0.0131 ± 0.0019	0.72
D_2^{Geneva}	0.015 - 0.03	6.33 ± 0.22	0.82
$D_2^{Cambridge}$	0.012 - 0.16	0.102 ± 0.039	1.03

Large differences in x_μ and $x_\mu \neq 1$ $\Uparrow$

For only 1 variable, ρ_D is $x_\mu \approx 1$!!!

α_s from shape variables, x_μ free



Consistent values of α_s when x_μ free

x_μ		free	1
unweighted avg.	$\alpha_s(M_Z^2)$ χ^2/d.o.f.	0.1165 ± 0.0026 9.2/17	0.1243 ± 0.0080 40/16
weighted avg.	$\alpha_s(M_Z^2)$ χ^2/d.o.f.	0.1164 ± 0.0025 7.3/16	

ρ_D not used in weighted avg., since fully correlated with ρ_H and ρ_S

α_s from shape variables, 'optimized μ'

$$R(\alpha_s, Y) = \frac{\alpha_s}{2\pi} A(Y) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[B(Y) + 2\pi\beta_0 \ln\frac{\mu^2}{s} A(Y) \right]$$

- Principle of minimal sensitivity (PMS)
  choose x_μ such that (Stevenson, P.R.D.23(1981)2916)
  cross section is least sensitive to varying it, *i.e.*,
  $\frac{\partial\sigma}{\partial x_\mu} = 0 \quad , \qquad \sigma = \int_{Y_{\text{min}}}^{Y_{\text{max}}} R(Y, \mu)\, \mathrm{d}Y$ (?)

- Effective charges (ECH) (Grunberg, P.R.D.29(1984)2315)
  or fastest apparent convergence (FAC)
  (equivalent at $\mathcal{O}(\alpha_s^2)$)
  Choose x_μ such that $\mathcal{O}(\alpha_s^2)$ term of $R(\alpha_s, Y)$ is 0

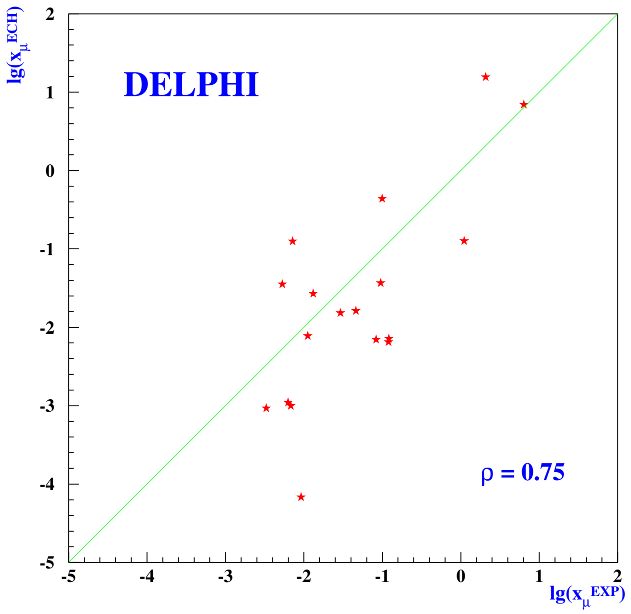
- Brodsky, Lepage and MacKenzie (BLM) (P.R.D.28(1983)228)
  Require effects of quark pairs to be absorbed in α_s.
  $\left.\frac{\partial}{\partial n_f}\right|_{n_f=5} \left[B(Y) + 2\pi\beta_0 \ln\frac{\mu^2}{s} A(Y) \right] = 0$

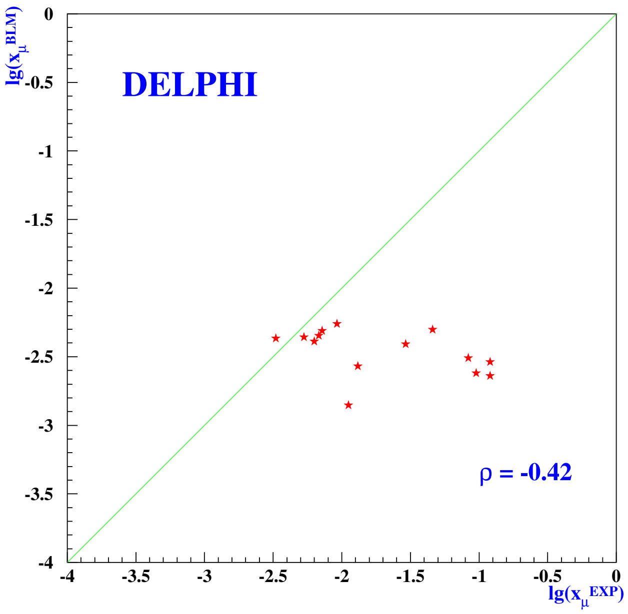
α_s from shape variables, 'optimized μ'

DELPHI weighted averages (17 shape variables):

method	$\alpha_s(M_Z^2)$	χ^2/d.o.f.
x_μ free	0.1164 ± 0.0025	7.3/16
PMS	0.1147 ± 0.0040	21/16
ECH	0.1148 ± 0.0038	18/16
BLM	0.1168 ± 0.0040	24/13
BLM	fits for 3 observables did not converge	

- Consistent values of α_s
- x_μ values for PMS and ECH correlate almost perfectly
  x_μ values for PMS and ECH correlate well with x_μ free
  but not for BLM
  $\Longrightarrow$ BLM disfavoured for α_s determination





α_s from shape variables, III

Calculate distribution of shape variable, $R(\alpha_s, Y)$,
using Resummation Calculations:
(Catani, Dokshitzer, Olsson, Trentadue, Turnock, Webber)

Leading and Next-to-Leading logs in perturbative expansion of $R(\alpha_s, Y)$ summed to all orders of α_s.

$$\text{LL}: \quad L \sum (\alpha_s L)^n \quad , \qquad L \equiv \ln \frac{1}{Y}$$

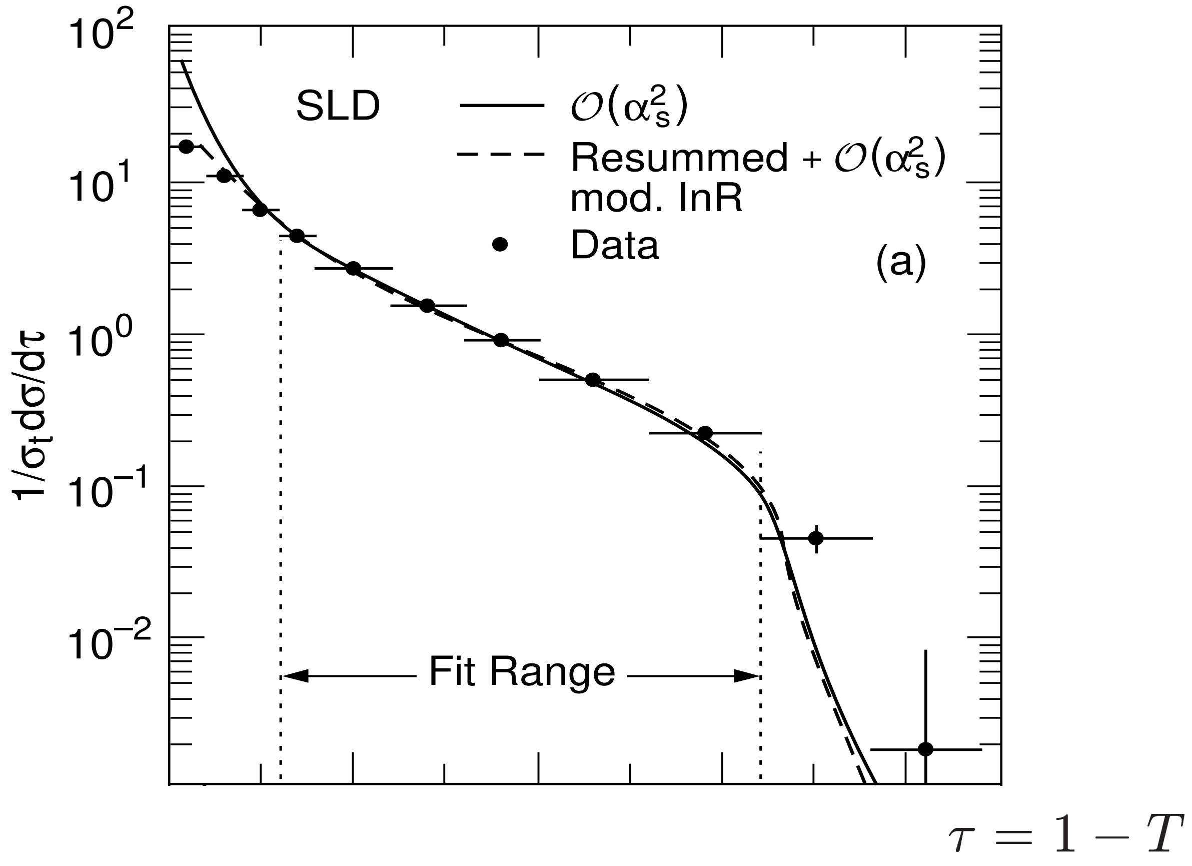
$$\text{NLL}: \quad \sum (\alpha_s L)^n$$

Most important in 2-jet region, where L is large

$\mathcal{O}(\alpha_s^2)$ M.E. and Resummed calculations can be combined.
Must avoid double counting
– Some theoretical uncertainty in this
There are several proposals

μ dependence should be smaller,
since NLL and all orders of α_s.
But NLL calculations not available for so many variables

α_s from shape variables, III



Resummation extends region of validity of calculation into 2-jet region (large T)

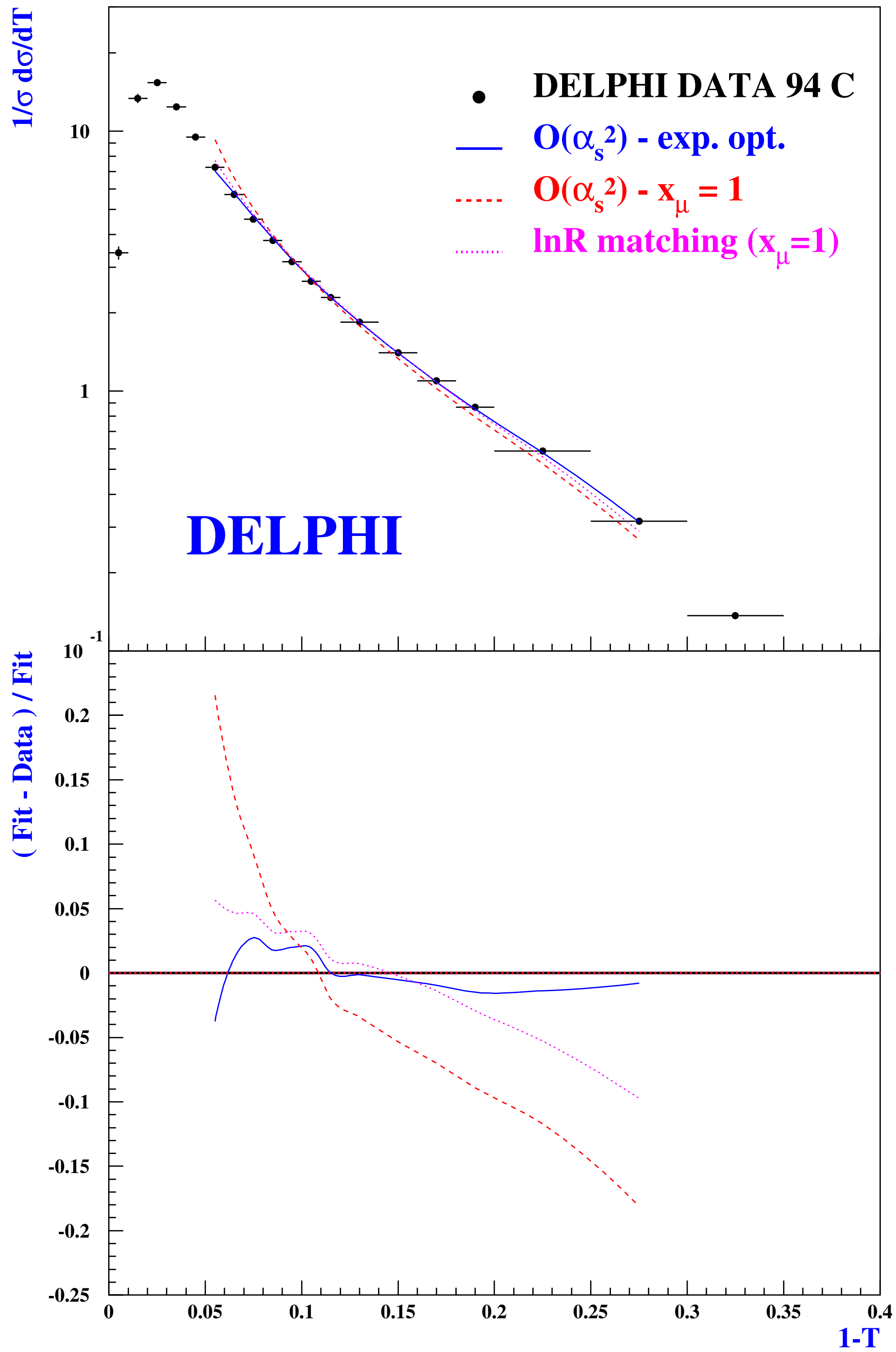
α_s from shape variables, III

But DELPHI finds worse χ^2/d.o.f. in this method

	χ^2/d.o.f.		
Observable	$\mathcal{O}(\alpha_s^2)$		$\mathcal{O}(\alpha_s^2)$ + NLLA
	x_μ free	$x_\mu = 1$	$x_\mu = 1$
$\tau = 1 - T$	1.24	25.9	9.5
C	1.02	12.0	15.2
B_H	0.96	1.67	8.4
B_T	1.19	8.1	11.9
ρ	0.90	13.7	7.6
D_2^{Durham}	0.72	2.57	1.70
avg.: $\alpha_s(M_Z^2)$	0.1164 ± 0.0025	0.124 ± 0.008	0.118 ± 0.003
χ^2/d.o.f.	7.3/16	40/16	3.1/5

Nevertheless, $\mathcal{O}(\alpha_s^2)$ + NLLA values for α_s from different observables are consistent χ^2/d.o.f. = 3.1/5
and consistent with value from $\mathcal{O}(\alpha_s^2)$, x_μ free

α_s from shape variables, III

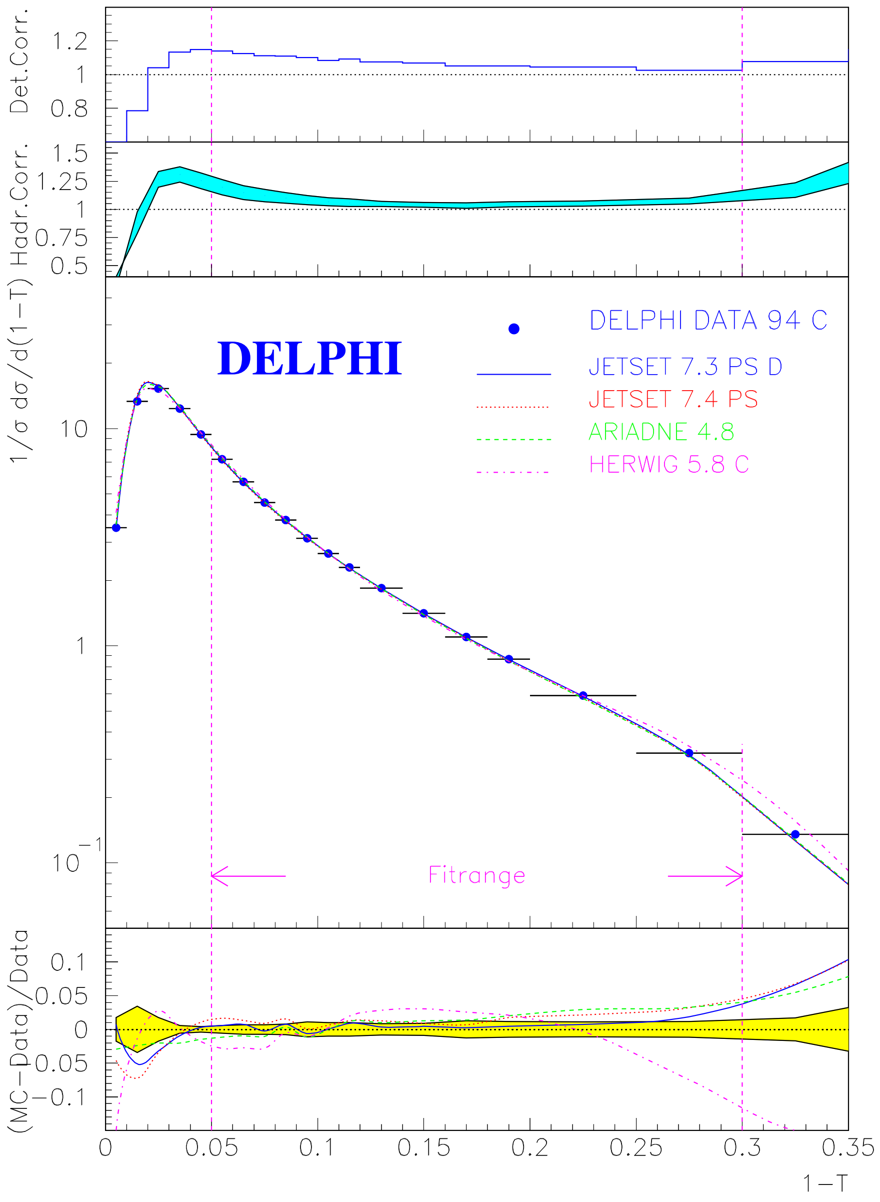


What is the best method is still not clear

Young graduate students take note

α_s from shape variables, hadronisation corrections

Shape variables are measured using hadrons
But calculations are for partons
So must correct – use MC models



Correction can be rather large, say $\mathcal{O}(25\%)$

MC models do not always agree

α_s from shape variables, power corrections

Moments, $< Y^n >$, of the distribution of shape variable Y

1. use all data
   hence particularly interesting where statistics are low (LEP-II)

2. perturbative QCD predictions from moments of the perturbative prediction,

$$< Y^n >^{\text{pert}} = \int Y^n \, R(\alpha_s, Y) \, \mathrm{d}Y$$

$$R(\alpha_s, Y) = \frac{\alpha_s}{2\pi} A(Y) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[B(Y) + 2\pi\beta_0 \ln\frac{\mu^2}{s} A(Y)\right]$$

3. For some variables, $\tau = 1 - T$, C, ρ_{H}, B_{T}, B_{W}, non-pert. effects (hadronisation) can be estimated from 'power corrections'. Then (Dokshitzer, Webber)

$$< Y^n > = < Y^n >^{\text{pert}} + < Y^n >^{\text{pow}}$$

α_s from shape variables, power corrections

$e.g.$, for $< Y > = < \tau >$ or $< C >$:

$$< Y >^{\mathrm{pow}} = a_Y \frac{4C_F}{\pi^2} \mathcal{M} \frac{\mu_I}{\sqrt{s}} \left[\alpha_0(\mu_I^2) - \alpha_s(\mu^2) - \frac{\beta_0}{2\pi} \alpha_s^2(\mu^2) \left(\ln \frac{\mu}{\mu_I} + \frac{K}{\beta_0} + 1 \right) \right]$$

where $$K = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{5}{9} n_f$$

and a_Y is a known factor, different for each Y

(For ρ_H, B_T, B_W, there are additional terms.)

Three new quantities:

1. $\mathcal{M}$ – the 'Milan' factor – accounts for two-loop effects
   $\mathcal{M} = 1.795$ for all Y (Dokshitzer *et al.*)

2. μ_I – infra-red matching scale ≈ 2 GeV

3. $\alpha_0(\mu_I^2)$ – a non-pert. parameter
   – contribution to $< Y >^{\mathrm{pow}}$ from below μ_I
   – same for all Y
   – for higher moments $< Y^n >$,
   parameters $\alpha_{n-1}(\mu_I^2)$
   and some n dependence in $< Y >^{\mathrm{pow}}$

α_s from shape variables, power corrections

The procedure:

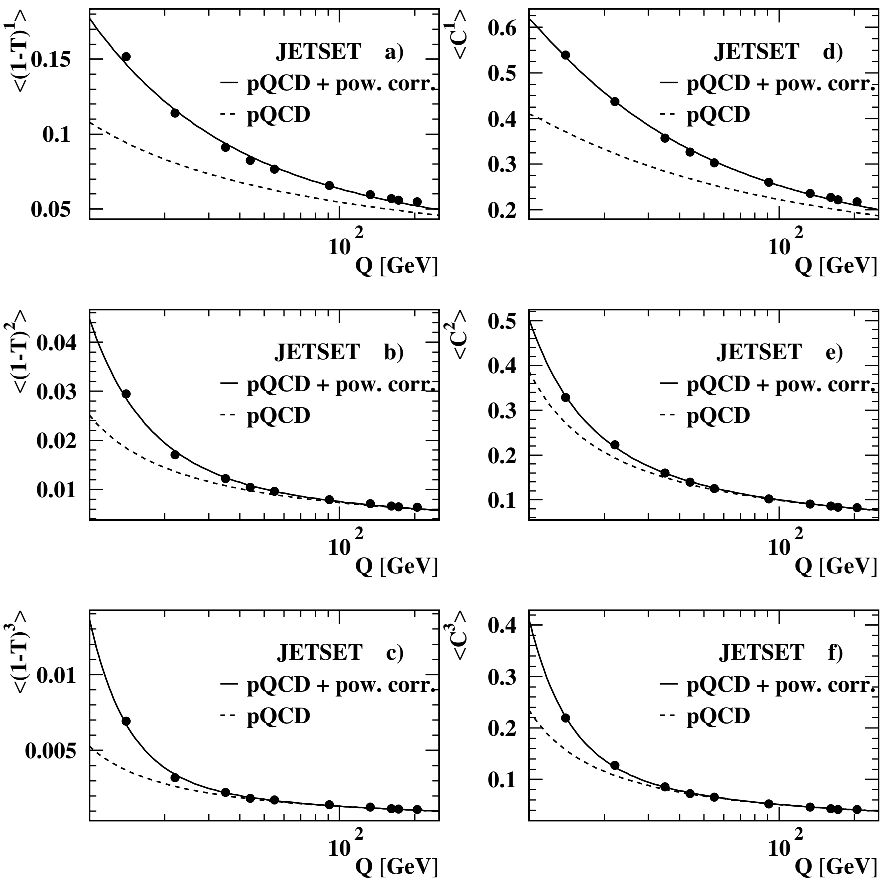
- fit each $< Y >$ *vs.* $\sqrt{s}$
  to determine $\alpha_s(M_Z^2)$ and $\alpha_0(\mu_I^2)$

- vary μ_I – α_s should not depend on μ_I

- $\alpha_0(\mu_I^2)$ should be same for all Y

- If this works, can fit all $< Y >$ dists. simultaneously

To illustrate, OPAL used JETSET MC as data:



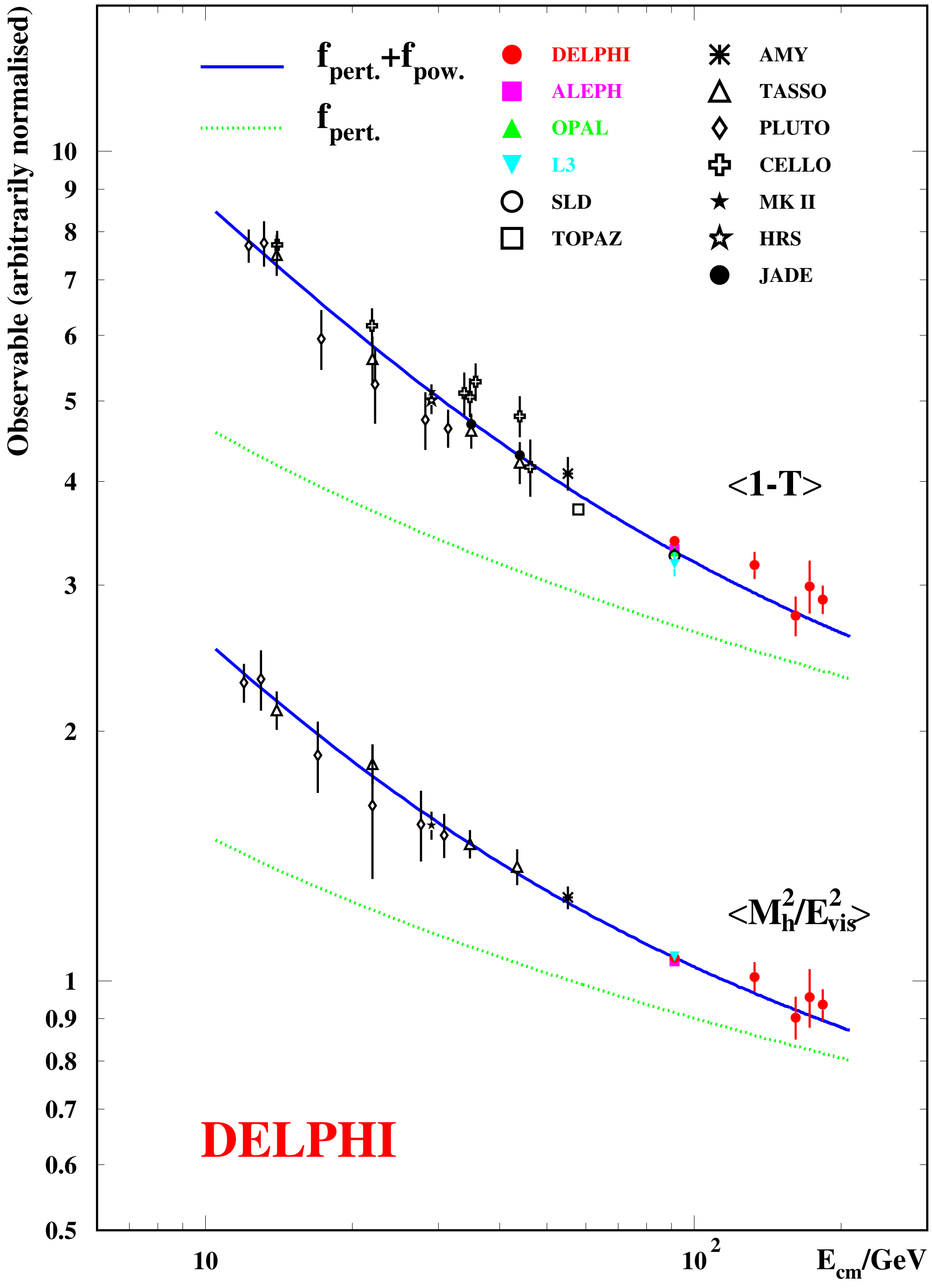
pQCD + power corrections agree with JETSET

α_s from shape variables, power corrections

Data:

$< 1 - T >$

$< \rho >$



	$< 1 - T >$	$< \rho >$
α_0 $\alpha_s(M_Z^2)$ χ^2/ndf	$0.494 \pm 0.008 \pm 0.003$ $0.1176 \pm 0.0016 \pm 0.0055$ $43/22$	$0.558 \pm 0.022 \pm 0.012$ $0.1172 \pm 0.0013 \pm 0.0035$ $2.0/11$
syst. err. dominated by scales $0.25 < x_\mu < 4$ and $1 < \mu_I < 3$ GeV		

α_s from shape variables, power corrections

Summary of results: (Vancouver)

	$\alpha_s(M_Z^2)$	α_0	x_μ
ALEPH	0.1168 ± 0.0048	0.451 ± 0.065	1.0
combined fit C, $1-T$ — a_0 values consistent			
DELPHI	0.1176 ± 0.0057 0.1172 ± 0.0037	0.494 ± 0.009 $(1-T)$ 0.558 ± 0.025 (ρ)	1.0 1.0
JADE	$0.1188^{+0.0044}_{-0.0034}$	0.325 - 0.616	1.0
fits to C, $1-T$ ρ, B_T, B_W — α_0 values inconsistent			
OPAL	0.1143 ± 0.0088 0.1158 ± 0.0107 0.141	0.322 ± 0.056 $(1-T)$ 0.249 ± 0.020 (C) as above	0.038 0.052 1.0

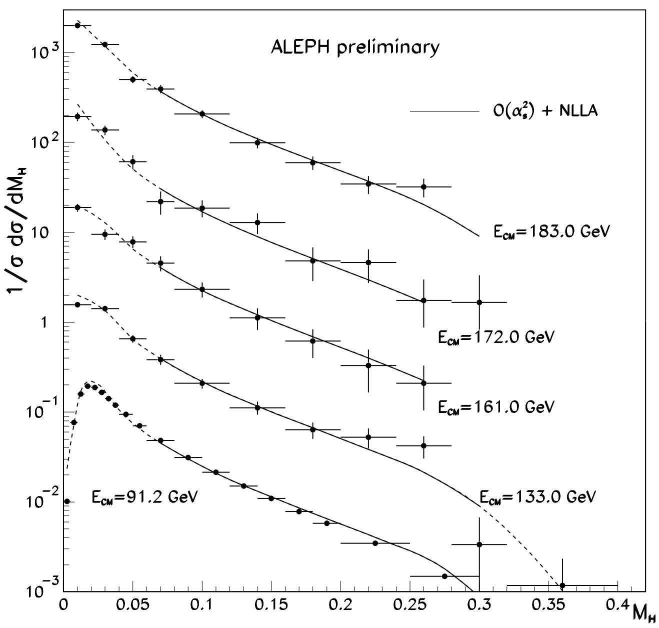
- α_0 universal only to $\sim 30\%$
- OPAL x_μ – fits very unstable with $x_\mu = 1$
  higher-order pert. terms large?
- consistent values of $\alpha_s(M_Z^2)$ except for OPAL $x_\mu = 1$

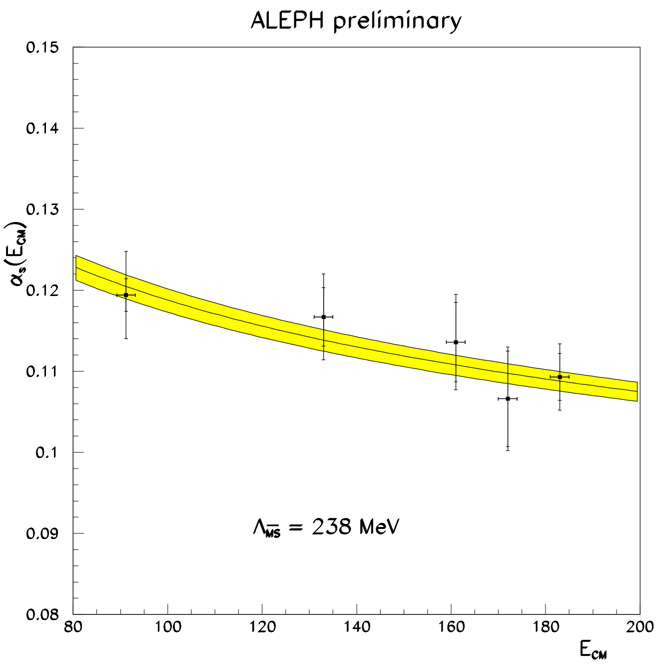
Still much work needed to understand this
Young graduate students take note

Does α_s run ?

Measure α_s at different E_{cms} in one experiment.
Systematic errors largely cancel in test of running

running seen by each LEP experiment
e.g., ALEPH: $91 < \sqrt{s} < 183$ GeV $\qquad \mathcal{O}(\alpha_s^2)$ + NLLA





Similar results for 4 other shape variables

Having seen running,
fit 5 shape variables and 5 energies simultaneously,
assuming QCD running:

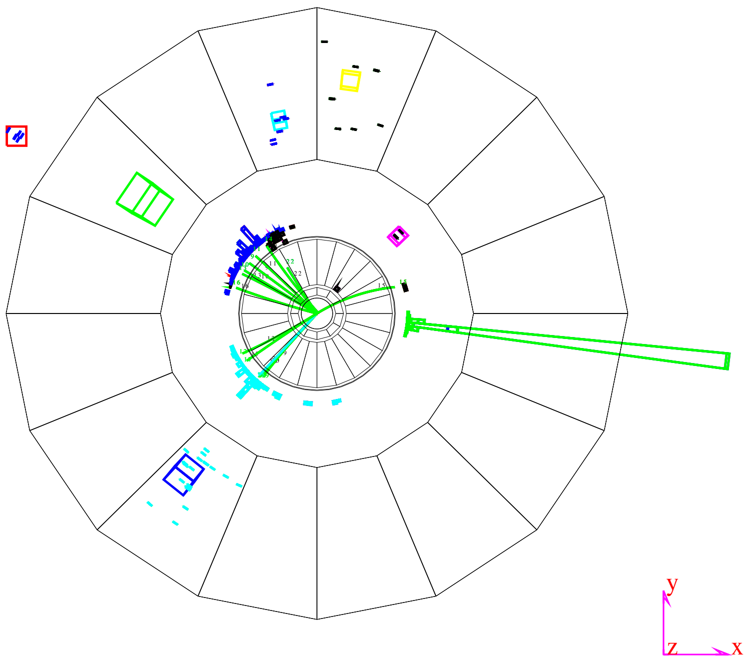
$$\alpha_s(M_Z^2) = 0.1217 \pm 0.0017 \pm 0.0044$$

Does α_s run ?

L3:

- Z data ($E_{\text{cms}} = M_Z$) (142.4 pb^{-1} from 1991–1995)

- Z data with isolated, energetic γ (mostly FSR, ISR)
  - isolated single γ identified by neural net based on shower-shape in BGO
  - Remove the γ from event
  - Boost event to CM of remaining hadronic system
  $s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$



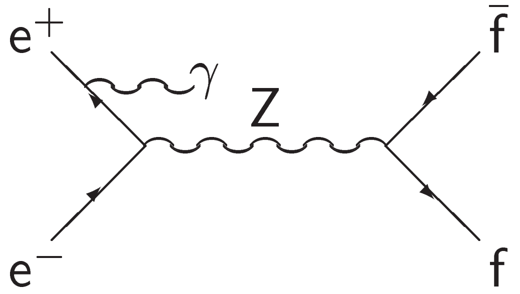
Efficiency: 27%–48%, Purity:68%–90%

For QCD, colour field
equivalent to hadronic event at $E_{\text{cms}} = \sqrt{s'} < M_Z$

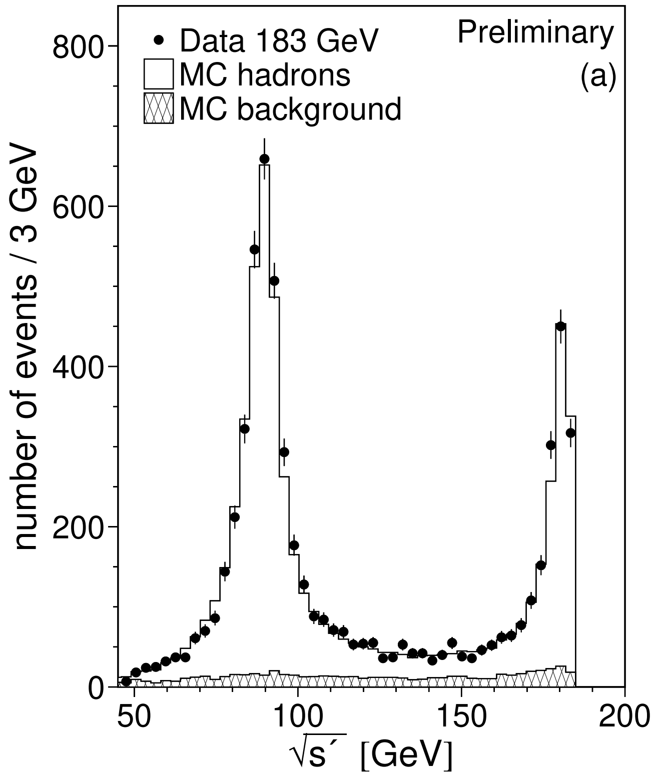
- LEP-2 data at $E_{\text{cms}} > M_Z$
  - Reject “Radiative Return” events
  - Reject W^+W^- events

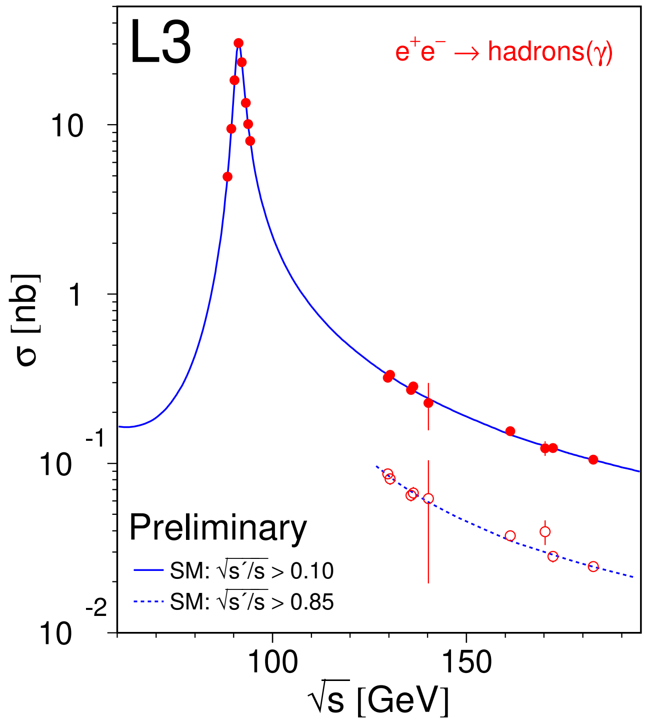
LEP-2: Radiative return to Z

$e^+e^- \to \gamma/Z \to f\bar{f}(\gamma)$, $\qquad s = E^2_{cms} \gg M^2_Z$:
Photon radiated from initial e^- or e^+ (ISR)



Large fraction of $e^+e^- \to f\bar{f}(\gamma)$ is radiative return to Z:
Mass $m_{f\bar{f}} = \sqrt{s'}$ of $f\bar{f}$ System: $\quad s' = s - 2\,E_\gamma\sqrt{s} \approx M^2_Z$

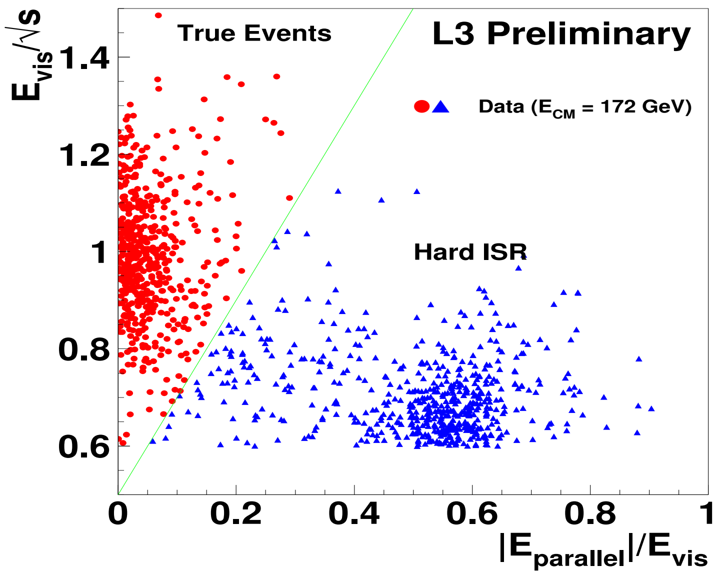




Reject events with high-energy photon
or large $\sum E_\parallel$

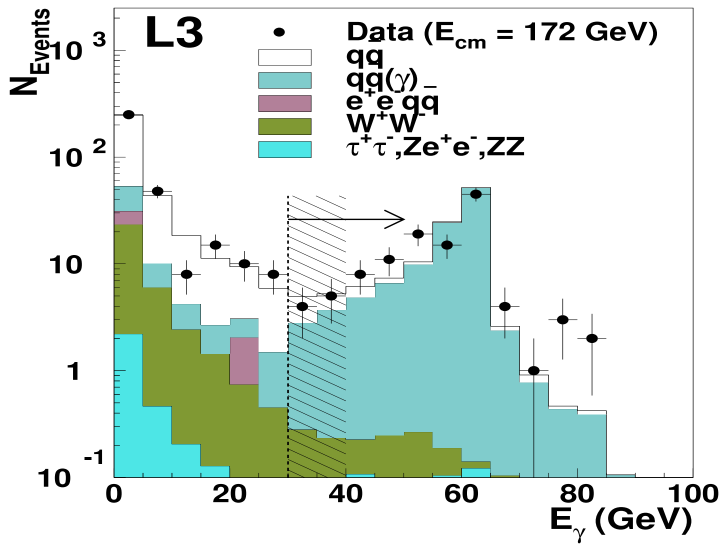
Rejection of radiative return to Z

Reject radiative returns to Z by demanding



low longitudinal energy imbalance :

$$(\mathrm{E_{vis}}/\sqrt{s}) > 2.0\ (|\ \mathrm{E}_{\|}\ |\ /\mathrm{E_{vis}}) + 0.5$$

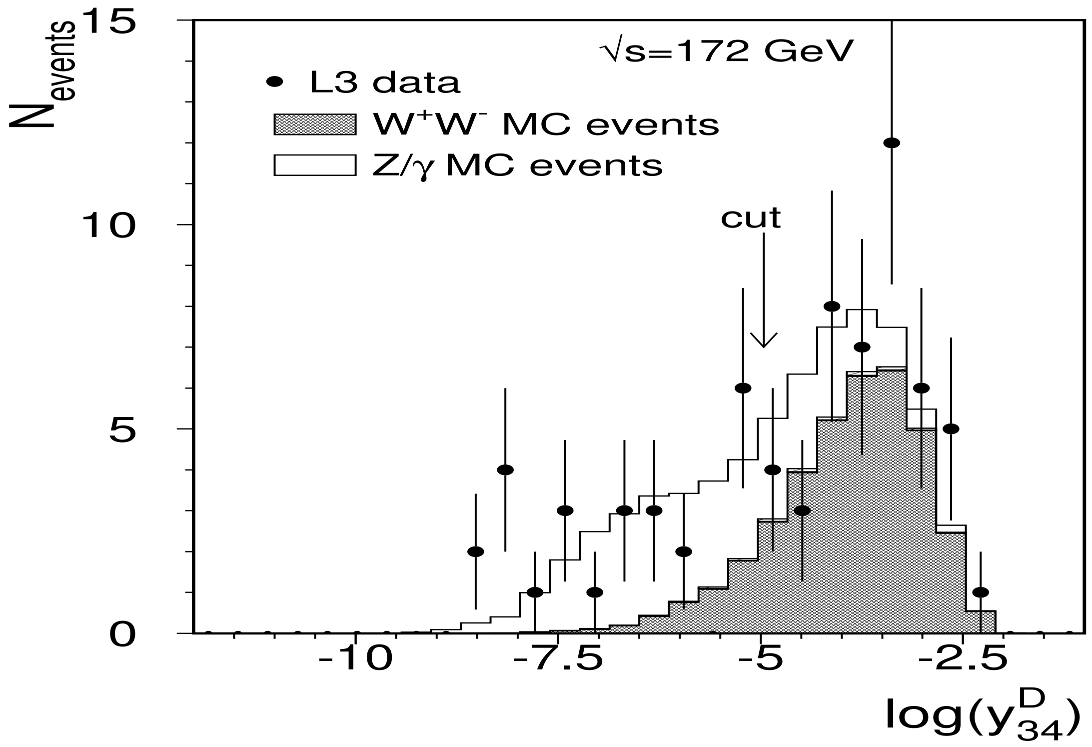


no energetic photon seen in the detector : $\mathrm{E}_\gamma < 30$ GeV

LEP-2: Rejection of W^+W^- events

Identify $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$ events by

- Require high multiplicity
- Force into 4-jet configuration (Durham algorithm)
- Select good 4-jet events by cuts on E of most and least energetic jets
- Cut on jet resolution parameter y_{34}
  (value of y_{cut} at which 3-jet event becomes 4-jet)

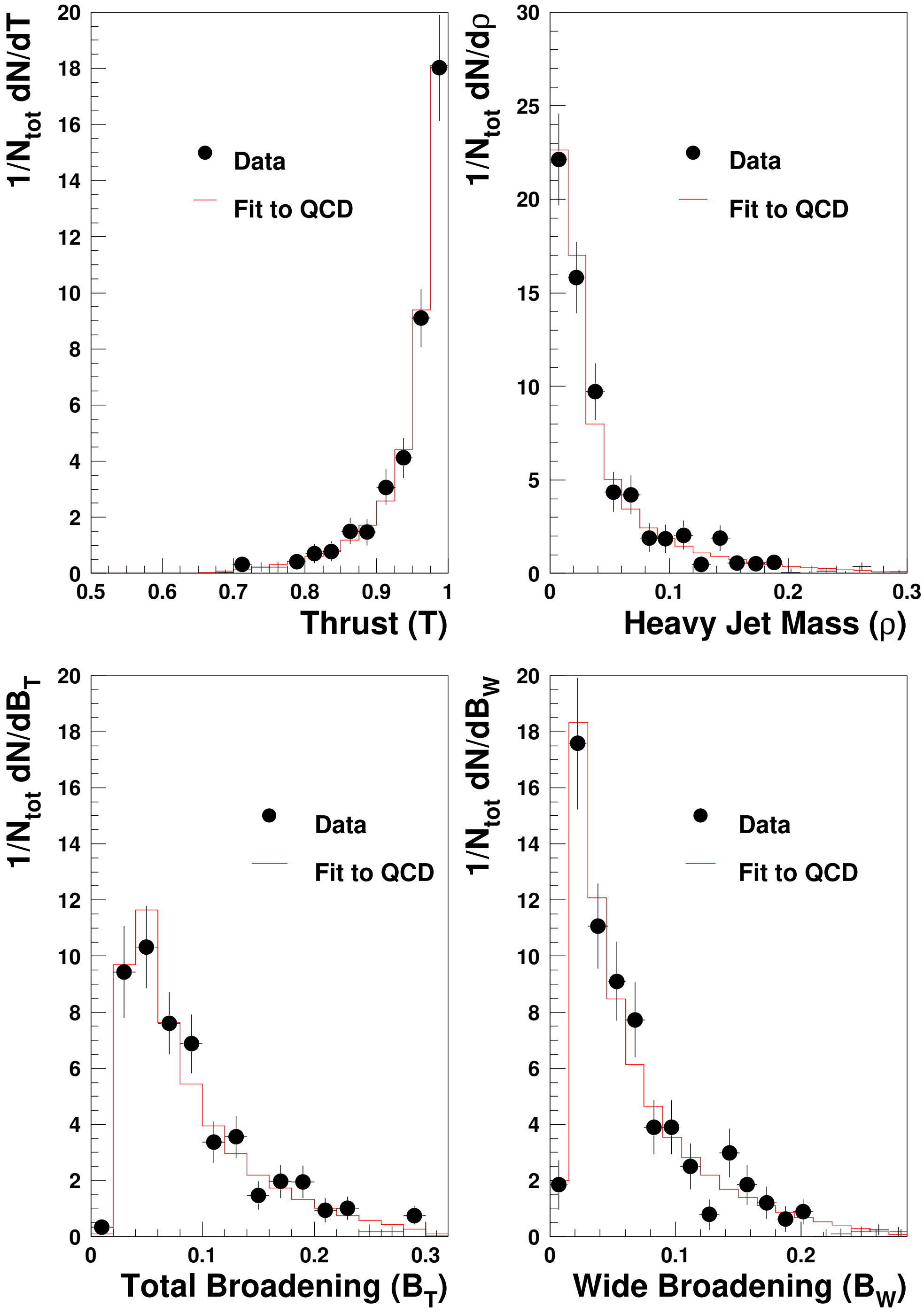


Selection efficiency of $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q} \approx 90\%$

Reject these events.

α_s from shape variables, III

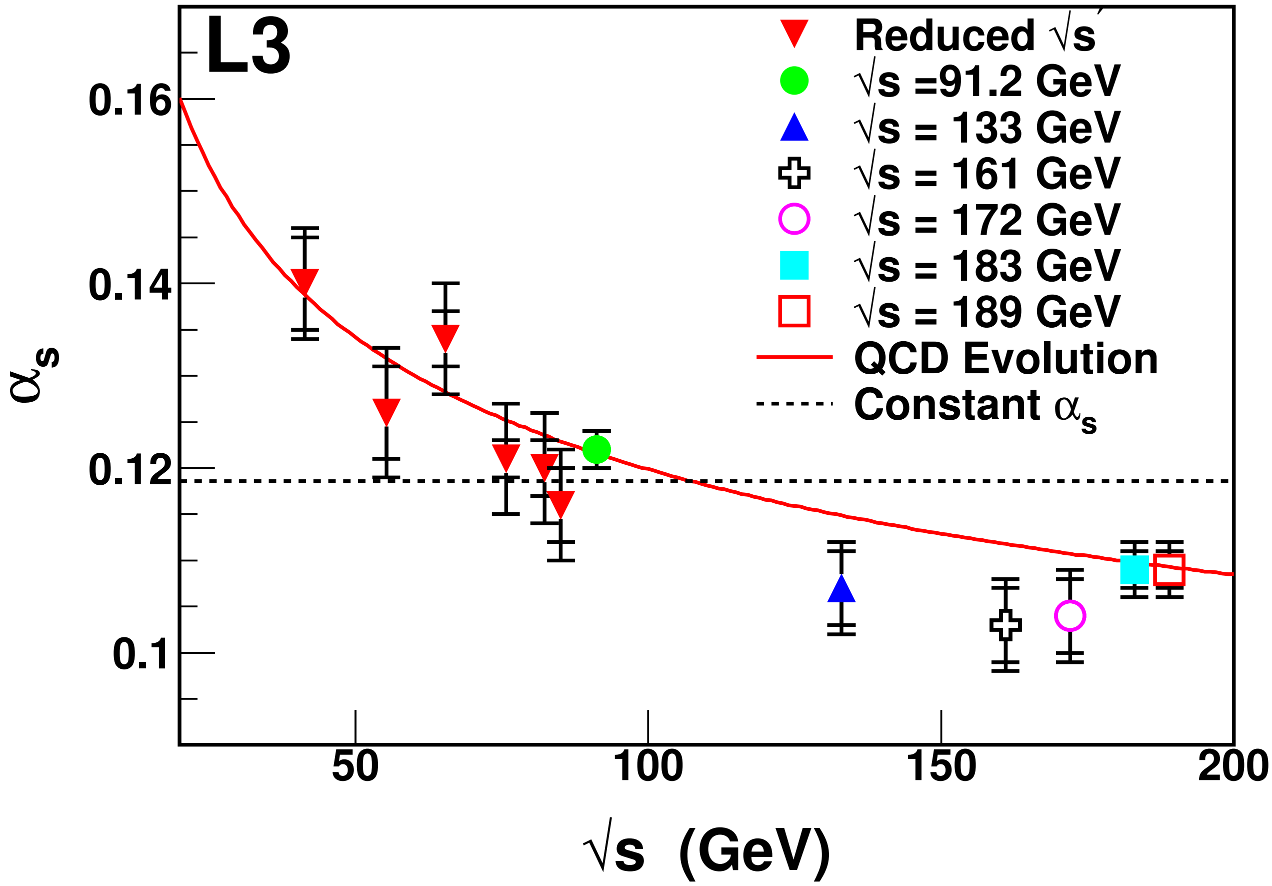
L3 data at 172 GeV



Reasonable fits to anal. dists. ($\mathcal{O}(\alpha_s^2)$ + NLLA)
with one free parameter, α_s
Mean of 4 α_s values:

$$\alpha_s(172\,\text{GeV}) \quad = \quad 0.104 \pm 0.006 \pm 0.005$$

L3: Does α_s run ?



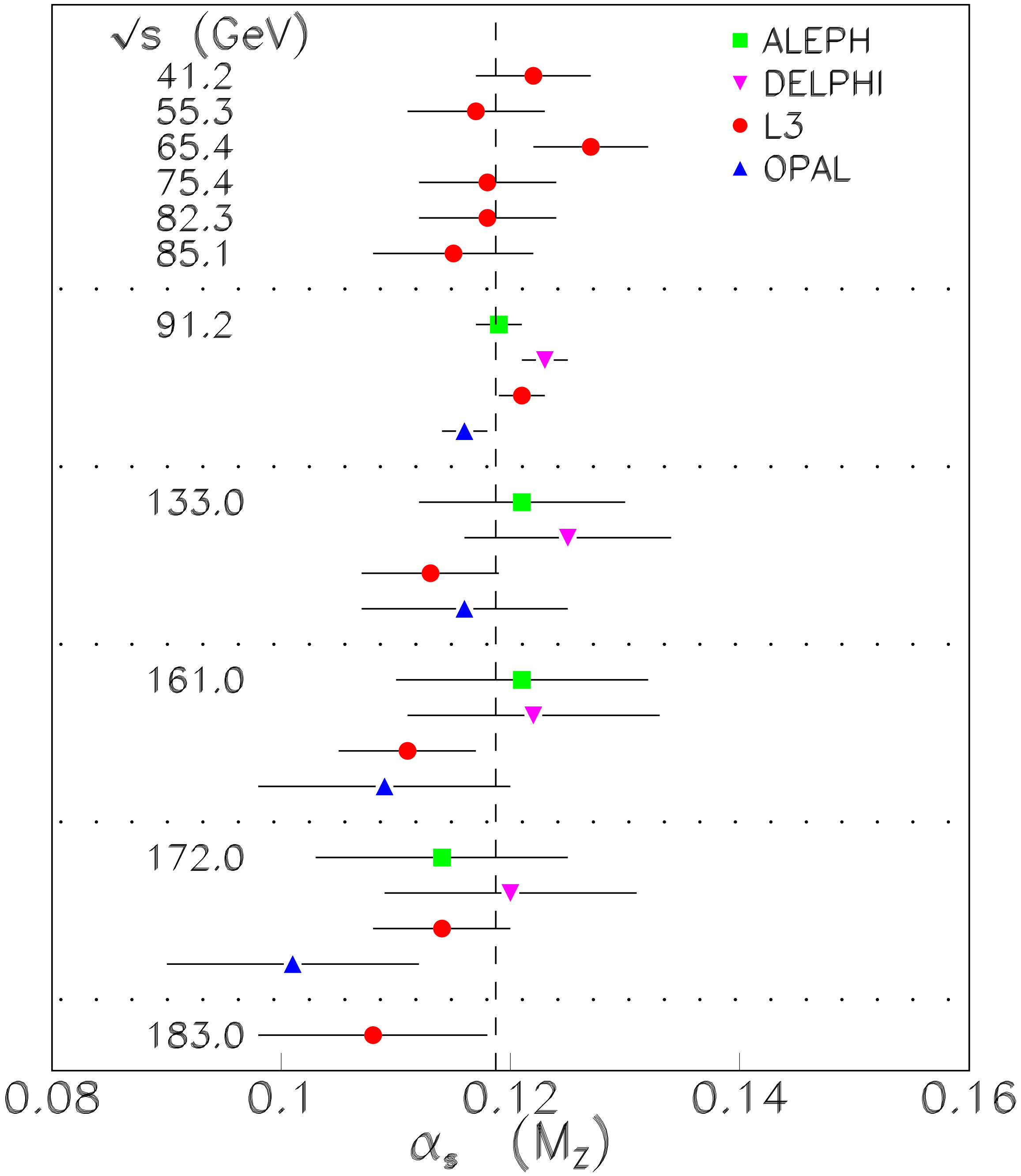
Fit to the 3-loop QCD evolution equation, using experimental errors only, gives

$$\alpha_s(M_Z^2) = 0.1216 \pm 0.0017 \text{ (expt) } \pm 0.0058 \text{ (theory)}$$

$$\chi^2 = 16.9 \text{ for } 10 \text{ DoF}, \quad \text{CL} = 0.08$$

α_s from shape variables, III

Other LEP expts. have also analyzed shape variables using $\mathcal{O}(\alpha_s^2)$ M.E. + Resummed LL, NLL.



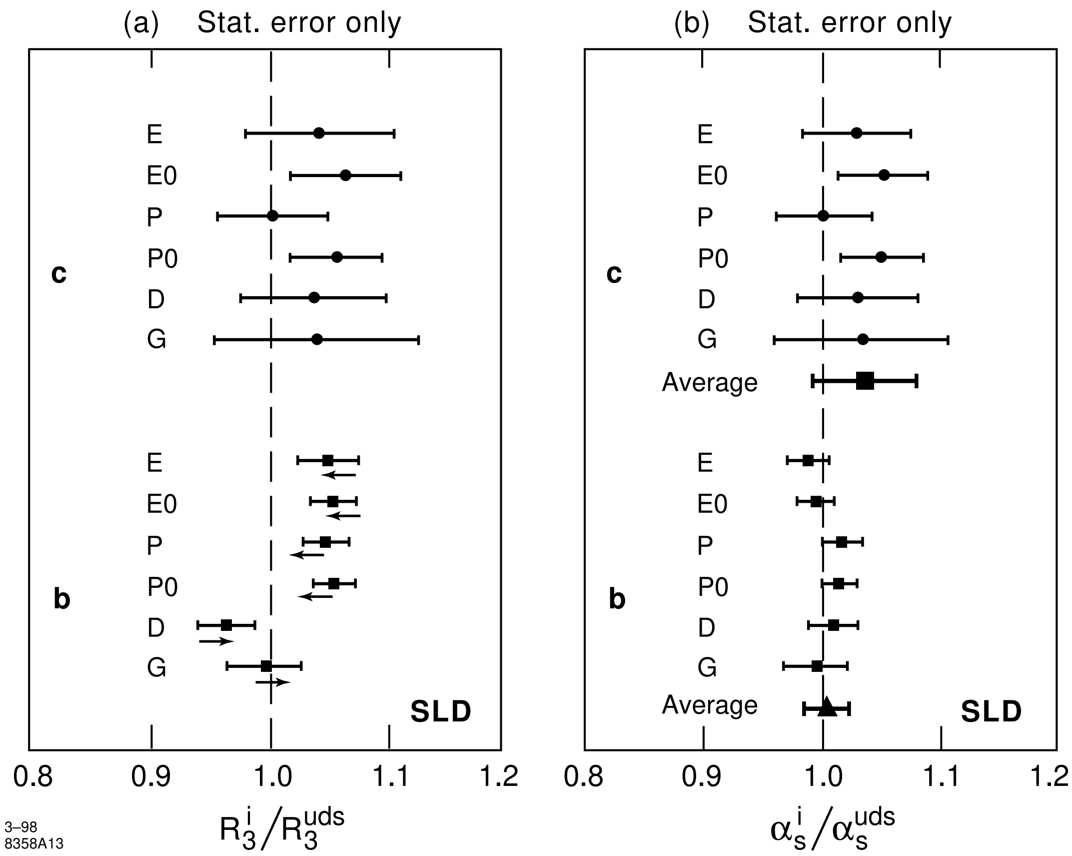
Avg.: $\alpha_s(M_Z^2) = 0.1188 \pm 0.0009$ (expt)
± 0.0066 (theory)

Flavour independence of α_s

Important test of QCD: any deviation $\rightarrow$ new physics

SLD SLAC-PUB-7660 (1998)

- Measure α_s for different flavour tagged samples:
  - decay length
  - inv. mass of particles from decay vertex
  - $\Longrightarrow$ high purity uds,c,b samples
- using 3-jet fraction, R_3, from various jet algorithms
  - b mass effects are not negligible
  - $R_3 \rightarrow \alpha_s$ using recent NLO calculations including mass effects



$$\frac{\alpha_s^c}{\alpha_s^{uds}} = 1.036 \pm 0.043\,(\text{stat})^{+0.041}_{-0.045}\,(\text{syst})^{+0.020}_{-0.018}\,(\text{th})$$

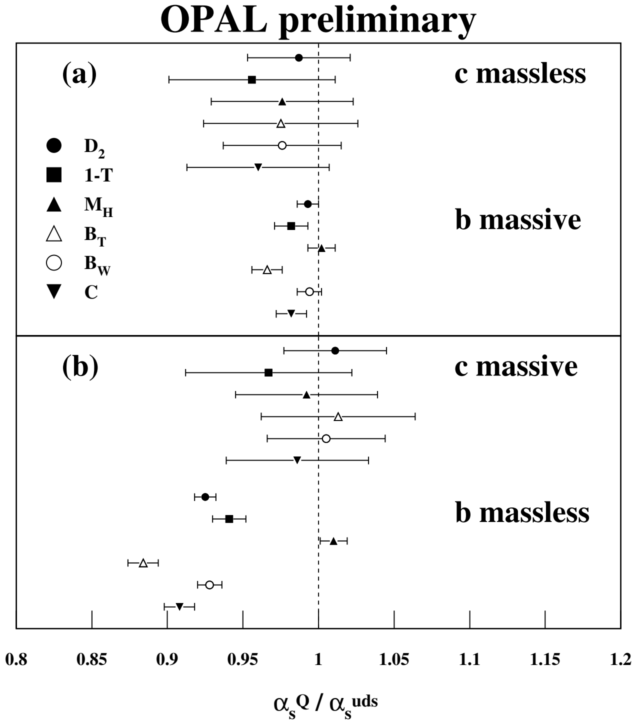
$$\frac{\alpha_s^b}{\alpha_s^{uds}} = 1.004 \pm 0.018\,(\text{stat})^{+0.026}_{-0.031}\,(\text{syst})^{+0.018}_{-0.029}\,(\text{th})$$

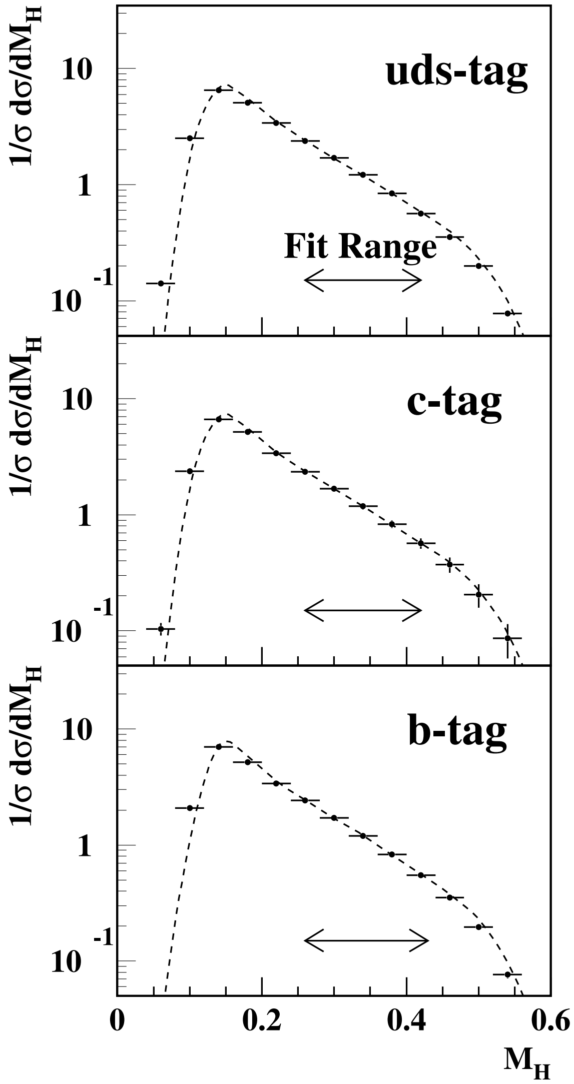
Flavour independence of α_s

OPAL: (ICHEP98 - 295)

tags with decay length and $D^{\star\pm}$ for c sample

- Measure for each tagged sample -- T,ρ,B_T,B_W,D_2 and C
- fit with NLO predictions including mass effects for c and b contributions





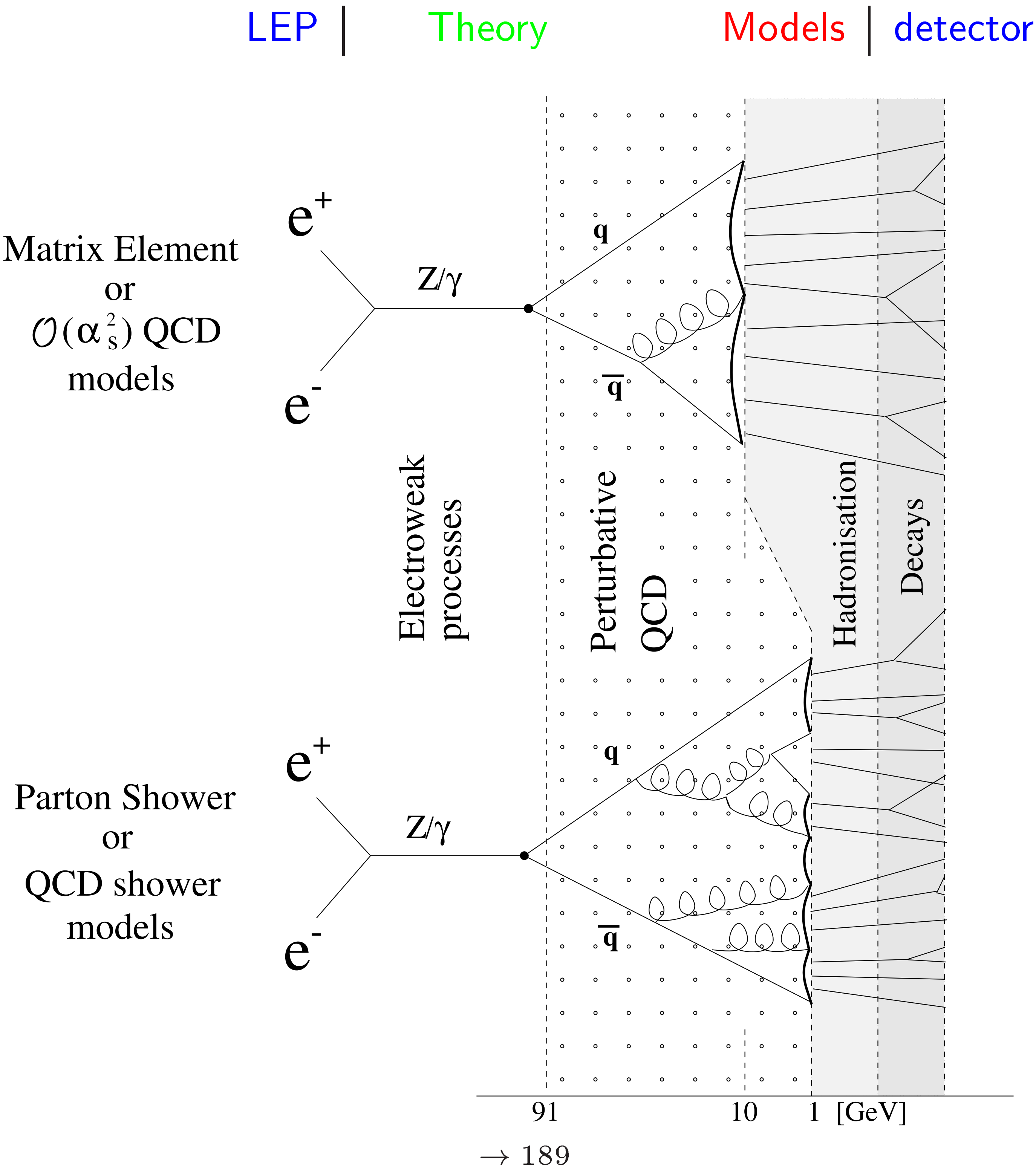
$$\frac{\alpha_s^c}{\alpha_s^{uds}} = 1.002 \pm 0.017\,(\text{stat}) \pm 0.025\,(\text{syst}) \pm 0.009\,(\text{th})$$

$$\frac{\alpha_s^b}{\alpha_s^{uds}} = 0.988 \pm 0.005\,(\text{stat}) \pm 0.004\,(\text{syst}) \pm 0.011\,(\text{th})$$

b mass effects clearly seen at a level of 5-8%

All results consistent with flavour independence of α_s

Monte Carlo models



Monte Carlo models

MC models are important:

- to correct data – e.g., hadronisation corrections, calculation of efficiencies, acceptances

- to test theoretical ideas – e.g., LLA, NLLA calculations are generally for massless quarks (or $s \to \infty$). MCs incorporate masses, E conservation, etc. (although perhaps not correctly)

- to gain insight by using (toy) models – e.g., in non-perturbative regions.

MCs for $e^+e^- \to$ hadrons:

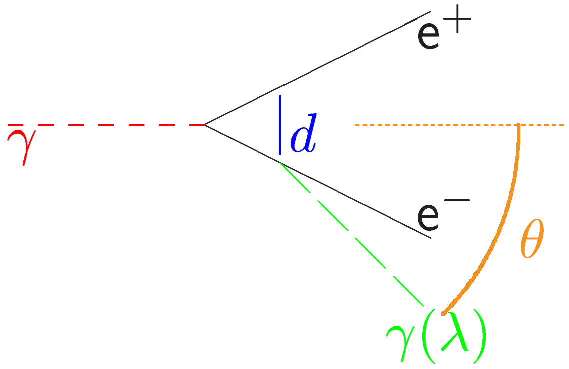
PYTHIA JETSET	$\mathcal{O}(\alpha_s^2)$ M.E.	string frag. (JETSET)
PYTHIA JETSET	∼MLLA shower	string frag. (JETSET)
ARIADNE	∼MLLA shower	string frag. (JETSET)
HERWIG	∼MLLA shower	cluster frag.
COJETS	∼LLA shower	independent frag.

Parton Showers

LLA $\Longrightarrow$ each branching independent
prob. of branching from Altarelli-Parisi eq.
Hence, easily programmable in MC

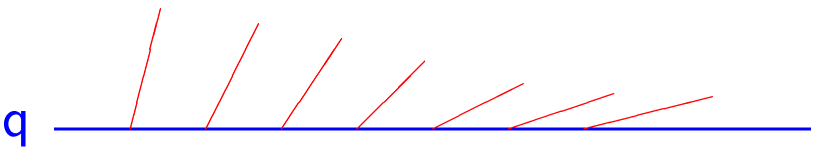
But, there are coherence effects

Angular Ordering:
QED (Chudakov effect)
if $\lambda_\perp = \lambda \sin\theta < d$
then radiation is suppressed
since γ cannot resolve the e$^-$,
$i.e.$, charge that γ would 'see'
is $Q = Q_\gamma = 0$



Quantitatively described by destructive interference between radiation from e$^+$ and e$^-$

QCD: $\gamma \to$ g e $\to$ q
More complicated since colour charge, $Q_{\rm g} \neq 0$
But still interference leads to
suppression of soft gluons at large angles
$\Longrightarrow$ angular ordering



successive radiation at smaller angles: $\theta_1 > \theta_2 > \theta_3 > \ldots$

Consequences of QCD Colour Coherence

- Angular Ordering
- Suppression of soft gluons

$\Longrightarrow$ MLLA incorporating all leading double and single logs for angular averaged observables

To incorporate in MC parton showers:
generate as in LLA (independently) but restrict angles of g emission:
from line 1 only if $\theta_{1k} < \theta_{12}$
from line 2 only if $\theta_{2k} < \theta_{12}$
$\Longrightarrow$ ϕ dependence.

Hadronisation

The parton shower is perturbative
So it is stopped when the virtual mass of a parton becomes less than some cut-off, Q_0, (~ 1 GeV)

What do you do after turning off the shower?

Hadronisation models:

- Independent Fragmentation

– Each parton hadronises independently

– g $\rightarrow$ q$\overline{\text{q}}$ according to Altarelli-Parisi
or g treated like a q or $\overline{\text{q}}$ (equal probability)

– q$\overline{\text{q}}$ pair is created out of 'sea'
with $p_\perp$ from a Gaussian dist. ('limited $p_\perp$')

– q $+ \overline{\text{q}}_{\text{sea}} \rightarrow$ meson, with fraction z of E of parent q, according to 'fragmentation function', $F(z)$.

– The remaining $\overline{\text{q}}_{\text{sea}}$ has fraction $(1-z)$ of E of parent q.

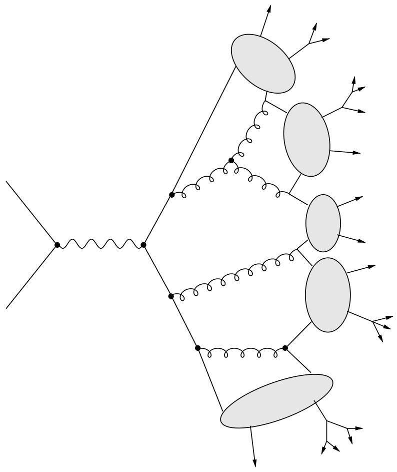
– Iterated until remaining E is less than some value

– Uses E rather than virtuality of q
q remains on mass shell $\Longrightarrow$ E,$\vec{p}$ not conserved
– must be fixed up later

– ignores colour fields of nearby partons

Hadronisation

- Cluster Fragmentation (HERWIG)

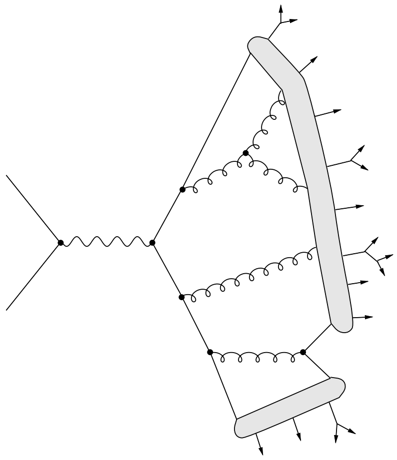


– g $\to$ q$\overline{\text{q}}$

– preconfinement of colour
  q$\overline{\text{q}}$ $\to$ colour – singlet cluster

– clusters decay (isotropically)

- Lund-string Fragmentation (JETSET)



– colour field between partons $\approx$ string, *i.e.*, thin tube of colour-field lines, between partons, which form a colour singlet

– As q, $\overline{\text{q}}$ at ends of string fly apart, string breaks,
  with q, $\overline{\text{q}}$ appearing at ends.

– Breaking of string is controlled by Gaussian $p_\perp$ and fragmentation fct., $F(z), z = E + p_{||}$

Most studies find better agreement of data with Lund-string Fragmentation than with Cluster Fragmentation.

Monte Carlo models

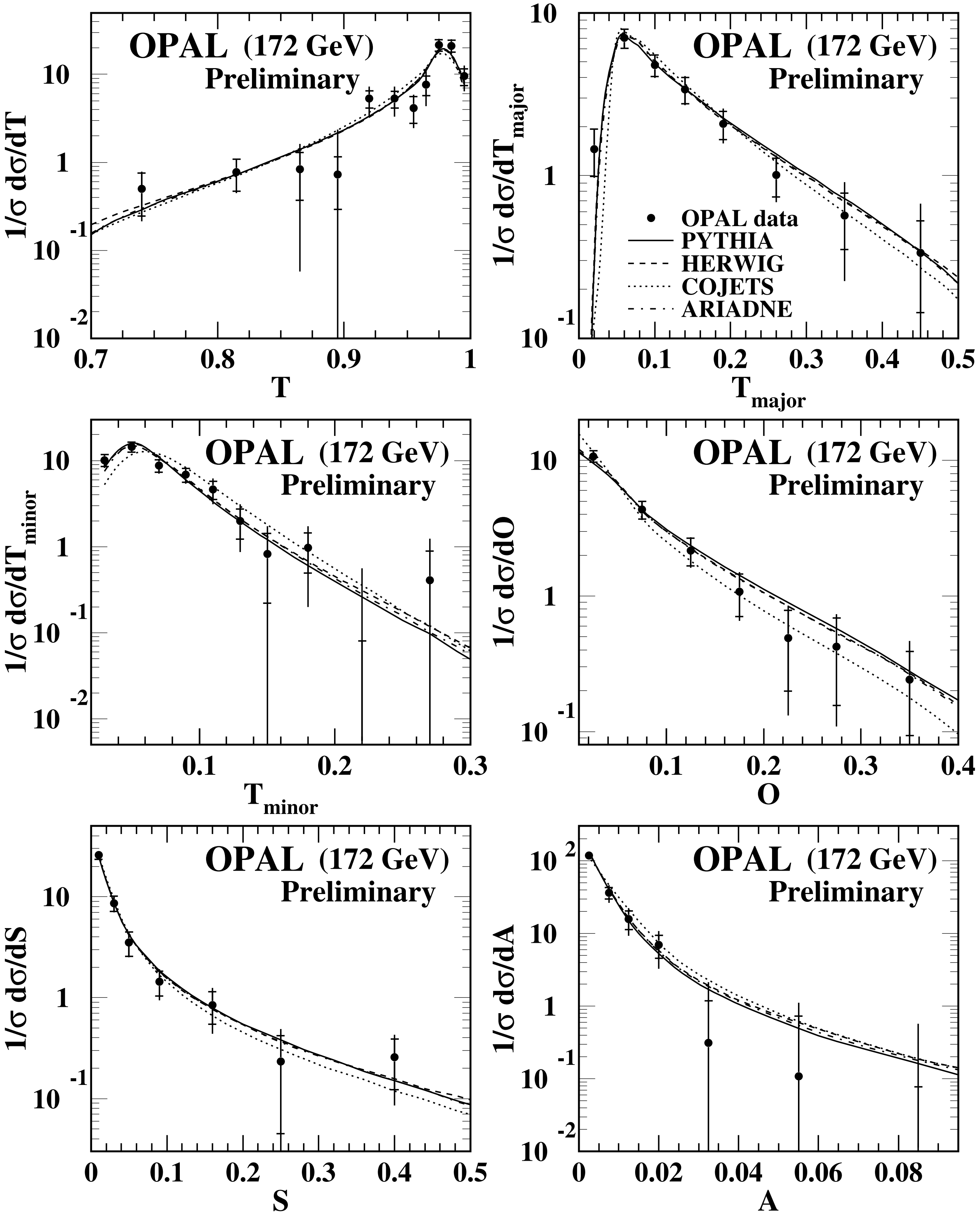
To see how well the MC models describe the data,

compare MC tuned at Z (LEP-I), to event shape variables Y

1. Y dist. to MC at high energy
2. evolution of $< Y >$ with $\sqrt{s}$

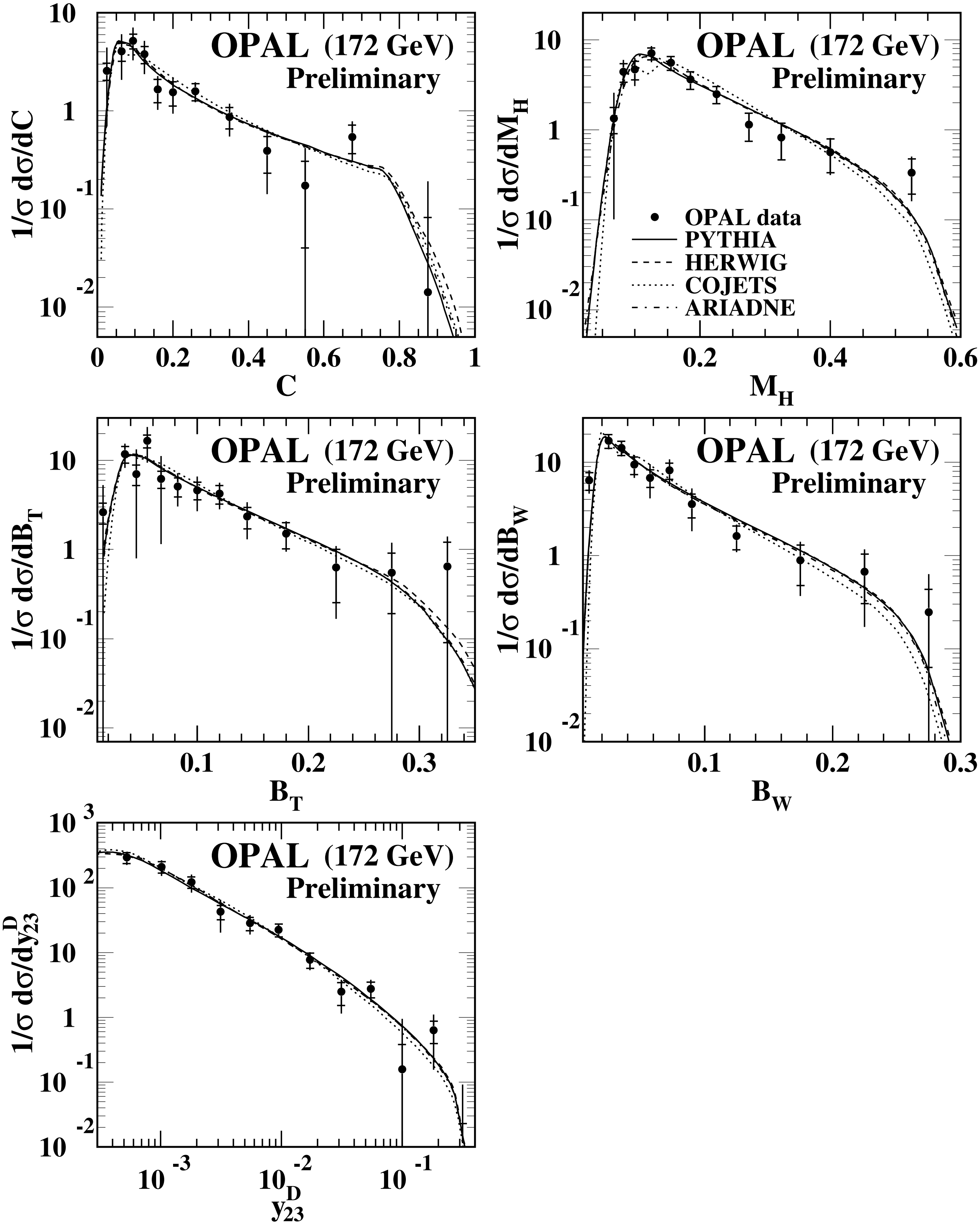
many variables – will not show all

Shape Variables



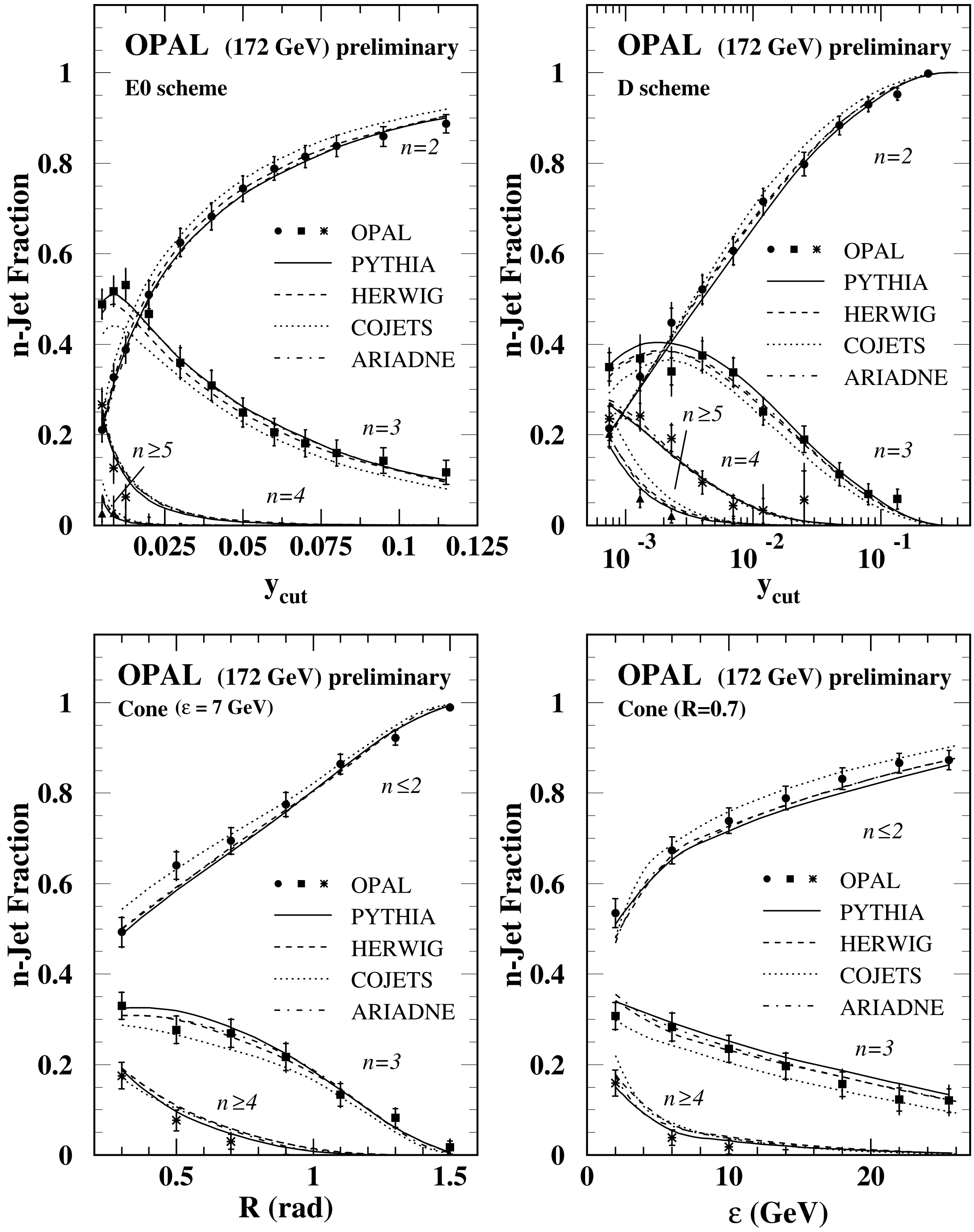
Good agreement

Shape Variables



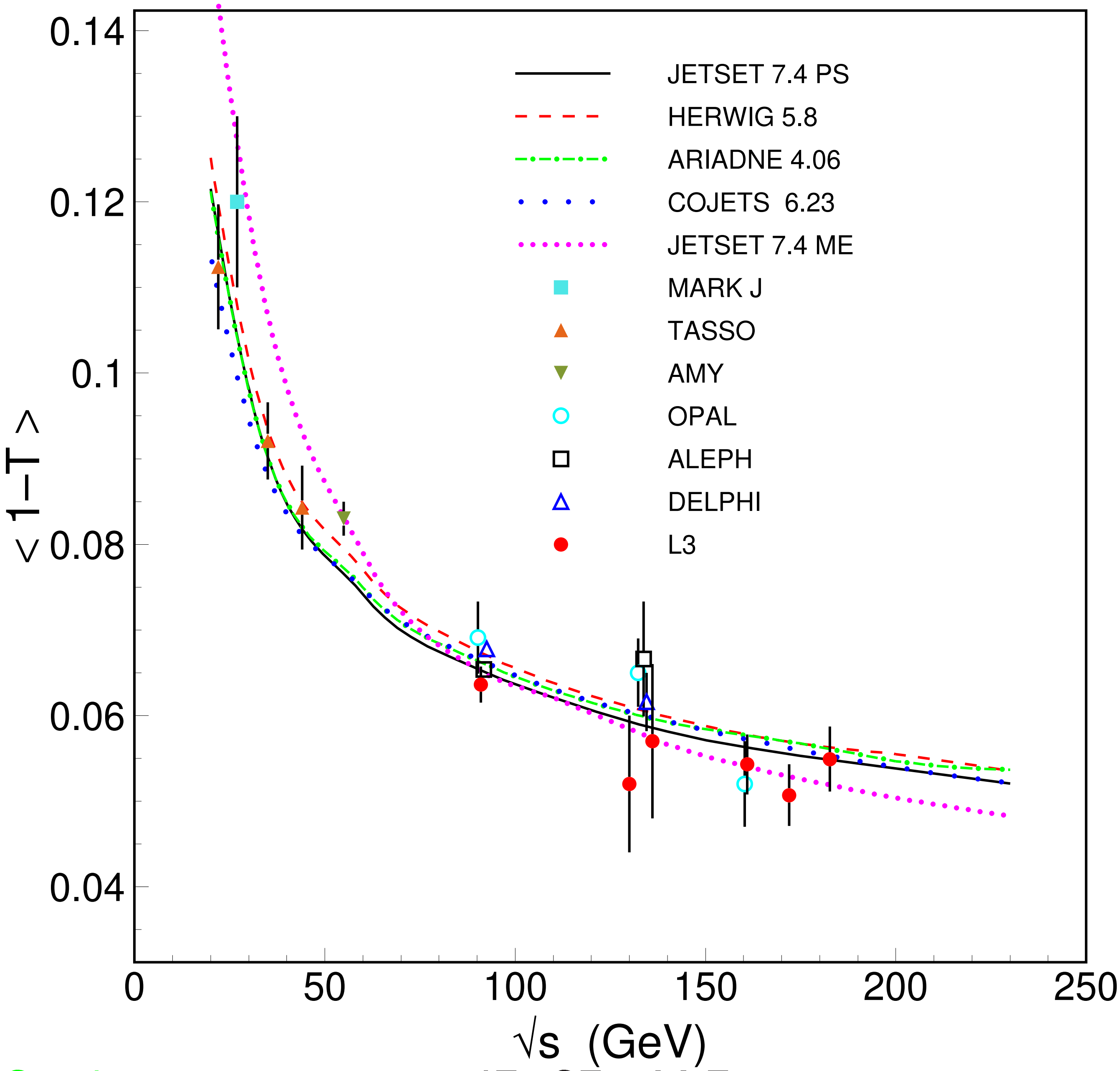
Good agreement

Jet Fractions



Good agreement

Shape Variables – Evolution



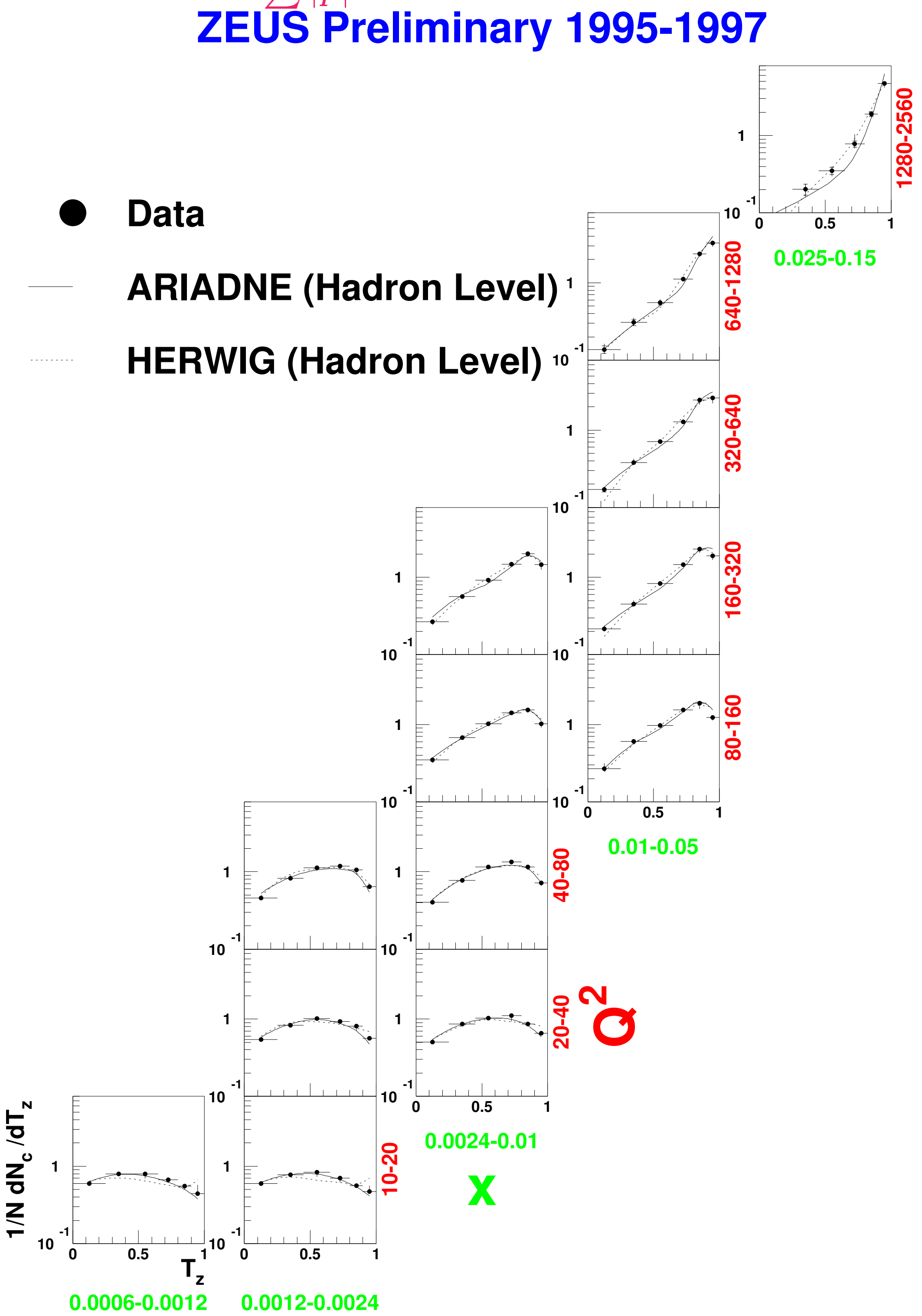
Good agreement except JETSET M.E.

Other shape variables – even better

Shape Variables – Evolution

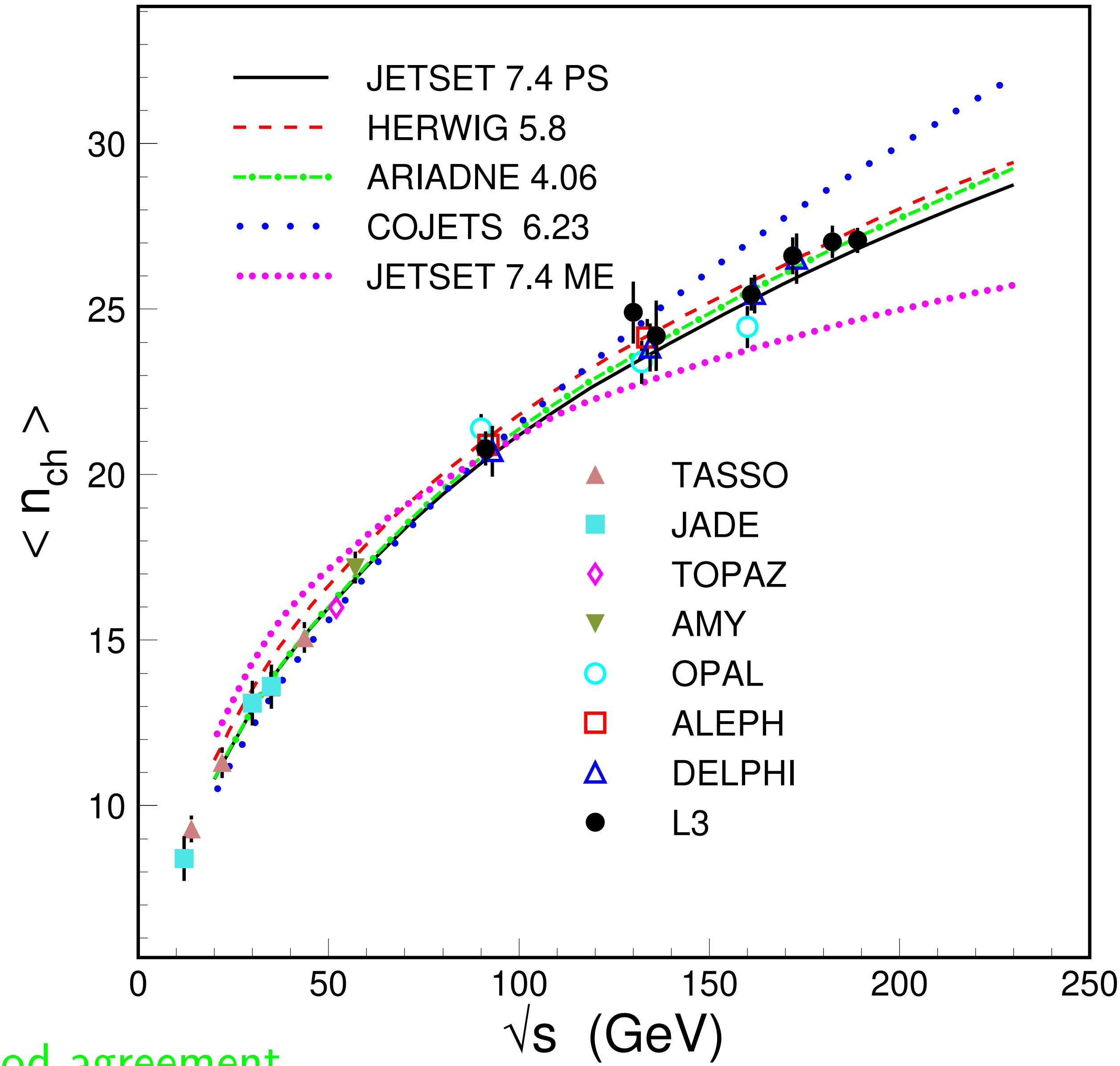
ZEUS shape variables (in Breit frame) (ICHEP98 808)

e.g., thrust, $T_z = \frac{\sum |\vec{p} \cdot \vec{n}|}{\sum |\vec{p}|}$, $\vec{n}$ along virtual photon



Good agreement with MCs for both x and Q^2 dependence

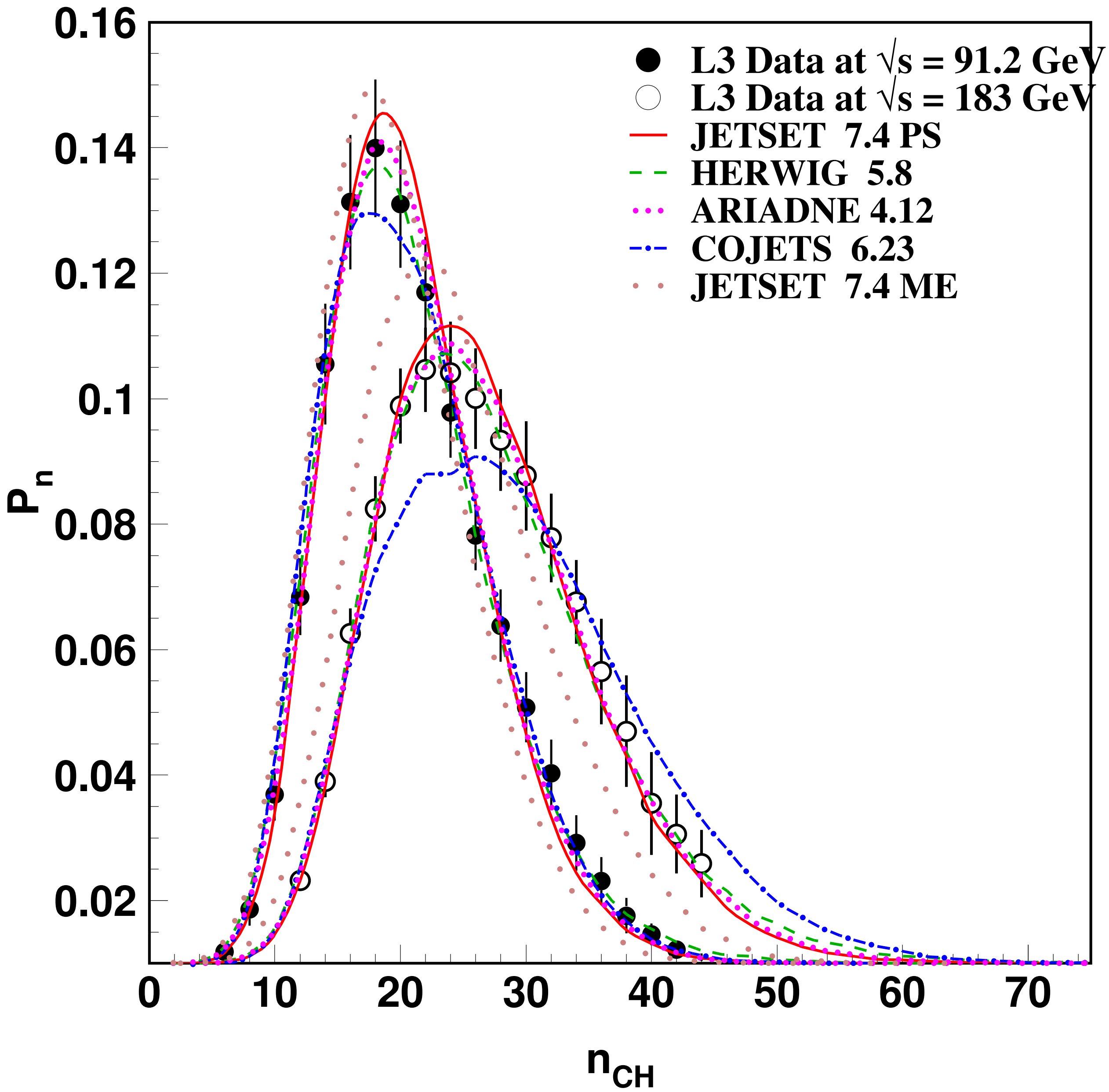
Charged-Particle Multiplicity



Good agreement
except for COJETS, JETSET M.E.

- COJETS disagrees because it has only LLA shower.
  Need MLLA shower, which incorporates
  soft gluon destructive interference.
- JETSET M.E. disagreement not surprising
  String fragmentation must simulate soft gluons.
  Number of soft gluons depends on E_{cm},
  so must be retuned at each E_{cm}.

Charged-Particle Multiplicity



Good agreement
except for COJETS, JETSET M.E.

Analytical calculations and LPHD

Besides MC,
there are Analytical Calculations in various approximations:

DLA	–double log approximation
LLA	–leading log approximation
MLLA	–modified leading log approximation
NLLA	–next-to-leading log approximation
NNLLA	–next-to-next-to-leading log approximation

But note:

- These calculations are for gluon radiation.
- Application to particles requires assumption of Local Parton Hadron Duality (LPHD):
$$f_{\text{particle}}(Y) \propto f_{\text{parton}}(Y)$$
an appealing, if simple, idea – more later

Jets at perturbative scales ($\gtrsim 1$ GeV) do not need LPHD
although hadronisation corrections are needed

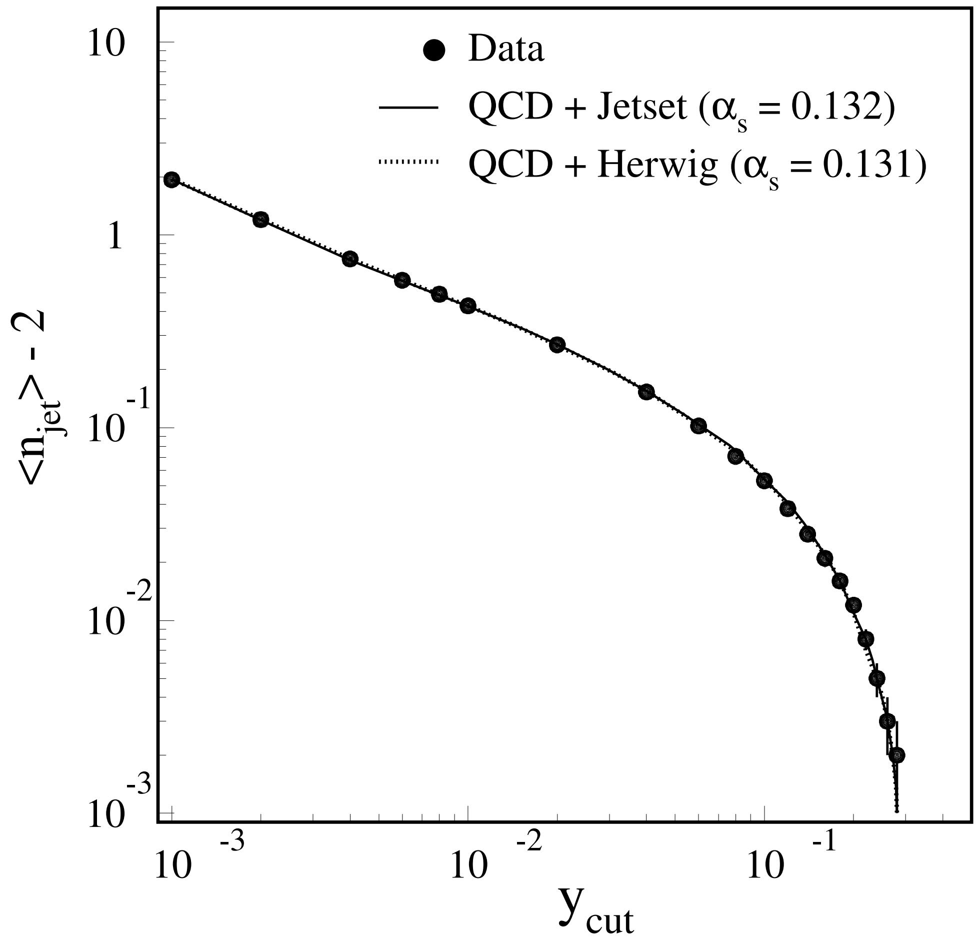
Recall jet-finder definitions, *e.g.*,
Durham: $y_{ij} = \frac{\min(E_i^2 E_j^2)}{s}(1-\cos\theta_{ij})$
Durham $sy_{ij} \approx p_{\mathrm{t}}^2$ of softer particle w.r.t. harder particle
p_{t}^2 is a reasonable scale for a jet
Hence, scale $= E_{\text{cms}}\sqrt{y_{\text{cut}}}$

Sub-jet Multiplicities

Mean jet multiplicity is calculable in perturbative QCD.

On Z, $y_{\text{cut}} = 10^{-3} \Rightarrow$ scale $M_Z\sqrt{10^{-3}} \approx 2.8$ GeV

$y_{\text{cut}} = 10^{-1} \Rightarrow$ scale $M_Z\sqrt{10^{-1}} \approx 28$ GeV

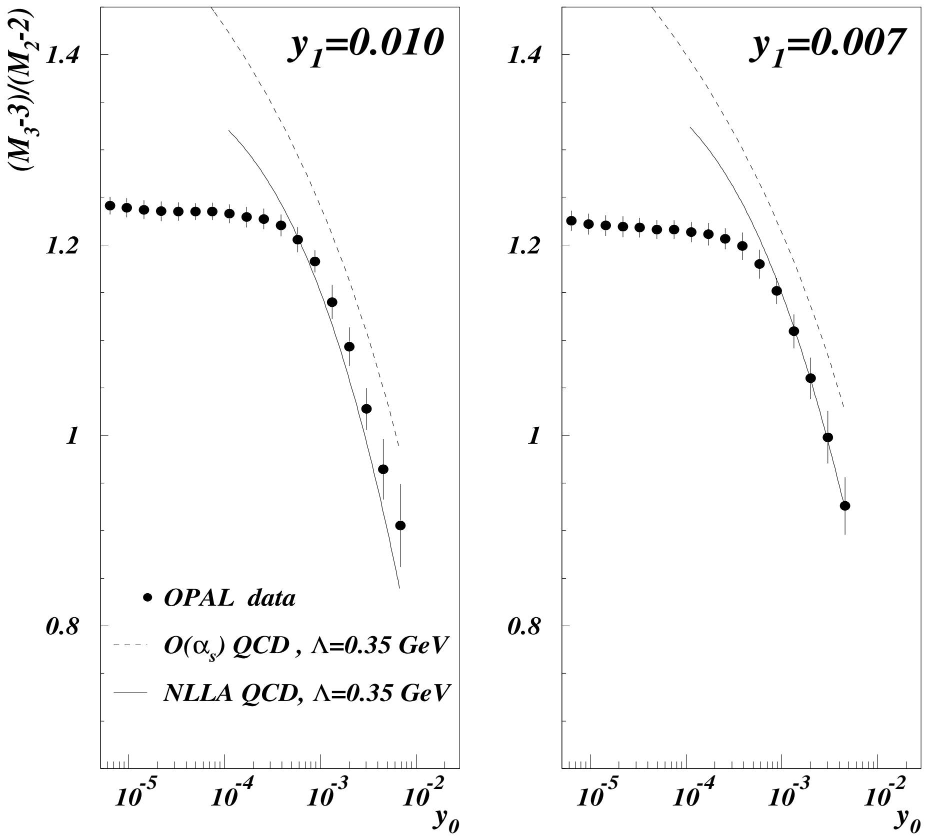


Good agreement

Jet Multiplicities – q/g

Coherence $\Longrightarrow$ gluons closer to jet axis
Therefore, smaller values of $y_{\rm cut}$ are needed to resolve the radiated gluons as jets.
Effect different for g, q jets, since different colour charges

- Decide if 2- or 3-jet event using $y_{\rm cut} = y_1$
- For $y_{\rm cut} = y_0 < y_1$, find number of jets
  – for 2-jet events, $M_2(y_0)$
  – for 3-jet events, $M_3(y_0)$



In pert. region: Good agreement with NNLA

Single-Particle Spectra

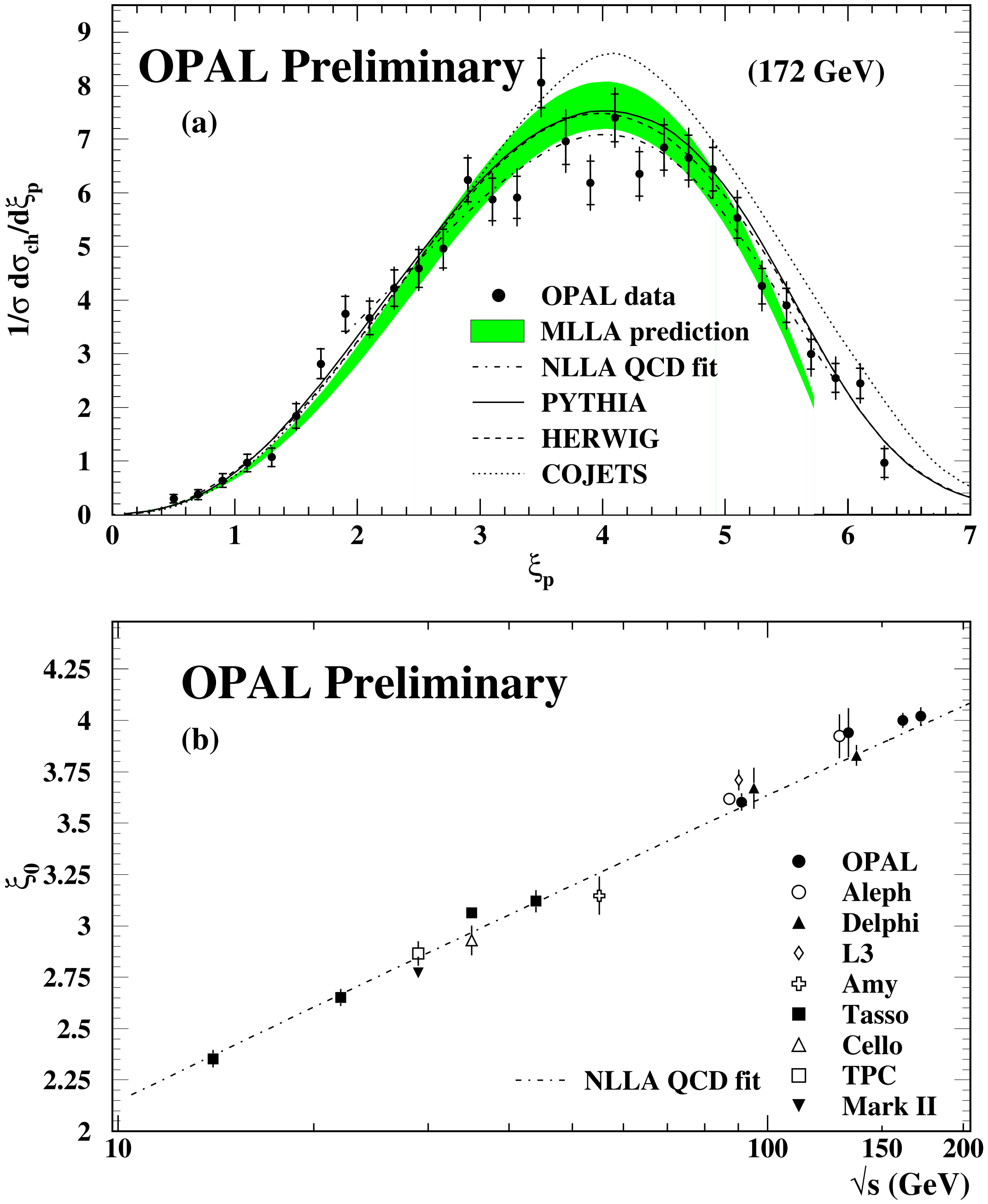
"Classic" evidence of gluon interference:

Gluon interference $\Longrightarrow$ suppression of soft gluons
Assuming LPHD,
$\Longrightarrow$ suppression of soft particles

For fragmentation studies (q $\rightarrow$ hadrons), one looks at distribution of $x_p = \frac{2p}{E_{\text{cms}}}$ ($= \frac{p}{E_{\text{jet}}}$ for 2-jet event)
To study soft-particle production, look at $\xi_p = \ln \frac{1}{x_p}$

At lowest order, for massless particles, expect a Gaussian distribution.
Phase space and non-zero masses cause some suppression of small-x (large ξ)
Additional suppression from gluon interference

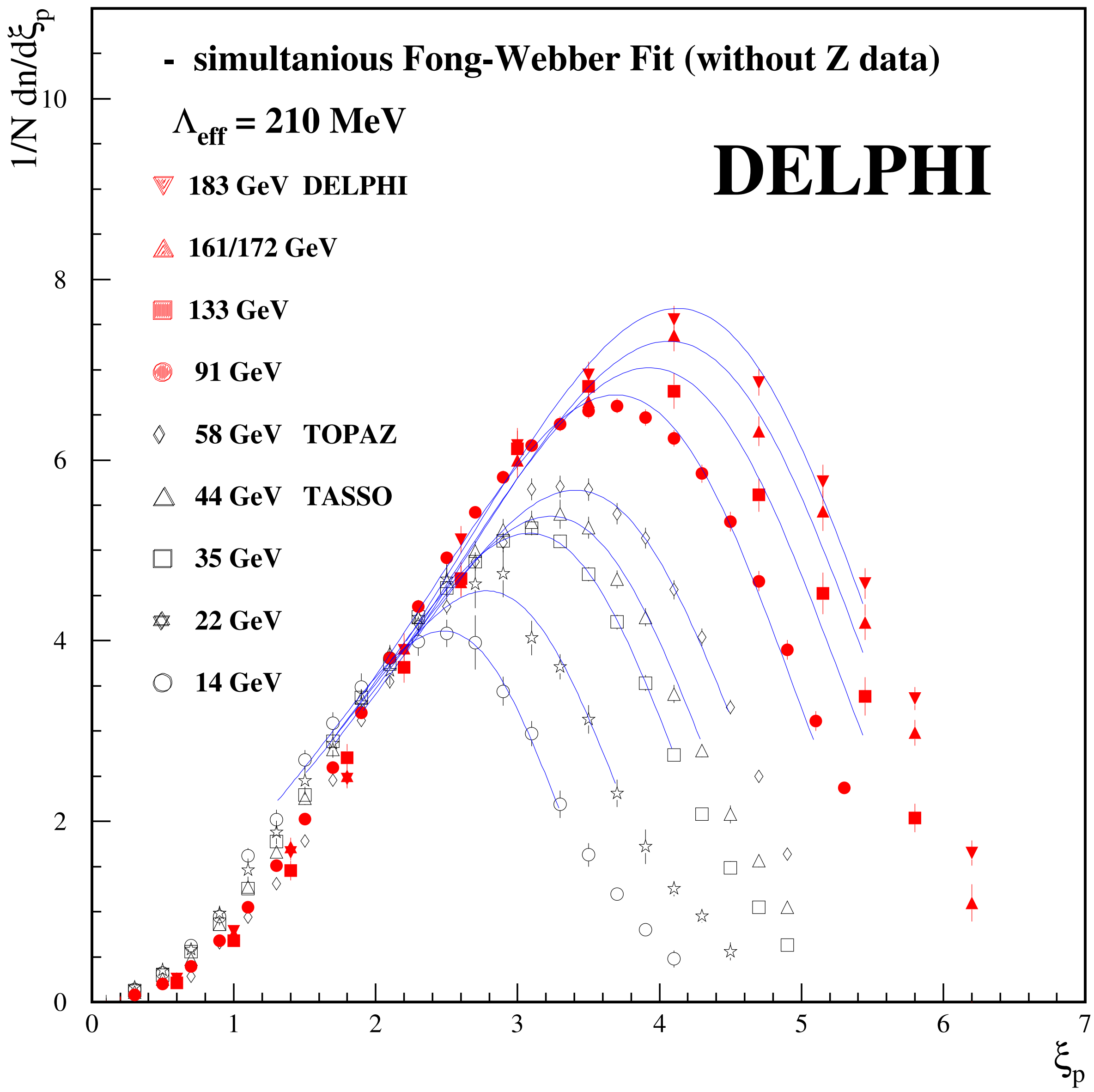
Single-Particle Spectra



(a) COJETS (LLA) far off
PYTHIA, HERWIG, MLLA a bit too high at max.
MLLA drops a bit too fast at high $\xi_p = -\ln x_p = \frac{2\,p}{E_{\text{cms}}}$
NLLA Good agreement (Fong, Webber Nucl.Phys.B355(1992)54)

(b) NLLA describes well ξ_0 evolution with $\sqrt{s}$.
($\xi_0 = \xi$ at which d$\sigma/$dξ_p is max.)

Single-Particle Spectra



Single-Particle Spectra

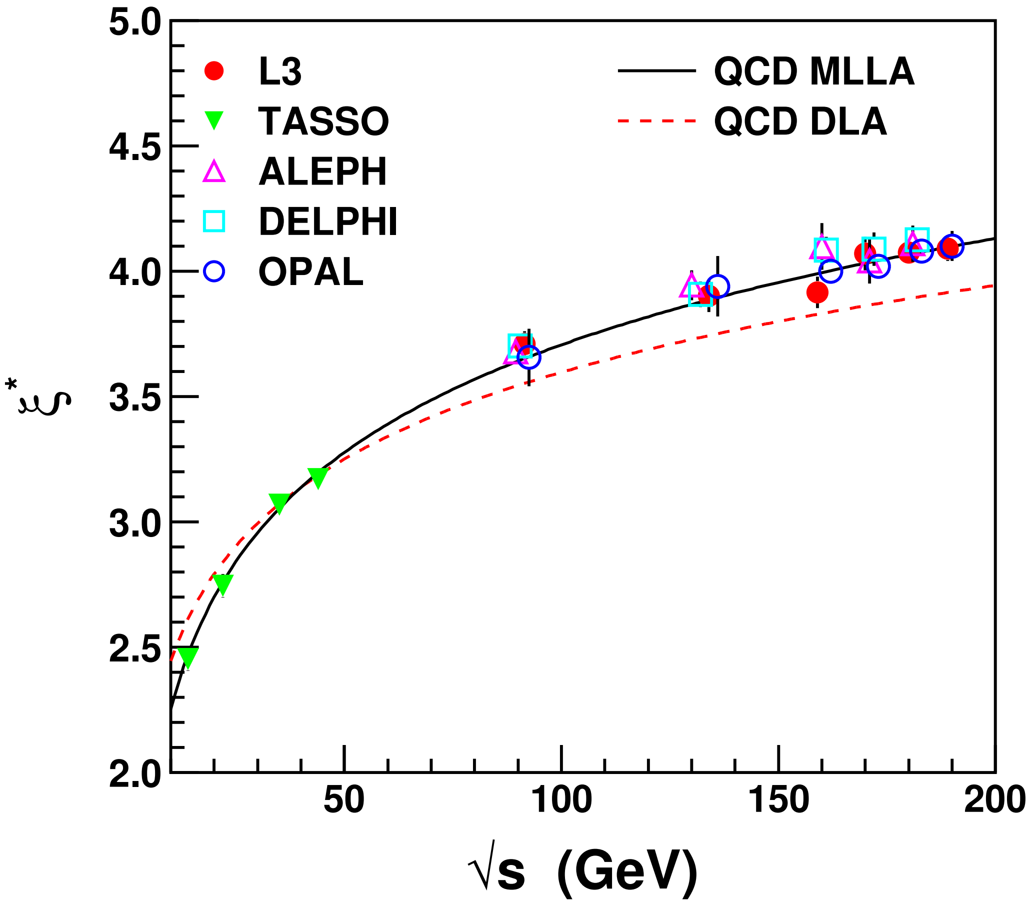
Predictions for energy evolution of $\xi^\star$:

LLA $\quad \xi^\star = Y \quad , \quad$ where $Y = \ln\left(\frac{\sqrt{s}}{2\Lambda_{\text{eff}}}\right)$

DLA $\quad \xi^\star = Y \ \left(\frac{1}{2}\right)$

MLLA $\quad \xi^\star = Y \ \left(\frac{1}{2} + \sqrt{\frac{C}{Y}} - \frac{C}{Y} + \mathcal{O}(Y^{-3/2})\right)$

where C is known fct. of number of colours, flavours, *etc.*

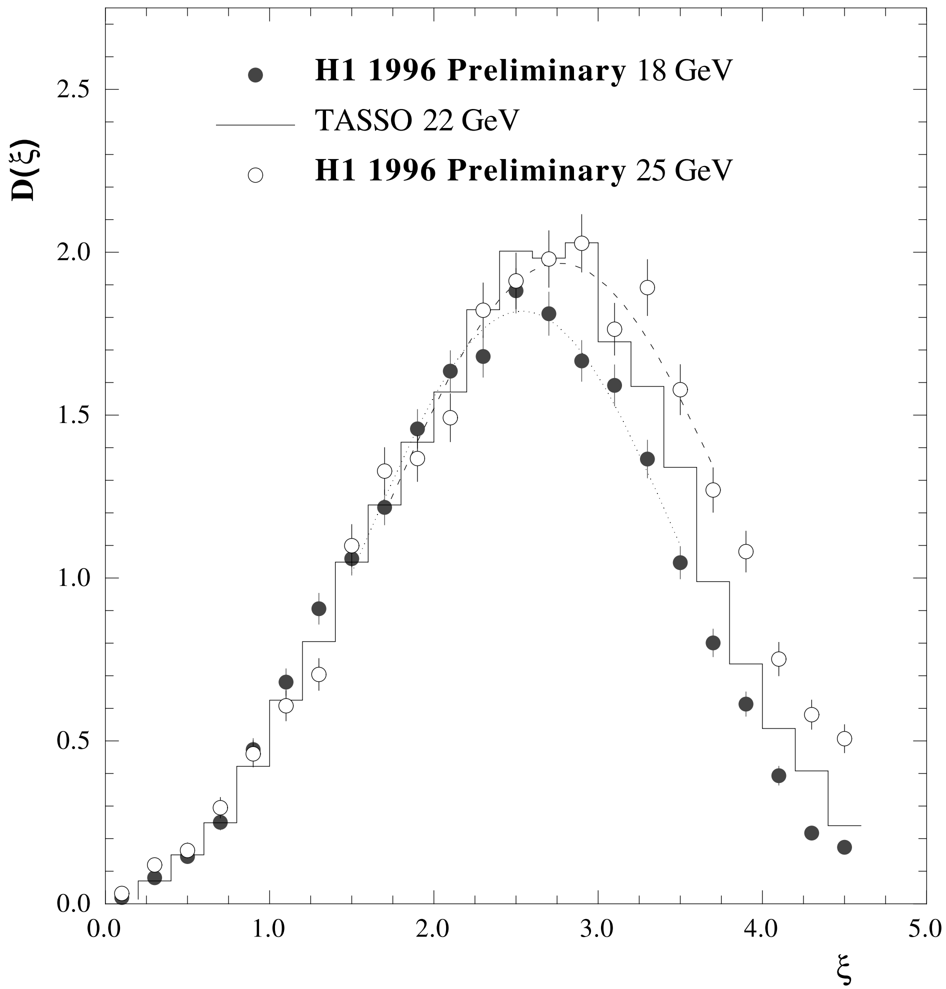


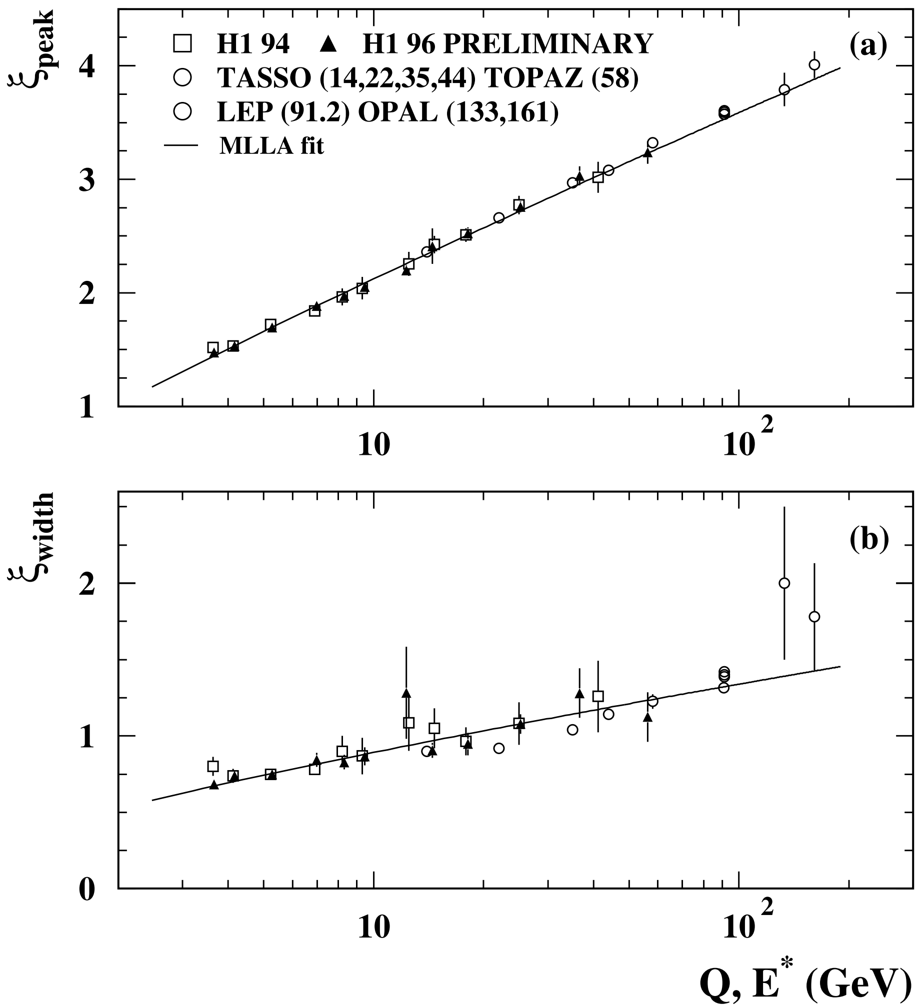
Good agreement with MLLA

Clear evidence for its gluon interference

Single-Particle Spectra

Consistent results from H1 in Breit frame:





Quark-Gluon jet differences

Larger gluon colour charge $\left(\text{g/q: } \frac{C_\mathsf{A}}{C_\mathsf{F}} = \frac{9}{4}\right)$

$\Longrightarrow$ gluons radiate more soft gluons than quarks do

LL: $\frac{<N_\mathrm{g}>}{<N_\mathrm{q}>} = \frac{C_\mathsf{A}}{C_\mathsf{F}} = \frac{9}{4}$

$\Longrightarrow$ more particles in a gluon jet than in a quark jet

LL+LPHD: $\frac{<N_\mathrm{g}>}{<N_\mathrm{q}>} = \frac{C_\mathsf{A}}{C_\mathsf{F}} = \frac{9}{4}$

$\Longrightarrow$ gluons in a gluon jet are softer than in a quark jet

$\Longrightarrow$ particles in a gluon jet are softer than in a quark jet

But comparison of q and g jets is difficult

and previous results: $\frac{<N_\mathrm{g}>}{<N_\mathrm{q}>} \approx 1.0 - 1.25 \neq \frac{9}{4}$

Quark-Gluon jet differences

Comparison is difficult. To use $e^+e^- \to q\bar{q}g$ events,

- want unbiased method of identifying q
  use b-tagging – usually decay vertex

- want $E_q \approx E_g$ select
  - "Mercedes" topology:
    $E_1 \approx E_2 \approx E_3$ (jets 1 and 2 with b-tags)
  - "Y" topology:
    $E_1 > E_2 \approx E_3$ (jet-2 with b-tag, assume jet 1 is q)

- But soft gluon interference in PS depends on angles between q, g jets

- Analytical calculations of properties of q, g jets are for
  - $q\bar{q}$ events and for gg events.
    q or g jet is then 1 hemisphere of the event.
  - But, we do NOT have gg events.

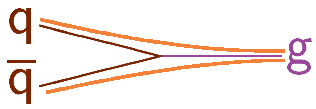
2 new approaches to using $e^+e^- \to q\bar{q}g$ events:

- OPAL: Events which approximate gg events.

- DELPHI: Different scales depending on jet angles for q,g.

Charged Particle Multiplicity — q/g

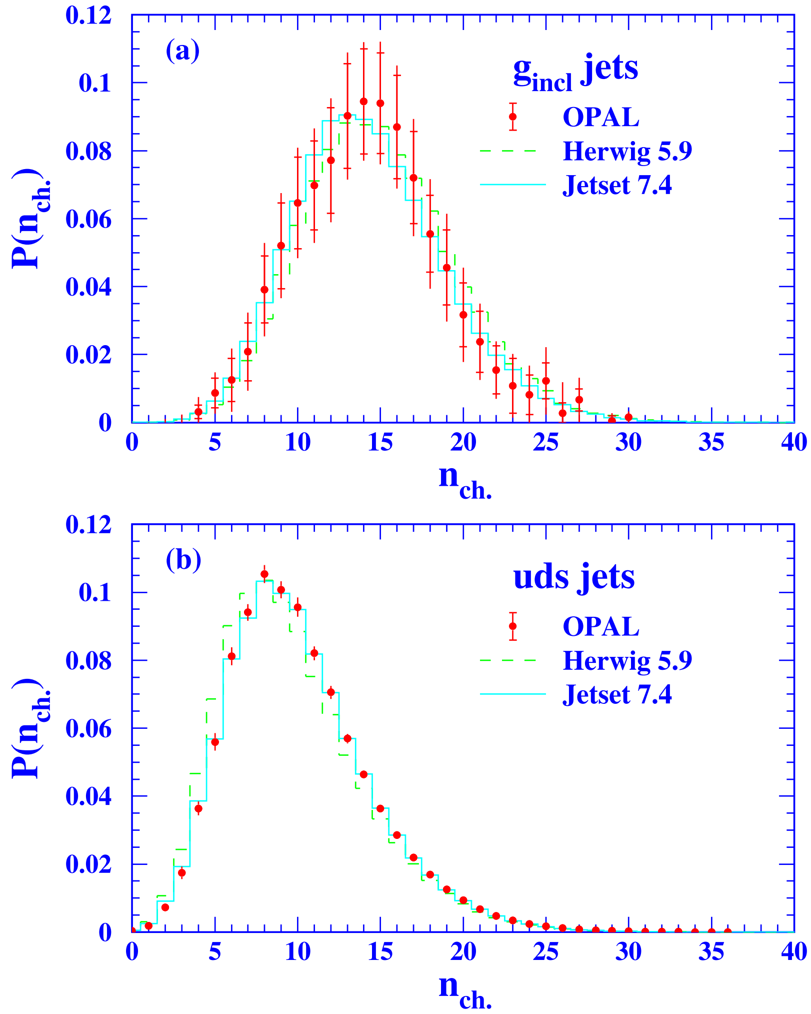
OPAL selects g jet in $q\bar{q}g$ events where $q\bar{q}$ close together (10-50°) and both tagged as b (294 events).



Then colour field ≈ that in a "gg" event.

OPAL compares the g with a q from a 2-jet ($q\bar{q}$) event with a light-quark tag (since QCD calculations are for massless quarks)

$\langle E_g \rangle = 41.2$ GeV (from MC) $\approx$ $E_q = E_{beam} = 45.6$ GeV



Good agreement with JETSET, HERWIG

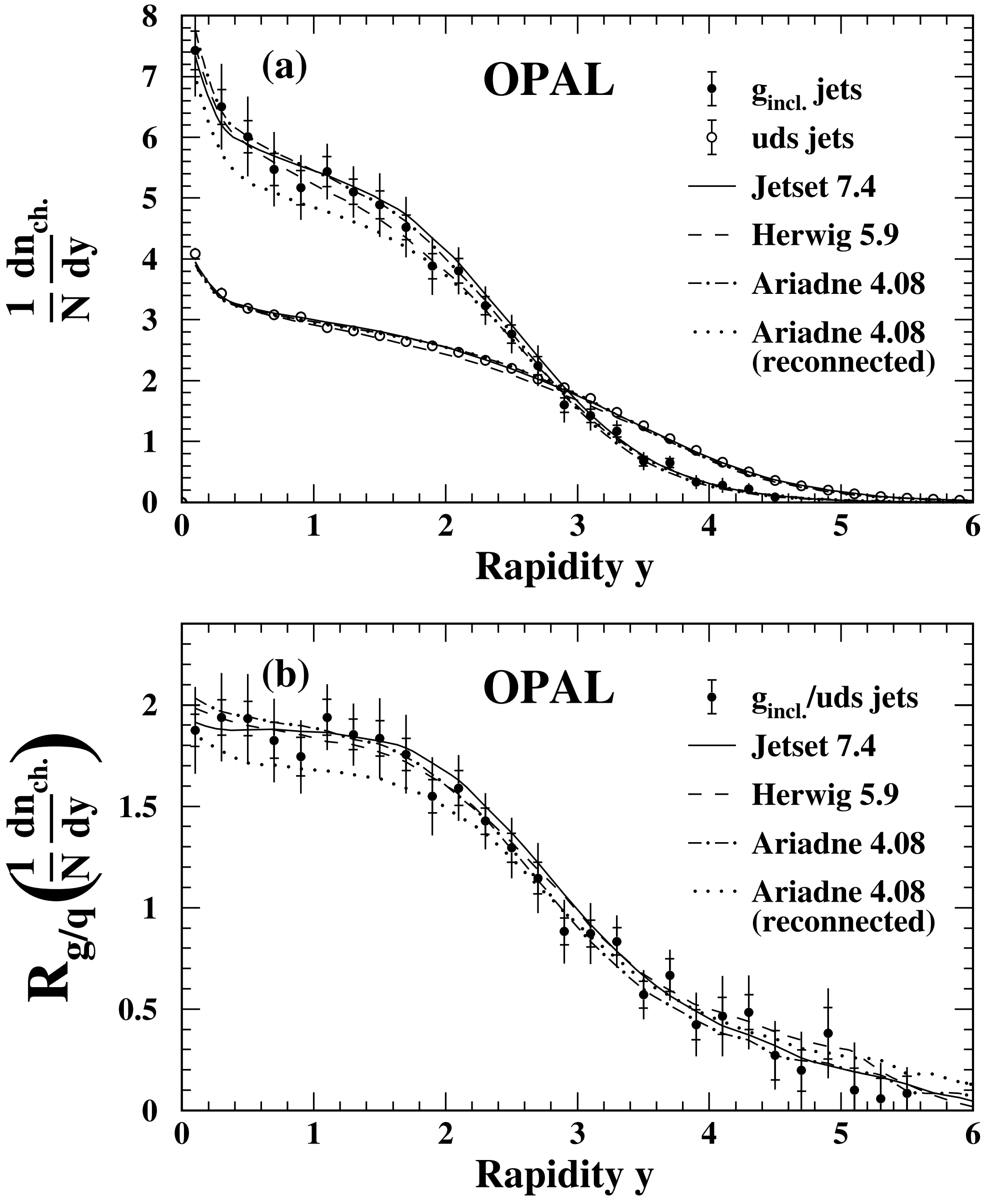
Correcting q mult. to 41.2 GeV using QCD $\langle n_{ch} \rangle$ evolution,

$$r \equiv \frac{\langle n_{ch} \rangle_g}{\langle n_{ch} \rangle_{uds}} = 1.494 \pm 0.025 \pm 0.051$$

Charged Particle Multiplicity — q/g

But $\quad r \equiv \frac{\langle n_{\text{ch}}\rangle_{\text{g}}}{\langle n_{\text{ch}}\rangle_{\text{uds}}} = 1.494 \pm 0.025 \pm 0.051 \neq 2.25$

However, $r = C_{\text{A}}/C_{\text{F}} = 2.25$ is an *asymptotic* prediction for *soft* gluons.
So, OPAL looks as function of rapidity w.r.t. sphericity axis



For softest particles ($|y| < 2$), $\quad r = 1.815 \pm 0.038 \pm 0.062$
closer to 2.25

Charged Particle Multiplicity — q/g

DELPHI adopts a (EPS97 conf., ICHEP98 - 146)
"hardness scale" for quarks: (Dokshitzer, Diakonov, Trojan)

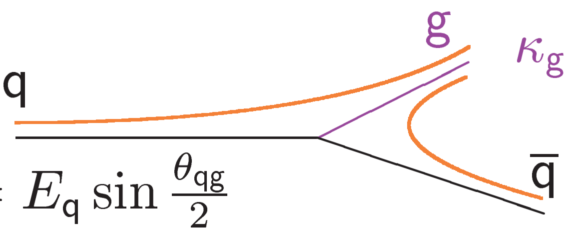
$$\kappa_{\mathsf{DDT}} = E_{\mathsf{jet}} \sin\frac{\theta}{2}$$

$\theta =$ angle between jet and closest other jet

$\kappa_{\mathsf{DDT}} \approx p_{\mathsf{t}}$ of process producing the parton

For 2-jet event, $\theta = 180° \Longrightarrow \kappa = \kappa_{\mathsf{DDT}} \equiv E_{\mathsf{jet}} = E_{\mathsf{cms}}/2$

For gluons:



$$\kappa_{\mathsf{q}} = E_{\mathsf{q}} \sin\frac{\theta_{\mathsf{qg}}}{2}$$

$$\kappa_{\bar{\mathsf{q}}} = E_{\bar{\mathsf{q}}} \sin\frac{\theta_{\bar{\mathsf{q}}\mathsf{g}}}{2}$$

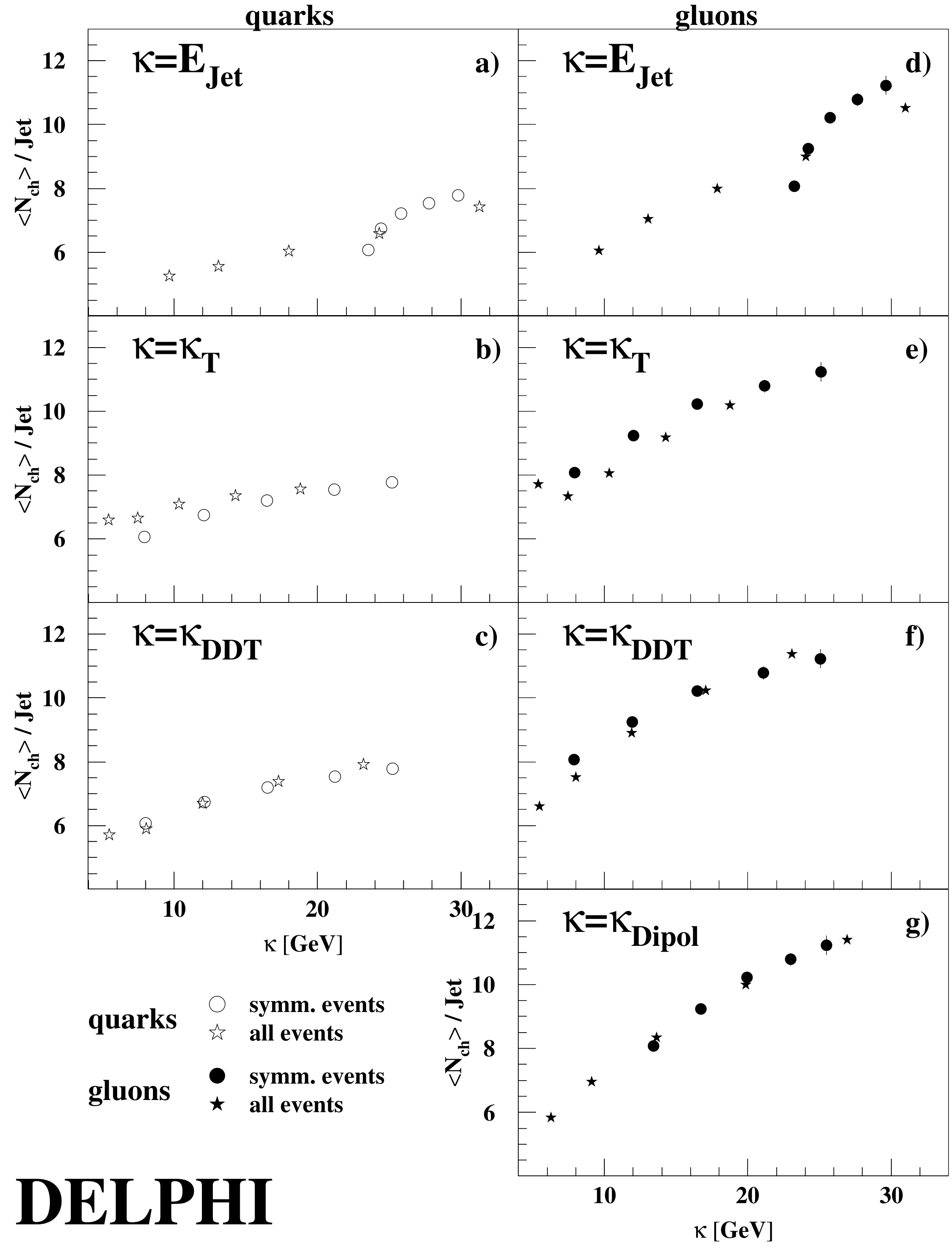
From dipole model colour fields, angular dependence of κ_{g} should be some kind of average of κ_{q}, $\kappa_{\bar{\mathsf{q}}}$

– scale usually enters calculations via logs

– $\therefore$ use "average of logs" $\Rightarrow$ harmonic mean:

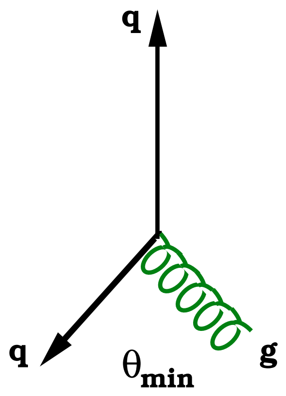
$$\kappa_{\mathrm{g}} = \kappa_{\mathsf{dipole}} \equiv E_{\mathrm{g}} \sqrt{\sin\frac{\theta_{\mathsf{qg}}}{2} \sin\frac{\theta_{\bar{\mathsf{q}}\mathsf{g}}}{2}}$$

Charged Particle Multiplicity — q/g



Topological dependence is taken into account by κ_{DDT} for quarks and κ_{DDT}, or somewhat better, κ_{dipole} for gluons

Charged Particle Multiplicity — q/g



Using symmetric ("Y") events:
$E_\mathrm{q} \approx E_\mathrm{g}$
and κ_DDT for both q and g

Fit to
NNLO analytic prediction (Gaffney & Mueller, Nucl.Phys.B250(1985)1109)
+ LPHD assumption
+ constant for non-perturbative effects

$$\begin{aligned} < N_\mathrm{q} > &= N + N_\mathrm{q}^\mathrm{non-pert} \\ < N_\mathrm{g} > &= N r(\kappa) + N_\mathrm{g}^\mathrm{non-pert} \\ \text{where } N &= K \alpha_\mathrm{s}^b \exp \frac{c}{\sqrt{\alpha_\mathrm{s}}} [1 + \mathcal{O}(\sqrt{\alpha_\mathrm{s}})] \\ r(\kappa) &= \frac{C_\mathrm{A}}{C_\mathrm{F}} (1 - r_1\gamma_0 - r_2\gamma_0^2) \end{aligned}$$

r_1, r_2 are the NL, NNL terms
b, c, r_1, r_2 are known functions of C_A, C_F, n_f, α_s.

Thus 3 fit params.: K, $N_\mathrm{q}^\mathrm{non-pert}$, $N_\mathrm{g}^\mathrm{non-pert}$

Also take $\frac{C_\mathrm{A}}{C_\mathrm{F}}$ as fit param. to measure it

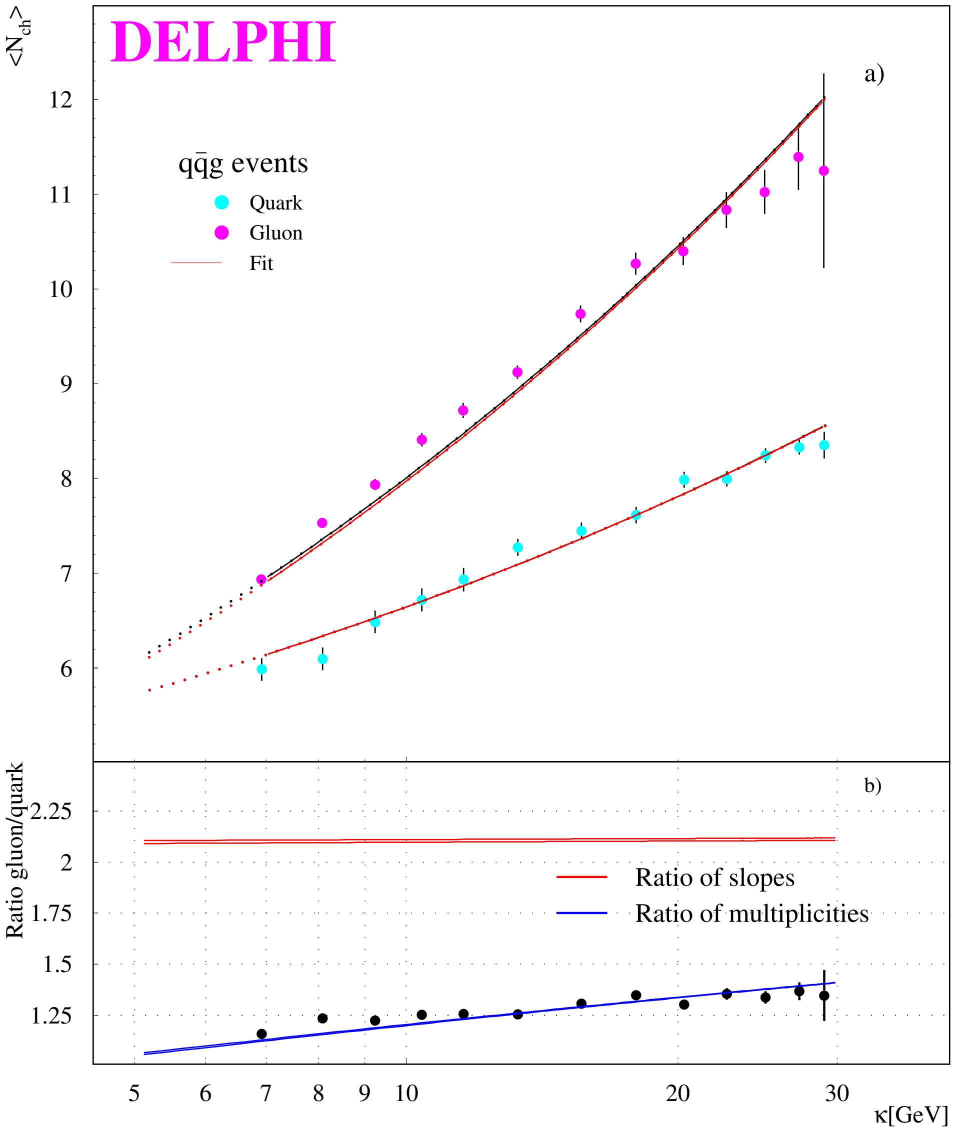
Charged Particle Multiplicity — q/g

Good fit

Success for NLO with

κ_{DDT}

+ LPHD



larger multiplicity for g, increasing with κ

Ratio of slopes directly related to $\frac{C_A}{C_F}$

	$\frac{C_A}{C_F}$	
Fit:	2.26 ± 0.10	
SU(3)	2.25	agrees well
ALEPH 4-jet	$2.20 \pm 0.09 \pm 0.13$	(CERN-PPE/97-002)
DELPHI 4-jet	$2.51 \pm 0.25 \pm 0.16$	(EPS97 236)

Charged Particle Multiplicity — q/g

Note: ratio of slopes less sensitive to non-pert. effects:

With ansatz

$$
\begin{aligned}
N_\mathrm{q}(\kappa) &= N(\kappa) + N_\mathrm{q}^{\mathrm{non-pert}} \\
N_\mathrm{g}(\kappa) &= N(\kappa) r(\kappa) + N_\mathrm{g}^{\mathrm{non-pert}}
\end{aligned}
$$

Non-pert. terms	are	independent of κ
$r(\kappa) \approx \frac{C_\mathrm{A}}{C_\mathrm{F}}$	$\sim$	independent of κ

Hence, derivatives have no non-pert. terms and

$$
\frac{\frac{\mathrm{d}N_\mathrm{q}(\kappa)}{\mathrm{d}\kappa}}{\frac{\mathrm{d}N_\mathrm{g}(\kappa)}{\mathrm{d}\kappa}} \approx \frac{C_\mathrm{A}}{C_\mathrm{F}}
$$

Charged Particle Multiplicity — q/g

Can improve determination of $C_{\mathsf{A}}/C_{\mathsf{F}}$
using MLLA prediction for multiplicity of a 3-jet event:

(Dokshitzer *et al.*, Sov.J.Nucl.Phys. 47(1988)881)

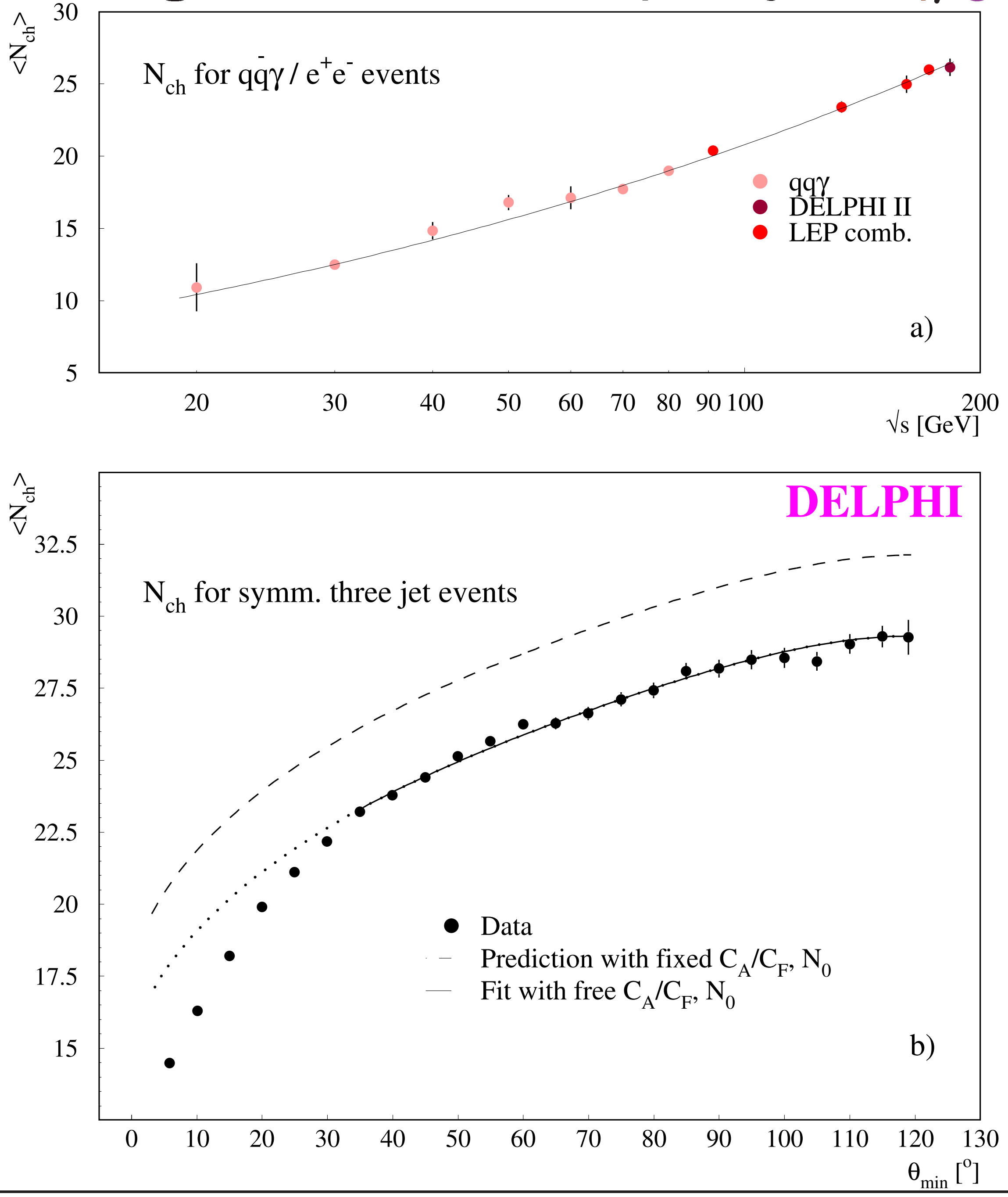
$$
\begin{aligned}
N_{\mathrm{q\bar{q}g}} &= N_{\mathrm{q\bar{q}}}(2E^{\star}_{\mathrm{q}}) + r(p^{\star}_{\mathrm{g}\perp}) \left(\frac{1}{2} N_{\mathrm{q\bar{q}}}(p^{\star}_{\mathrm{g}\perp}) - N^{\mathrm{non-pert}}_{\mathrm{q}} \right) \\
N_{\mathrm{q\bar{q}}\gamma} &= N_{\mathrm{q\bar{q}}}(2E^{\star}_{\mathrm{q}})
\end{aligned}
$$

where $E^{\star}_{\mathrm{q}}$, $p^{\star}_{\mathrm{g}\perp}$ are in rest-frame of $\mathrm{q\bar{q}}$

Avoids necessity of assigning particles to jets

Fit $N_{\mathrm{q\bar{q}g}}$ and $N_{\mathrm{q\bar{q}}\gamma}$

Charged Particle Multiplicity — q/g



	C_A/C_F	
Fit:	$2.266 \pm 0.053(\text{st.}) \pm 0.055(\text{sys.}) \pm 0.096(\text{th.})$	
SU(3)	2.25 agrees well – most precise measurement	
ALEPH 4-jet	$2.20 \pm 0.09 \pm 0.13$	(CERN-PPE/97-002)
DELPHI 4-jet	$2.51 \pm 0.25 \pm 0.16$	(EPS97 236)

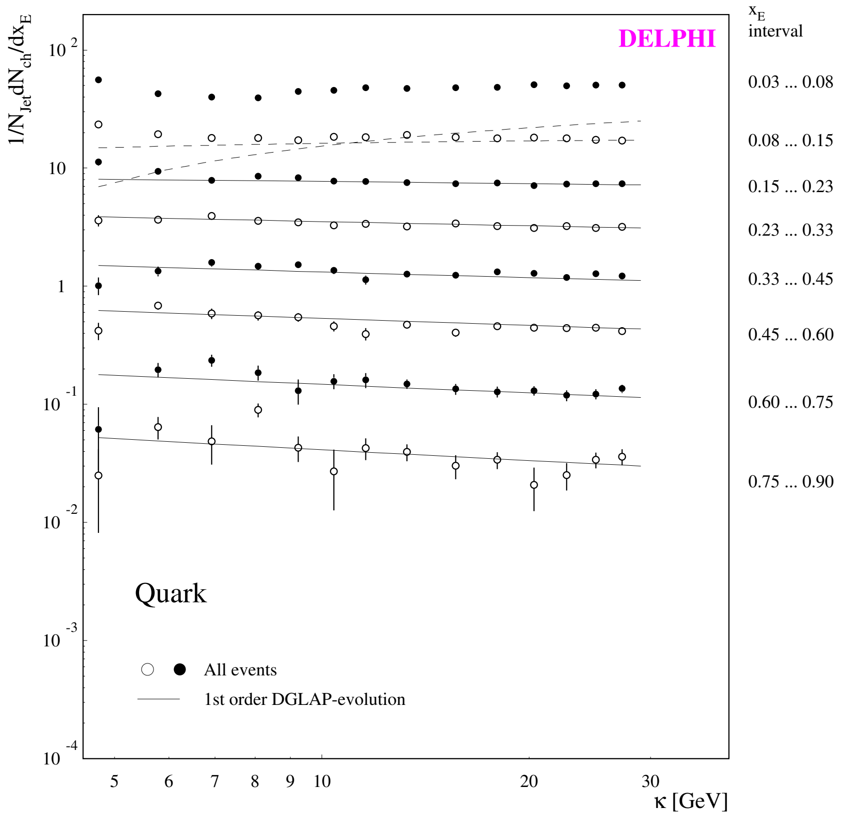
Fragmentation functions — q/g

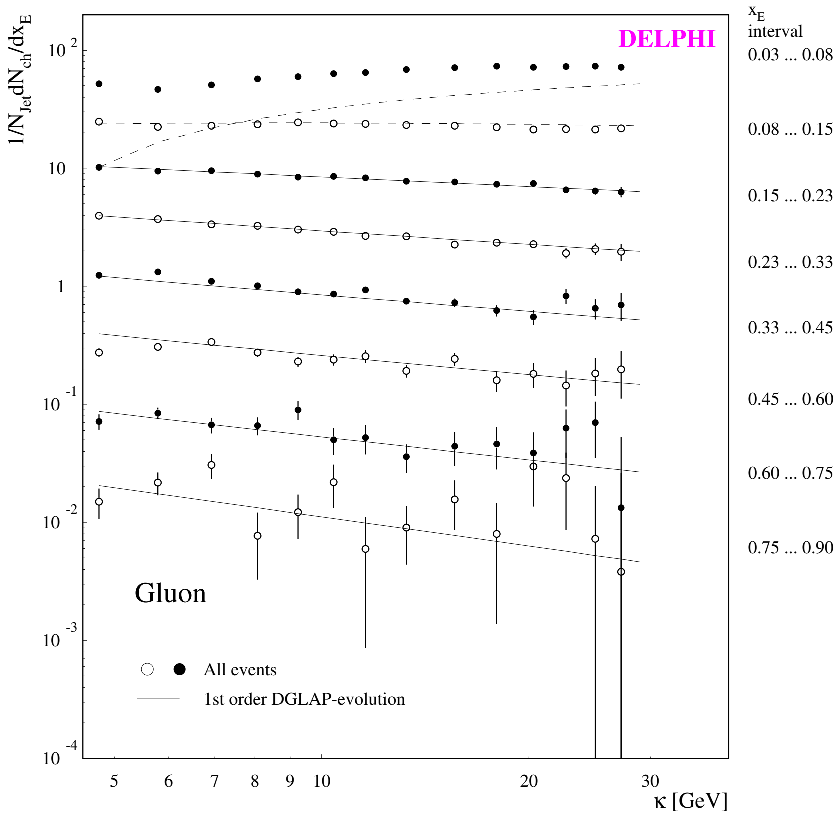
frag. fct. of q, g to stable hadrons, $D(x_\mathsf{E}, Q^2)$, $\quad x_\mathsf{E} = \frac{E_\mathsf{hadron}}{E_\mathsf{jet}}$

As for structure fcts. of hadrons, QCD predicts $D(x_\mathsf{E}, Q^2)$ at LO by the DGLAP eqs.

DELPHI uses $Q^2 = \kappa$ Good agreement with DGLAP

q

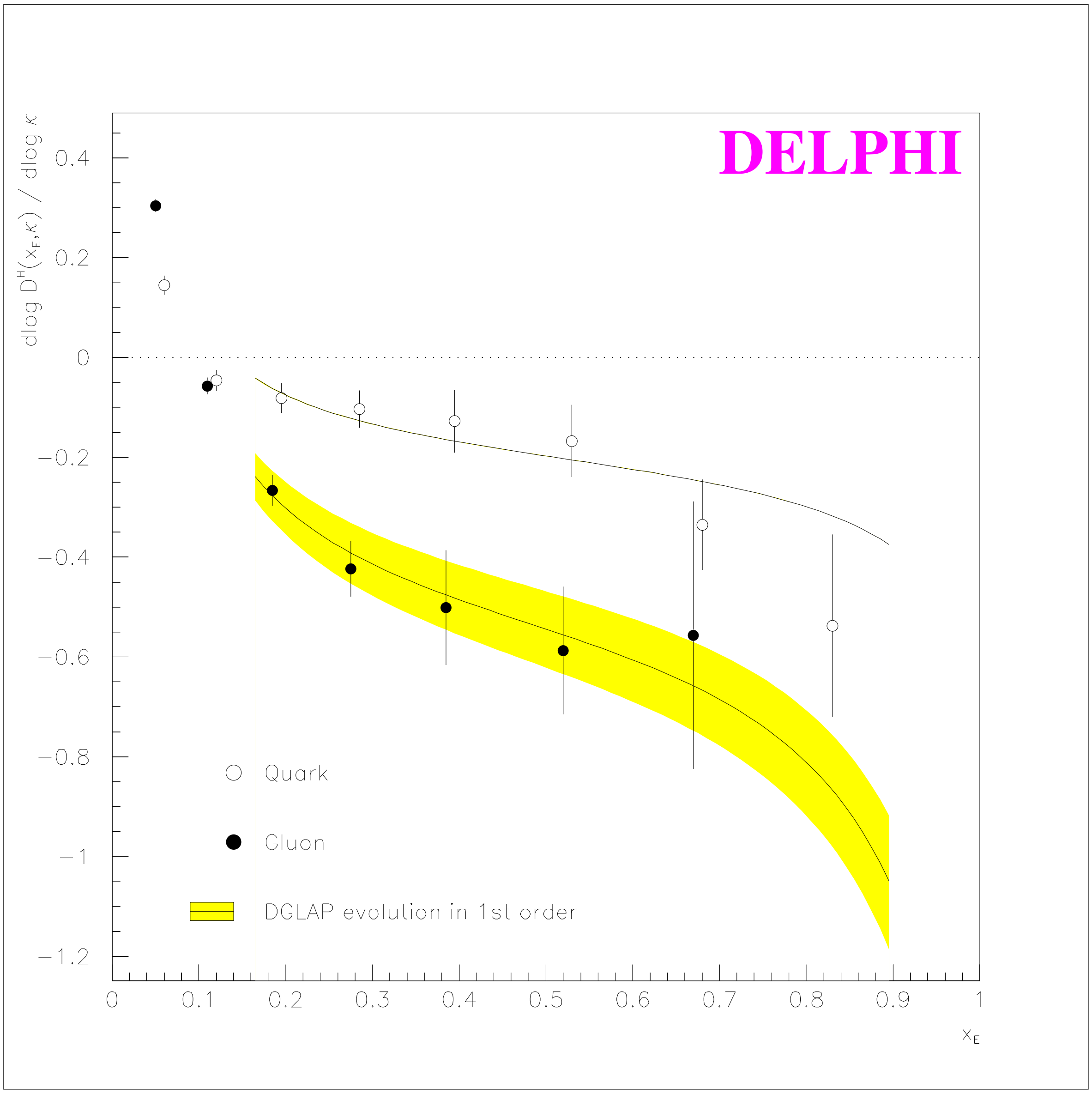




g

Fragmentation functions — q/g

$$\frac{\mathrm{d}\ln D(x_{\mathsf{E}},\kappa)}{\mathrm{d}\ln\kappa}$$



scaling violation for g larger than for q, as expected

Good agreement with DGLAP

Sensitive to colour factors: $\frac{C_{\mathsf{A}}}{C_{\mathsf{F}}} = 2.44 \pm 0.21(\text{stat.})$

Fluctuations

Usually studied using Factorial and Cumulant moments of the charged particle multiplicity distribution, P_n

$$F_q = \frac{\sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n}{\left(\sum_{n=1}^{\infty} n\, P_n\right)^q}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \binom{q-1}{m} K_{q-m} F_m$$

and their ratio

$$H_q = \frac{K_q}{F_q}$$

F_q measures q-particle correlations
K_q measures genuine q-particle correlations

- "Statistical Fluctuations" $=$ Independent production
  P_n is Poissonian $\qquad F_q = 1$
- Correlations imply $\qquad F_q > 1$

The normalization $\left(\sum_{n=1}^{\infty} n\, P_n\right)^q$
can be done differently,
and various "corrections" can be used.

NFMs and Intermittency

Divide phase space into M bins (binsize $= \delta$),
and average NFMs over the bins

- "Statistical Fluctuations" $=$ Independent production
  P_n is Poissonian $\qquad F_q(\delta) = 1$

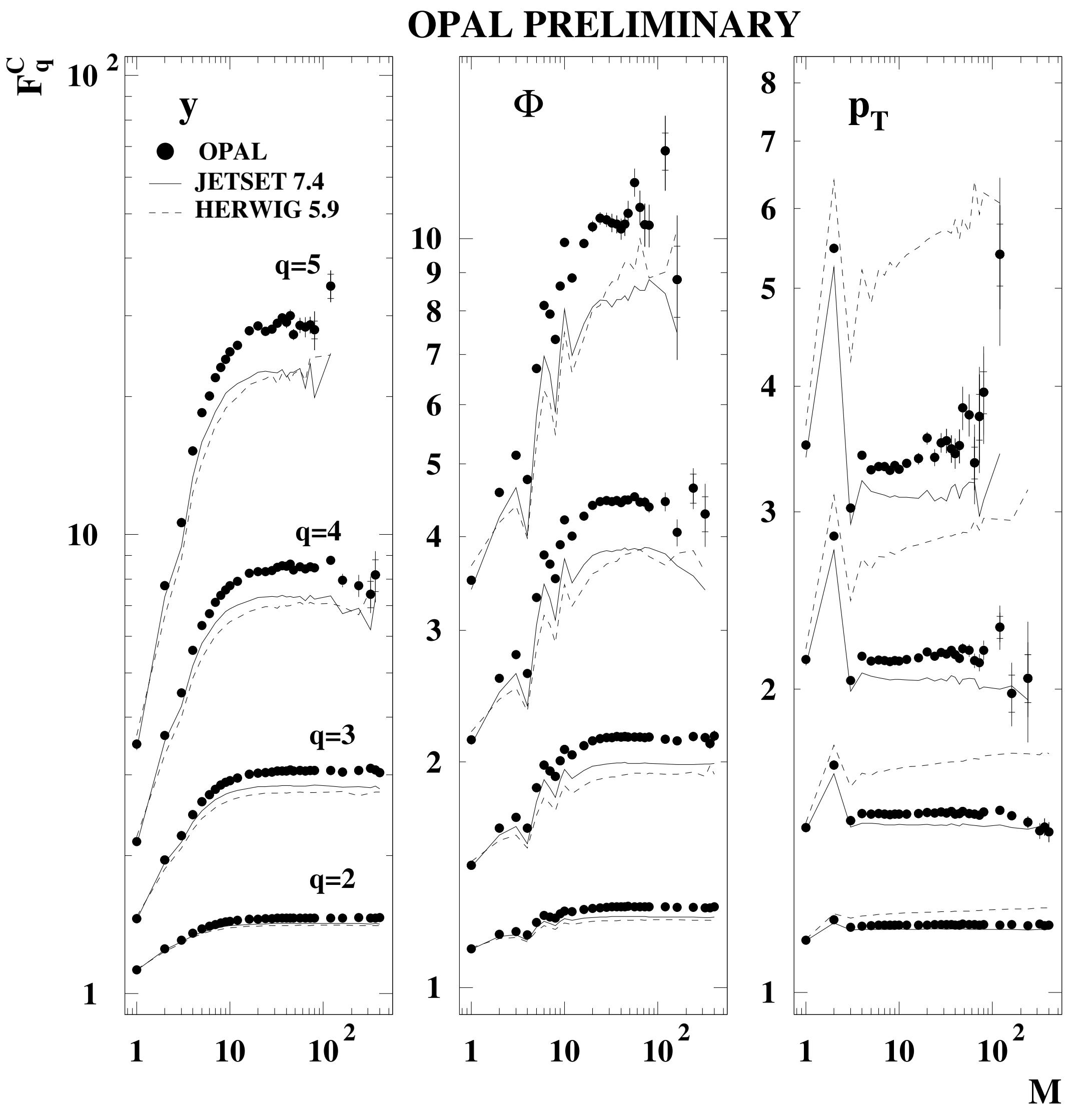
- Correlations imply $F_q(\delta) > 1$

- Dynamical Fluctuations, scale-invariant correlations
  Intermittency $\qquad F_q(\delta) \propto \delta^{-\phi_q} \quad , \qquad \phi_q > 0$

- anomalous fractal dimensions, $\qquad d_q = \frac{\phi_q}{q-1}$

Local Particle Fluctuations – 1-dim

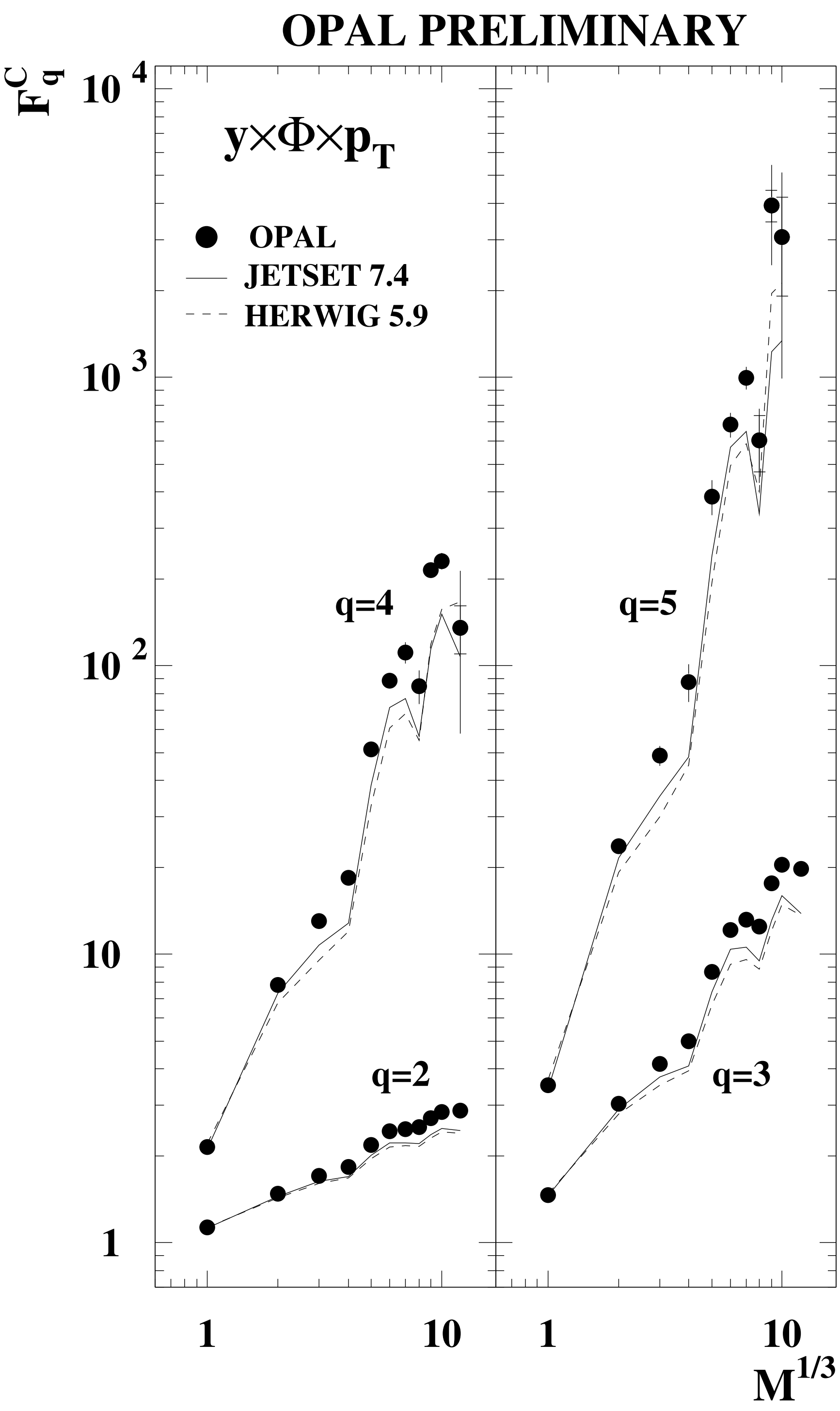
OPAL (paper 297) Factorial Moments in 1 dimension:
(y, p_T, ϕ w.r.t. sphericity axis) ($|y| < 2$, $-2.4 < \ln p_T < 0.7$)



- $F_q(y)$ show power law followed by saturation
- MCs agree reasonably well with data
  but have slightly lower values
  except HERWIG: too high for p_T

Local Particle Fluctuations – 3-dim

OPAL (Vancouver 297) Factorial Moments in 3 dimensions:
(y, p_T, ϕ w.r.t. sphericity axis) ($|y| < 2,\ -2.4 < \ln p_\mathsf{T} < 0.7$)



Good agreement with MCs

Conclusions

- Basic assumptions of QCD are well tested, *e.g.*, vector gluons, SU(3).
- Perturbative QCD effects are verified, *e.g.*, running of α_{s}, gluon coherence effects
- Softer aspects are in qualitative agreement, *e.g.*, q/g differences, particle correlations

for transparencies:
`http://wwwinfo.cern.ch/~metzger/talks.html`

Smoking living salmon
Stuffing living geese
I'm a little Christian
Jesus brings me peace

L. Durrell, "Monsieur"