Bose-Einstein Correlations in e^+e^- Annihilation

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Introduction

q-particle density

$$\rho_q(p_1,...,p_q) = \frac{1}{\sigma_{\text{tot}}}\frac{\mathrm{d}^q\sigma_q(p_1,...,p_q)}{\mathrm{d}p_1...\mathrm{d}p_q}$$

where σ_q is inclusive cross section

Normalization:

$$\begin{aligned}\int \rho_1(p)\,\mathrm{d}p &= \langle n\rangle \\ \int \rho_2(p_1,p_2)\,\mathrm{d}p_1\,\mathrm{d}p_2 &= \langle n(n-1)\rangle\end{aligned}$$

In terms of "factorial cumulants", C

$$\begin{aligned}\rho_1(p_1) &= C_1(p_1) \\ \rho_2(p_1,p_2) &= C_1(p_1)C_1(p_2) + C_2(p_1,p_2) \\ \rho_3(p_1,p_2,p_3)) &= C_1(p_1)C_1(p_2)C_1(p_3) \\ &+ \textstyle\sum_{3\text{ perms}} C_1(p_1)C_2(p_2,p_3) \\ &+ C_3(p_1,p_2,p_3)\end{aligned}$$

"trivial" 3-particle correlations

"genuine" 3-particle correlations

2-particle correlations

$$C_2 = \rho_2(p_1,p_2) - C_1(p_1)C_1(p_2)$$

Convenient to normalize

$$R_q = \frac{\rho_q}{\prod_{i=1}^{q}\rho_1(p_i)} \qquad K_q = \frac{C_q}{\prod_{i=1}^{q}\rho_1(p_i)}$$

e.g.,

$$R_2 = 1 + \frac{C_2}{\rho_1(p_1)\rho_1(p_2)} = 1 + K_2$$

Introduction

To study BEC, not other correlations, replace $\prod_{i=1}^{q} \rho_1(p_i)$ by $\rho_0(p_1, ..., p_q)$, the q-particle density if no BEC (reference sample)

e.g., 2-particle BEC are studied in terms of

$$R_2(p_1, p_2) = \frac{\rho(p_1, p_2)}{\rho_0(p_1, p_2)}$$

Since 2-π BEC only at small $Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$, integrate over other variables

$$R_2(Q) = \frac{\rho(Q)}{\rho_0(Q)}$$

Assuming incoherent particle production and spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda |G(Q)|^2$$

where $G(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ is the Fourier transform of $S(x)$

$\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius r

$\Longrightarrow$

$$R_2(Q) \propto 1 + \lambda e^{-Q^2 r^2}$$

$$R_2(Q) \propto 1 + \lambda e^{-Q^2 r^2}$$

Assumes

- incoherent average over source
  λ tries to account for
  - partial coherence
  - multiple (distinguishable) sources, long-lived resonances
  - pion purity
- spherical (radius r) Gaussian density of particle emitters
  seems unlikely in e^+e^- annihilation
- static source, $i.e.$, no t-dependence
  certainly wrong

Nevertheless, this Gaussian formula is the most often used parametrization
And it works fairly well
But do the resulting values of λ and r mean much?

When Gaussian parametrization does not fit well, can expand about the Gaussian (Edgeworth expansion).
Keeping only the lowest-order non-Gaussian term,
$\exp\left(-Q^2 r^2\right)$ becomes

$$\exp\left(-Q^2 r^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(Qr)\right]$$

(H_3 is third-order Hermite polynomial)

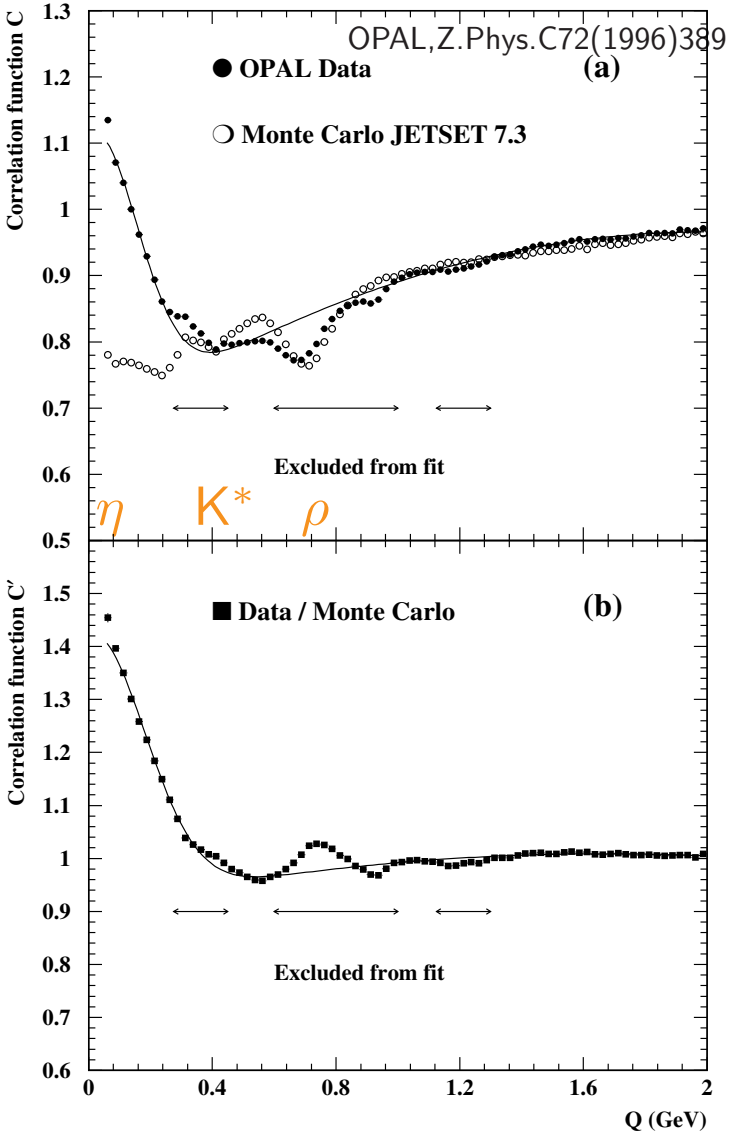
Experimental Problems I

I. Reference Sample, ρ_0
— it does NOT exist

Common choices:

1. $+,-$ pairs
   But different resonances than $+,+$
   – correct by MC. – But is it correct?
2. Monte Carlo — But is it correct?
3. Mixed events – pair particles from different events
   But destroys all correlations, not just BEC
   – correct by MC. – But is it correct?
4. Mixed hemispheres (for 2-jet events)
   – pair particle with particle reflected from opposite hemisphere
   But destroys all correlations
   – correct by MC. – But is it correct?

R_2



Experimental Problems II

II. Pion purity

1. mis-identified pions – K, p
   – correct by MC. – But is it correct?
2. resonances
   - long-lived affect λ
   BEC peak narrower than resolution
   - short-lived, $e.g.$, ρ, - affect r
   – correct by MC. – But is it correct?
3. weak decays
   - $\sim 20\%$ of Z decays are $b\bar{b}$
   like long-lived resonances,
   decrease λ

- per Z: 17.0 $\pi^{\pm}$, 2.3 $K^{\pm}$, 1.0 p

Origin of π^+ in Z decay	[%] (JETSET 7.4)
direct (string fragmentation)	16
decay (short-lived resonances) $\Gamma > 6.7\,\text{MeV}, \tau < 30\,\text{fm}$ (ρ, ω, K^*, Δ, ...)	62
decay (long-lived resonances) $\Gamma < 6.7\,\text{MeV}, \tau > 30\,\text{fm}$	22

Experimental Problems III, IV

III. Final-State Interactions

1. Coulomb
   - form not certain
   - for R_2, a few % in lowest Q bin
   - double if $+,-$ ref. sample
   - often neglected for R_2
   - but not negligible for R_3
2. Strong interaction - $S = 0$ $\pi\pi$ phase shifts can be incorporated together with Coulomb into the formula for R_2

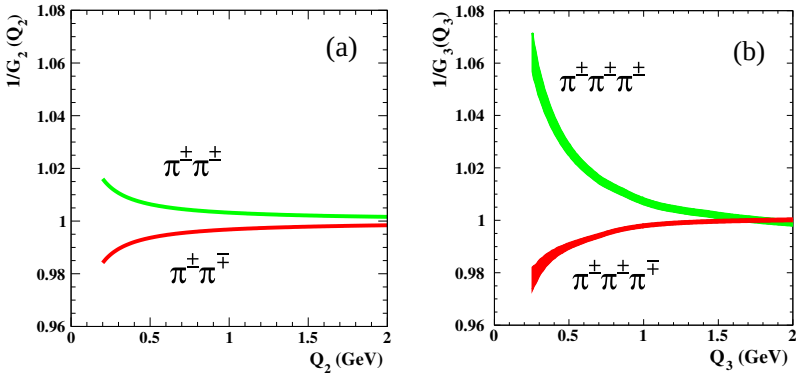
   Osada, Sano, Biyajima, Z.Phys. C72(1996)285)

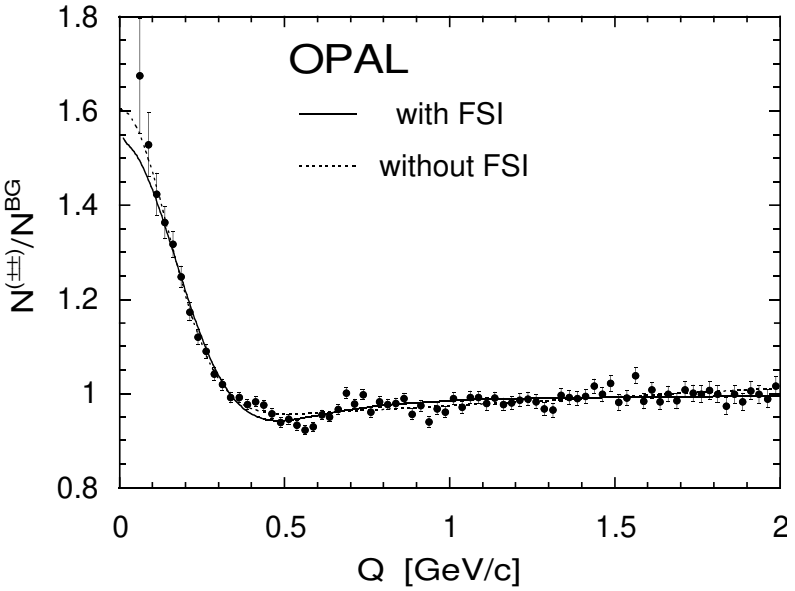
   - tends to increase λ and decrease r

     e.g., OPAL data:

     $\lambda_{\text{noFSI}} = 0.71,\ \lambda_{\text{FSI}} = 1.0$

     $r_{\text{noFSI}} = 1.34,\ r_{\text{FSI}} = 1.09\,\text{fm}$
   - Not used by experimental groups

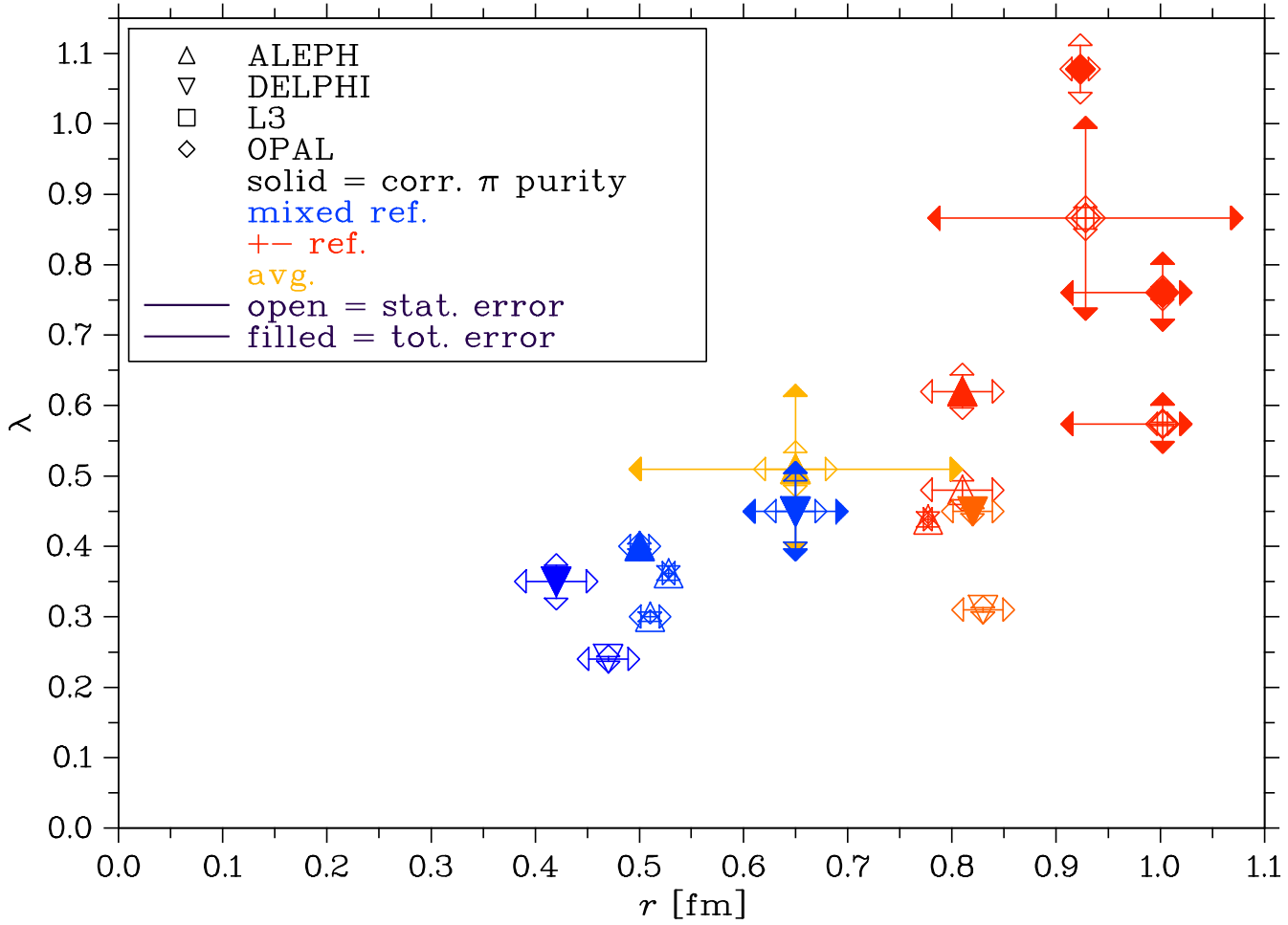




IV. Long-range correlations:

$R_2(Q) \propto (1 + \lambda e^{-Q^2 r^2})(1 + \delta Q)$

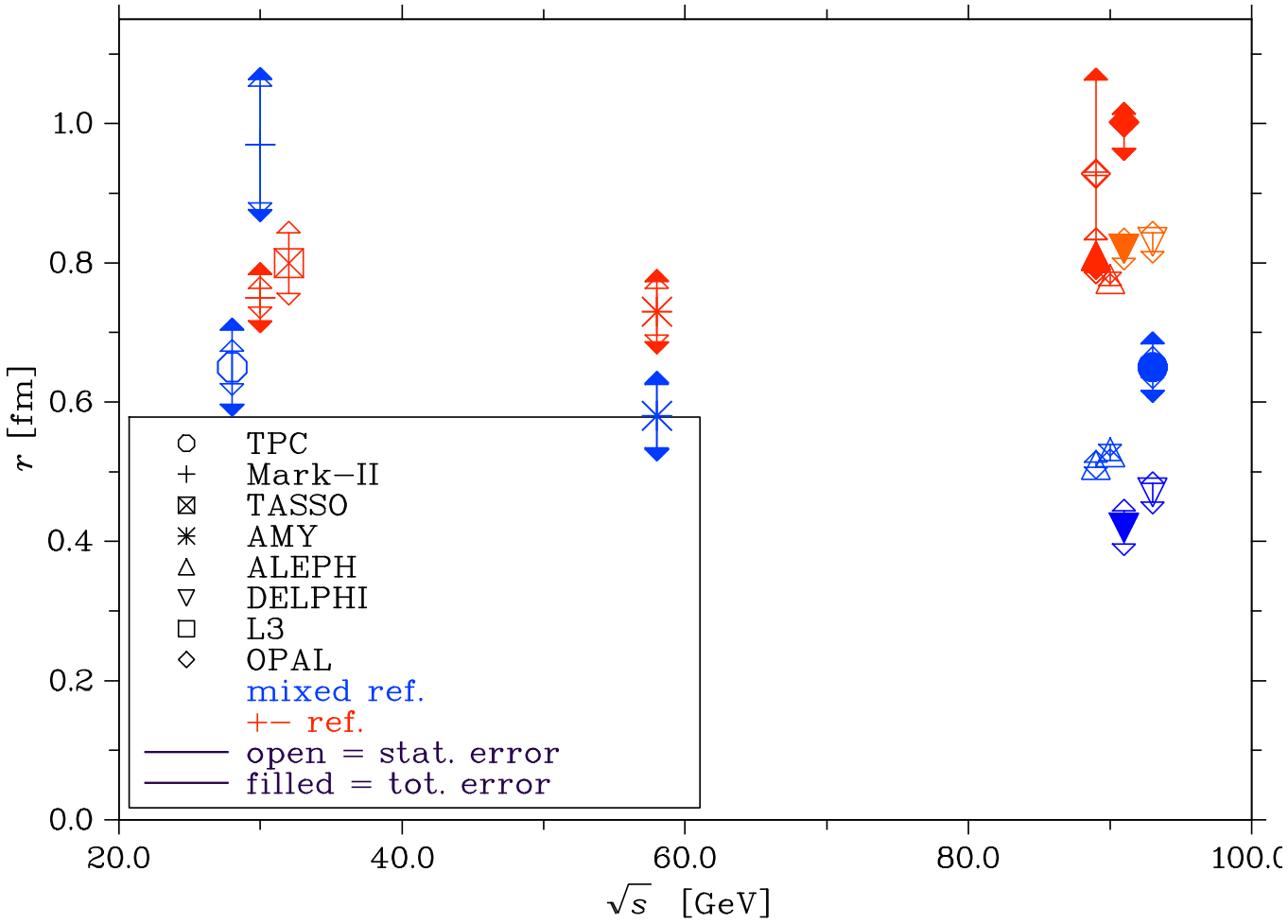
Results from R_2



– correction for π purity increases λ

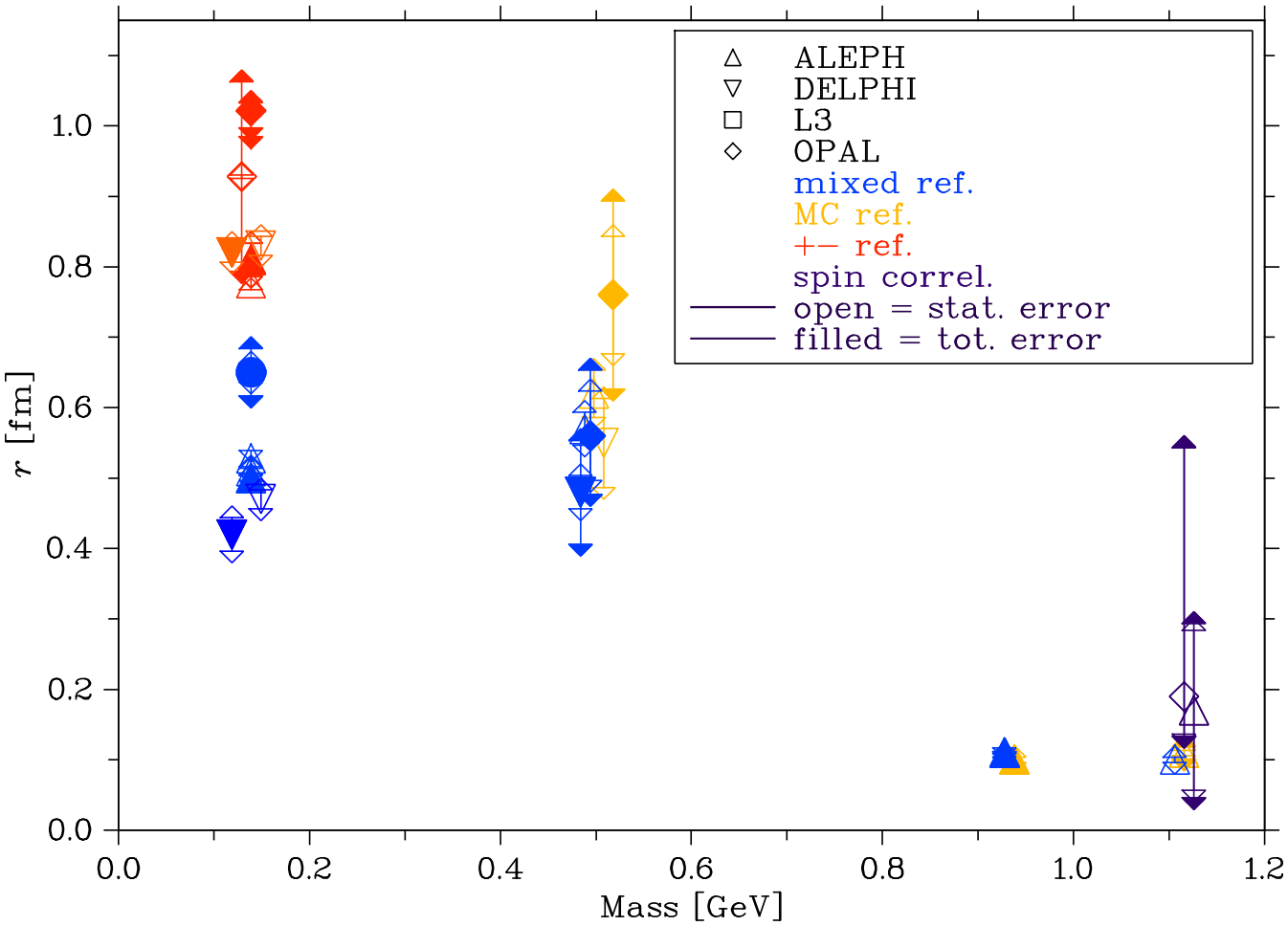
– mixed ref. gives smaller λ, r than +− ref.

$\sqrt{s}$ dependence of r



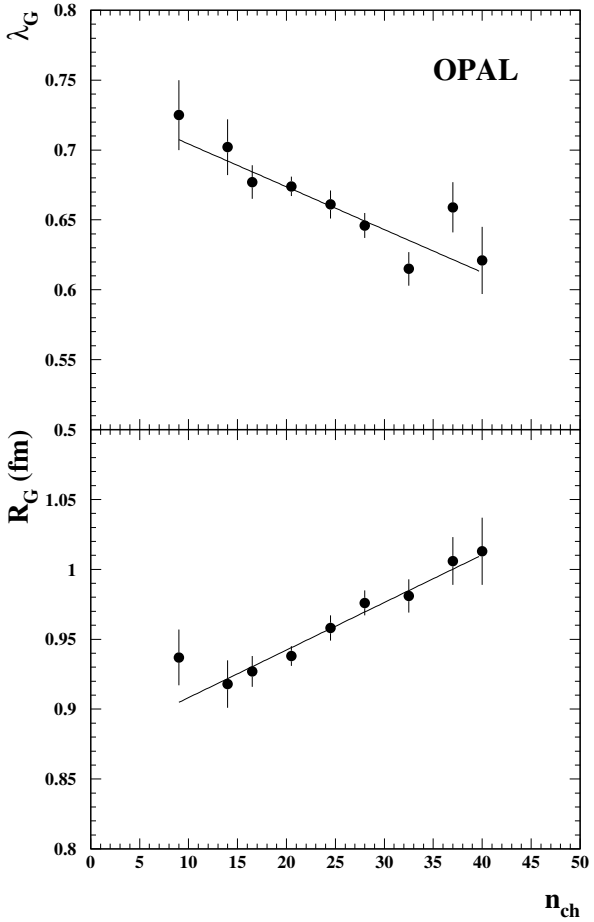
No evidence for $\sqrt{s}$ dependence

Mass dependence of r — BEC and FDC



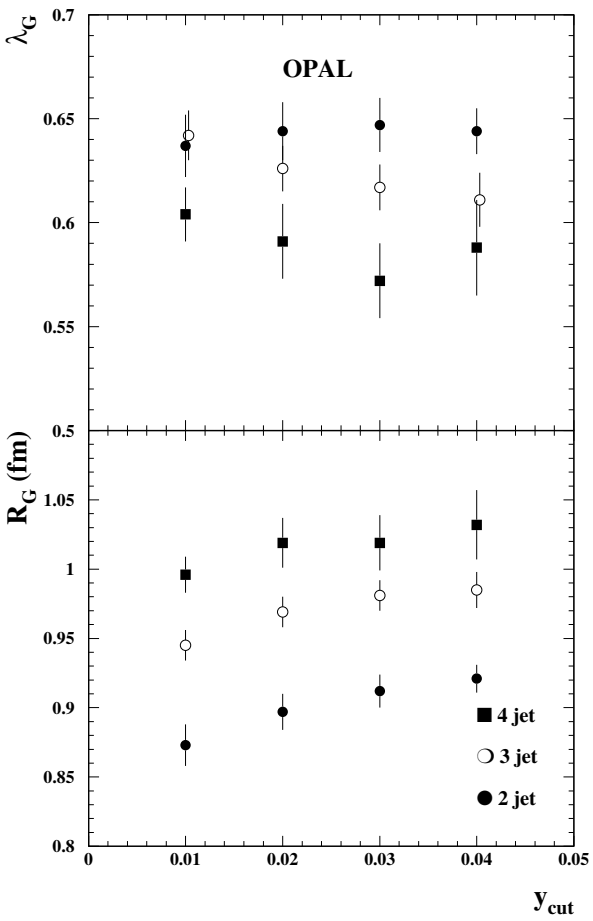
$r(\text{mesons}) > r(\text{baryons})$

Multiplicity/Jet dependence of r



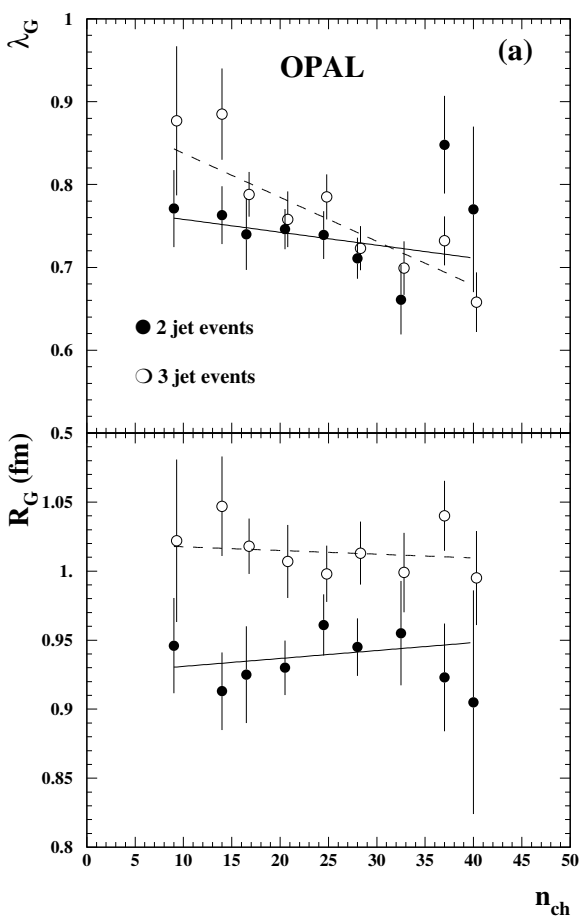
$\lambda \searrow$ with n_{ch}

$r \nearrow$ with n_{ch}



$\lambda \searrow$ with n_{jet}

$r \nearrow$ with n_{jet}



$\lambda_{\text{n-jet}} \approx$ indep. of n_{ch}

$r_{\text{n-jet}}$ indep. of n_{ch}

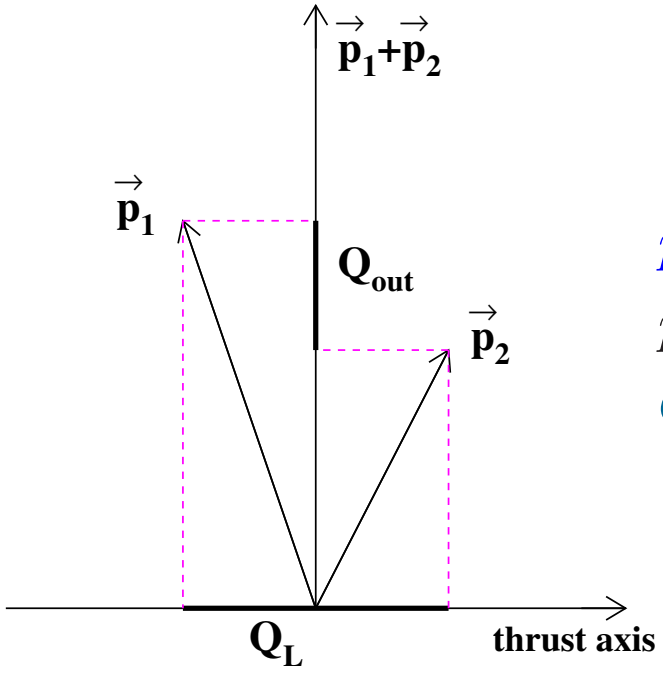
Multiplicity dependence is mainly due to number of jets.

Elongation of the source

The usual parametrization assumes a symmetric Gaussian source
But, there is no reason to expect this symmetry in $e^+e^- \rightarrow q\bar{q}$.
Therefore, we do a 3-dim. analysis in the Longitudinal Center of Mass System

LCMS:

Boost each π-pair along thrust axis



$p_{L1} = -p_{L2}$

$\vec{p}_1 + \vec{p}_2$ defines 'out' axis

$Q_{side} \perp (Q_L, Q_{out})$

the LCMS

Advantages of LCMS:

$$\begin{aligned} Q^2 &= Q_{\mathsf{L}}^2 + Q_{\mathsf{side}}^2 + Q_{\mathsf{out}}^2 - (\Delta E)^2 \\ &= Q_{\mathsf{L}}^2 + Q_{\mathsf{side}}^2 + Q_{\mathsf{out}}^2 (1 - \beta^2) \qquad \text{where } \beta \equiv \frac{p_{\mathsf{out}\,1} + p_{\mathsf{out}\,2}}{E_1 + E_2} \end{aligned}$$

Thus, the energy difference,
and therefore the difference in emission time of the pions
couples only to the out-component, Q_{out}.

Thus,
Q_{L} and Q_{side} reflect only spatial dimensions of the source
Q_{out} reflects a mixture of spatial and temporal dimensions.

Parametrization of R_2

Writing R_2 in terms of $\vec{Q} = (Q_{\mathrm{L}}, Q_{\mathrm{side}}, Q_{\mathrm{out}})$: $\quad R_2(\vec{Q}) = \frac{\rho_2(\vec{Q})}{\rho_0(\vec{Q})}$

We parametrize $R_2(\vec{Q})$ by a 3-dimensional Gaussian

$$R_2(Q_{\mathrm{L}}, Q_{\mathrm{out}}, Q_{\mathrm{side}}) = \gamma \cdot B \cdot (1 + \lambda G)$$

where

- $\gamma =$ normalization (≈ 1)
- $B = (1 + \delta Q_{\mathrm{L}} + \varepsilon Q_{\mathrm{out}} + \xi Q_{\mathrm{side}})$ describes large Q (long-range correlations)
- $\lambda =$ "incoherence", or strength of BE effect
- $G =$ azimuthally symmetric Gaussian:

$$G = \exp\left(-r_{\mathrm{L}}^2 Q_{\mathrm{L}}^2 - r_{\mathrm{out}}^2 Q_{\mathrm{out}}^2 - r_{\mathrm{side}}^2 Q_{\mathrm{side}}^2 + 2\rho_{\mathrm{L,out}} r_{\mathrm{L}} r_{\mathrm{out}} Q_{\mathrm{L}} Q_{\mathrm{out}}\right)$$

Elongation Results

parameter	Gaussian	Edgeworth
λ	$0.41 \pm 0.01^{+0.02}_{-0.19}$	$0.54 \pm 0.02^{+0.04}_{-0.26}$
r_{L} (fm)	$0.74 \pm 0.02^{+0.04}_{-0.03}$	$0.69 \pm 0.02^{+0.04}_{-0.03}$
r_{out} (fm)	$0.53 \pm 0.02^{+0.05}_{-0.06}$	$0.44 \pm 0.02^{+0.05}_{-0.06}$
r_{side} (fm)	$0.59 \pm 0.01^{+0.03}_{-0.13}$	$0.56 \pm 0.02^{+0.03}_{-0.12}$
$r_{\mathrm{out}}/r_{\mathrm{L}}$	$0.71 \pm 0.02^{+0.05}_{-0.08}$	$0.65 \pm 0.03^{+0.06}_{-0.09}$
$r_{\mathrm{side}}/r_{\mathrm{L}}$	$0.80 \pm 0.02^{+0.03}_{-0.18}$	$0.81 \pm 0.02^{+0.03}_{-0.19}$
κ_{L}	–	$0.5 \pm 0.1^{+0.1}_{-0.2}$
κ_{out}	–	$0.8 \pm 0.1 \pm 0.3$
κ_{side}	–	$0.1 \pm 0.1 \pm 0.3$
δ	$0.025 \pm 0.005^{+0.014}_{-0.015}$	$0.036 \pm 0.007^{+0.012}_{-0.023}$
ϵ	$0.005 \pm 0.005^{+0.034}_{-0.012}$	$0.011 \pm 0.005^{+0.037}_{-0.012}$
ξ	$-0.035 \pm 0.005^{+0.031}_{-0.024}$	$-0.022 \pm 0.006^{+0.020}_{-0.025}$
χ^2/DoF	2314/2189	2220/2186
C.L. (%)	3.1	30

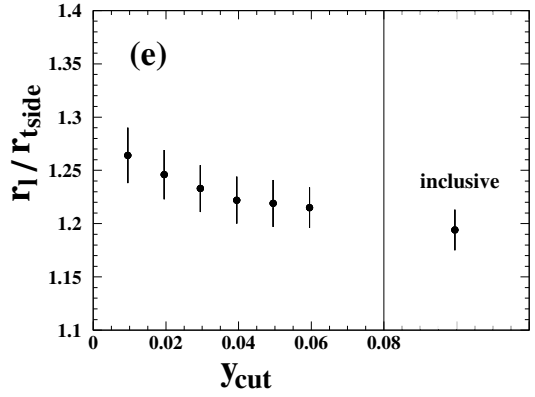
- $\rho_{\mathrm{L,out}} = 0$ So fix to 0.
- Edgeworth fit significantly better than Gaussian
- $r_{\mathrm{side}}/r_{\mathrm{L}} < 1$ more than 5 std. dev. Elongation along thrust axis
- Models which assume a spherical source are too simple.

Elongation Results

			Gauss / Edgeworth	2-D r_t/r_L	3-D r_{side}/r_L
DELPHI	mixed	2-jet	Gauss	$0.62\pm0.02\pm0.05$	—
ALEPH	mixed	2-jet	Gauss	$0.61\pm0.01\pm0.??$	—
	$+-$	2-jet	Gauss	$0.91\pm0.02\pm0.??$	—
	mixed	2-jet	Edgeworth	$0.68\pm0.01\pm0.??$	—
	$+-$	2-jet	Edgeworth	$0.84\pm0.02\pm0.??$	—
OPAL	$+-$	2-jet	Gauss	—	$0.82\pm0.02\pm{}^{+0.01}_{-0.05}$
L3	mixed	all	Gauss	—	$0.80\pm0.02\pm{}^{0.03}_{0.18}$
	mixed	all	Edgeworth	—	$0.81\pm0.02\pm{}^{0.03}_{0.19}$

~20% elongation along thrust axis

(ZEUS finds similar results in ep)



OPAL:

Elongation larger for narrower jets

3π BEC

Recall

$$\begin{aligned} \rho_3(p_1,p_2,p_3)) &= C_1(p_1)C_1(p_2)C_1(p_3) \\ &+ \textsf{“trivial” 3-particle correlations} \quad \sum_{3 \text{ perms}} C_1(p_1)C_2(p_2,p_3) \\ &+ \textsf{“genuine” 3-particle correlations} \quad C_3(p_1,p_2,p_3) \end{aligned}$$

or

$$\begin{aligned} \rho_3(p_1,p_2,p_3) &= \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) \\ &+ \sum_{\substack{3 \\ \text{perm}}} \left[\rho_1(p_1)\left(\rho_2(p_2,p_3) - \rho_1(p_2)\rho_1(p_3)\right)\right] \\ &+ C_3(p_1,p_2,p_3) \end{aligned}$$

3-particle BEC are studied in terms of

$$R_3(p_1,p_2,p_3) = \frac{\rho_3(p_1,p_2,p_3)}{\rho_0(p_1,p_2,p_3)}$$

3π BEC

Since BEC at small Q_3 $\qquad (Q_3^2 = M_{123}^2 - 9m_\pi^2 = Q_{12}^2 + Q_{23}^2 + Q_{13}^2)$

we use $\quad R_3(Q_3) = \dfrac{\rho(Q_3)}{\rho_0(Q_3)} \quad$ and $\quad R_2 = \dfrac{\rho(Q)}{\rho_0(Q)}$

$$R_3^{\text{noBEC}} = 1 \qquad \text{(def. of } \rho_0\text{)}$$

$$R_3^{\text{nongen}}(Q_3) = 1 + \sum_{\substack{3 \text{ perm} \\ Q_3}} \frac{\rho_1\rho_2}{\rho_0} - 3 \qquad = 1 + \sum_{\substack{3 \text{ perm} \\ Q_3}} [R_2(Q_{12}) - 1]$$

$$R_3^{\text{genuine}}(Q_3) = 1 + \frac{C_3(Q_3)}{\rho_0(Q_3)}$$

$$R_3(Q_3) = R_3^{\text{nongen}}(Q_3) + R_3^{\text{genuine}}(Q_3) - 1$$

$$R_3^{\text{genuine}}(Q_3) = 1 + R_3(Q_3) - R_3^{\text{nongen}}(Q_3)$$

3π BEC

Assuming static source density $f(x)$ in space-time,
with Fourier transform $G(Q) = \int \mathrm{d}x\, e^{iQx} f(x) = G e^{i\phi}$,

$$R_2(Q) = 1 + \lambda |G(Q)|^2 \quad , \qquad \lambda = 1$$

$$R_3(Q_3) = 1 + \underbrace{\lambda \left(|G(Q_{12})|^2 + |G(Q_{23})|^2 + |G(Q_{13})|^2 \right)}_{\text{from 2-particle BEC}}$$

$$+ \underbrace{2\lambda^{1.5}\, \Re\{G(Q_{12})G(Q_{23})G(Q_{13})\}}_{\text{from genuine 3-particle BEC}}$$

$$R_3^{\text{genuine}} = 1 + 2\lambda^{1.5}\, \Re\{G(Q_{12})G(Q_{23})G(Q_{13})\}$$

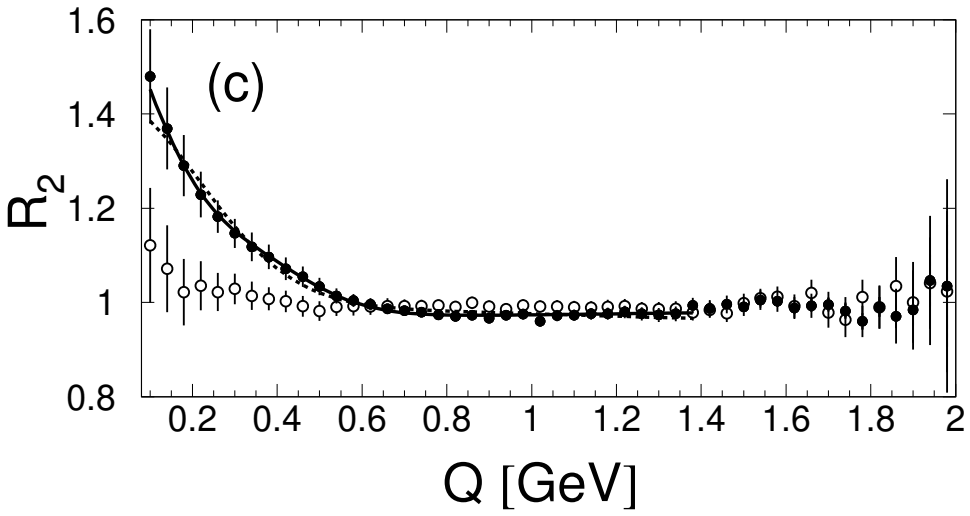
$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{(R_2(Q_{12}) - 1)(R_2(Q_{23}) - 1)(R_2(Q_{13}) - 1)}}$$

where $\omega = \cos(\phi_{12} + \phi_{23} + \phi_{13})$

$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{R_2(Q_3) - 1}} \qquad \text{if } f(x) \text{ is Gaussian}$$

Completely incoherent particle production implies $\lambda = 1$ $\omega = 1$

3π BEC



$$R_2 \propto 1 + \lambda \exp(-Q^2 r^2)$$

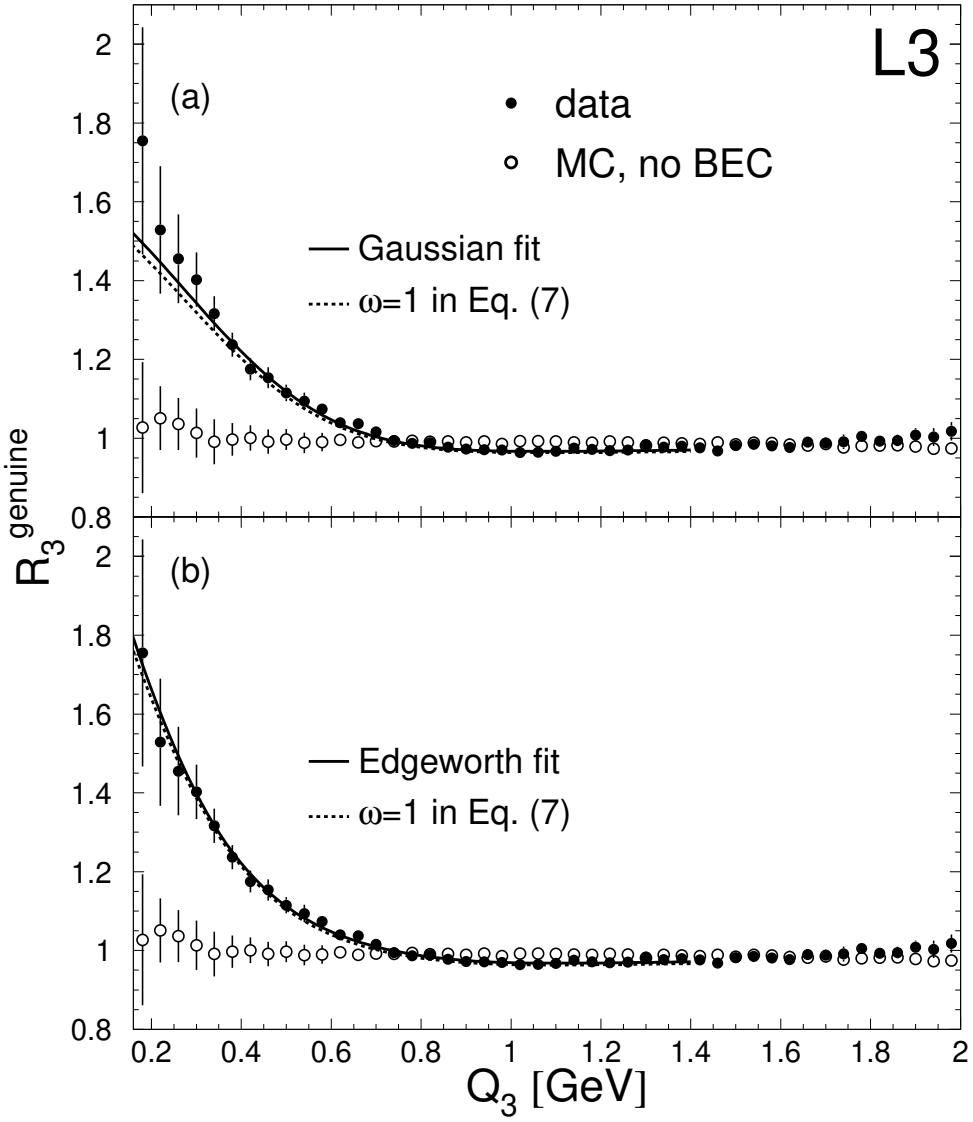
- - - Gaussian $\chi^2 = 60$, 29 dof

——Edgeworth $\chi^2 = 26$, 28 dof

$$R_3^{\text{genuine}} \propto 1 + \lambda^{1.5} \exp(-Q^2 r^2/2)$$

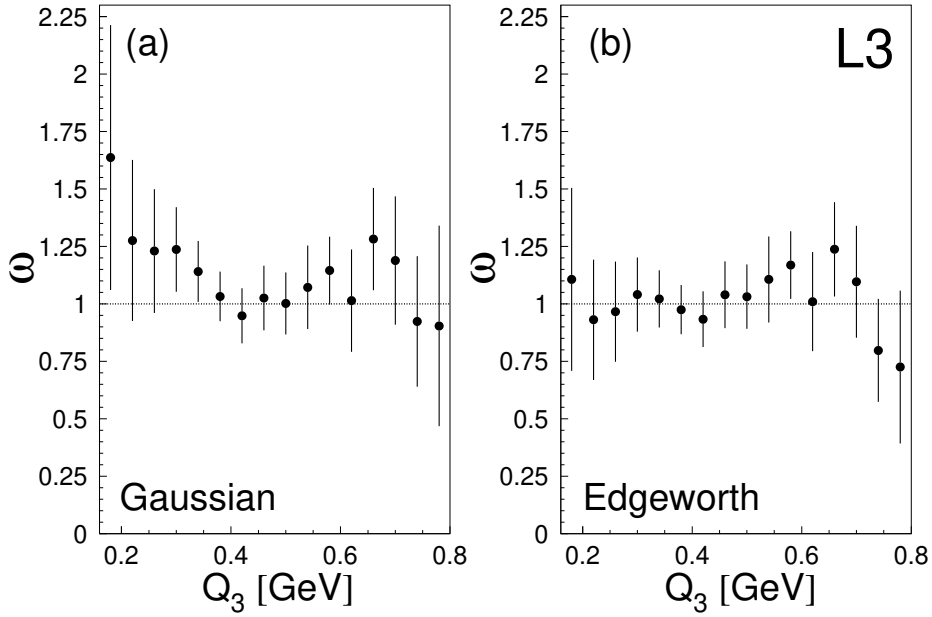
- - - Gaussian $\chi^2 = 30$, 27 dof

——Edgeworth $\chi^2 = 18$, 26 dof



$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{R_2(Q_3) - 1}}$$

Using R_3^{genuine} from data, R_2 from fit



Conclusion: Data consistent with $\omega = 1$,

$i.e.$, with completely incoherent pion production

Possibly a problem for string models!

3π BEC

L3:

from		Gaussian	Edgeworth
R_2	λ	$0.45 \pm 0.06 \pm 0.03$	$0.72 \pm 0.08 \pm 0.03$
R_3^{genuine}		$0.47 \pm 0.07 \pm 0.03$	$0.75 \pm 0.10 \pm 0.03$
R_2	r	$0.65 \pm 0.03 \pm 0.03$	$0.74 \pm 0.06 \pm 0.02$
R_3^{genuine}	(fm)	$0.65 \pm 0.06 \pm 0.03$	$0.72 \pm 0.08 \pm 0.03$

Values of λ, r from Gaussian, Edgeworth are different

Values of λ, r from R_2 and R_3^{genuine} are consistent.

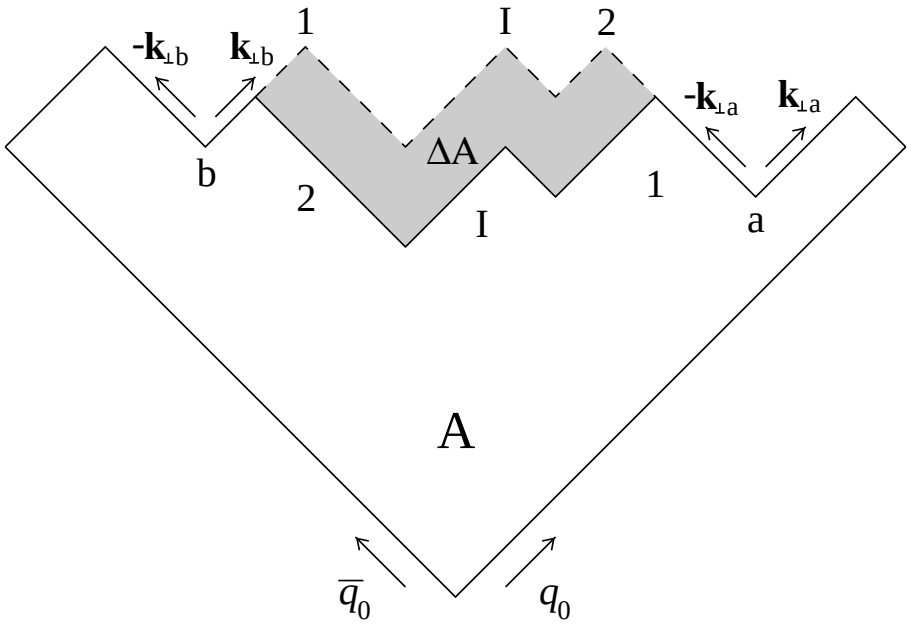
expt.		from R_2	from R_3 or R_3^{genuine}
MARK-II (29 GeV)	λ	$0.45 \pm 0.03 \pm 0.04$	$0.54 \pm 0.06 \pm 0.05$
	r	$1.01 \pm 0.09 \pm 0.06$	$0.90 \pm 0.06 \pm 0.06$
DELPHI	λ	$0.35 \pm 0.04 \pm 0.??$	$0.53 \pm 0.07 \pm 0.10$ gen
	r	$0.42 \pm 0.04 \pm 0.??$	$0.93 \pm 0.06 \pm 0.04$ gen
OPAL	λ	$0.76 \pm 0.03 \pm 0.05$	$0.79 \pm 0.01 \pm 0.06$ gen
	r	$1.00 \pm 0.02^{+0.02}_{-0.10}$	$0.82 \pm 0.01 \pm 0.04$ gen

Values of λ, r from R_2 and R_3 are fairly consistent.

BEC in String Models

Longitudinal BEC

- Different string configurations give same final state
- Matrix element to get a final state depends on area, A:
  $\mathcal{M} = \exp\left[(\imath\kappa - b/2)A\right]$
  where κ is the string tension
  and b is the decay constant
  $\kappa \approx 1\,\mathrm{GeV/fm}$ and $b \approx 0.3\,\mathrm{GeV/fm}$
- So, must sum all the amplitudes



Transverse BEC

- Transverse momentum via tunneling, also related to b

Using b from tuning of JETSET, predict

- BEC,
  including genuine 3-particle BEC
- $r_t < r_L$
- $r(\pi^0\pi^0) < r(\pi^+\pi^+)$

2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$

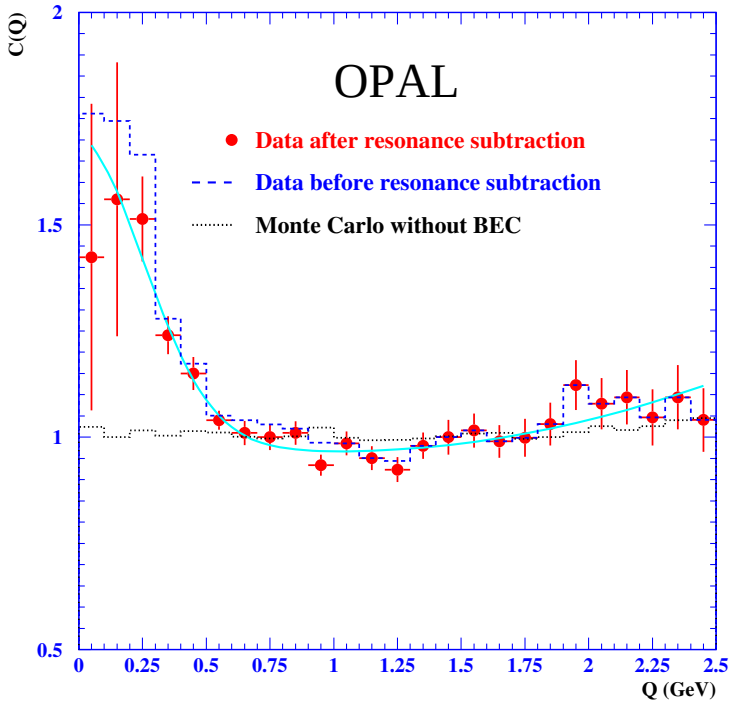
- Many measurements of BEC with charged π's
- but few with π^0's
  in e^+e^-: L3, P.L. B524 (2002) 55 OPAL, P.L. B559 (2003) 131

Selection:

OPAL	L3
$p_{\pi^0} > 1.0$ GeV	$E(\pi^0) < 6.0$ GeV
2-jet, $T > 0.9$	all events

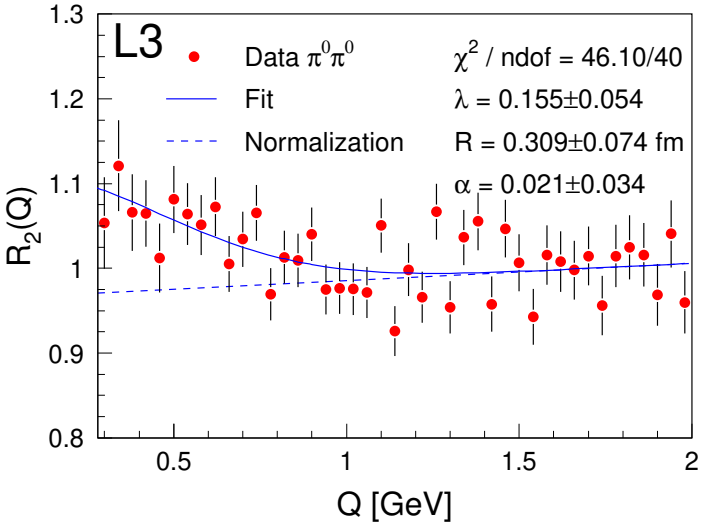
- Naively expect same BEC for $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$
- Hadronization with local charge conservation,
  e.g., string, $\Longrightarrow R_{00} < R_{\pm\pm}$
- But most π's from resonances — dilutes this effect.

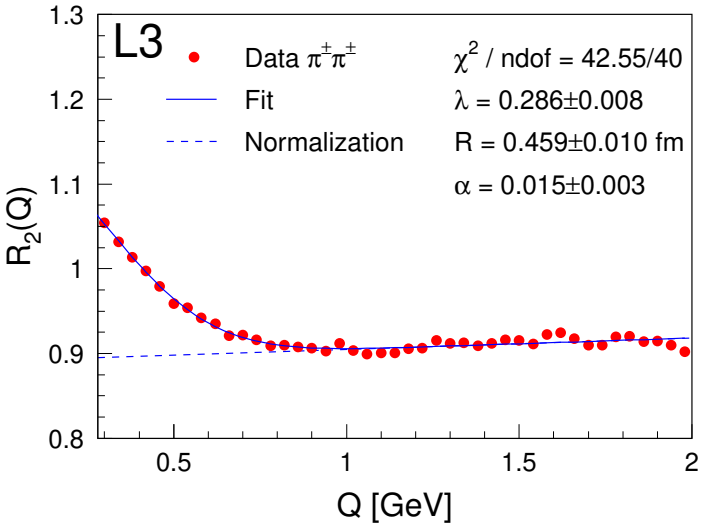
2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$



$\lambda = 0.55 \pm 0.10 \pm 0.10$

$R = 0.59 \pm 0.08 \pm 0.05$ fm





2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$

BEC from Z decays
Gaussian parametrization

	Expt.	ρ_0	R (fm)	λ
$\pm\pm$	OPAL	+−	$1.00^{+0.03}_{-0.10}$	0.57 ± 0.05
	L3	mix	0.65 ± 0.04	0.45 ± 0.07
	L3 3-π	mix	0.65 ± 0.07	0.47 ± 0.08
	L3 $E_\pi < 6$ GeV	MC	0.46 ± 0.01	0.29 ± 0.03
00	L3 $E_\pi < 6$ GeV	MC	0.31 ± 0.10	0.16 ± 0.09
	OPAL $E_\pi > 1$, 2-jet	MC	0.59 ± 0.09	0.55 ± 0.14

- L3: $R_{00} < R_{\pm\pm}$ and $\lambda_{00} < \lambda_{\pm\pm}$, both 1.5σ
- ALEPH, DELPHI find $R_{\pm\pm}(\text{mix})/R_{\pm\pm}(+-) \approx 0.68, 0.51$
  Applying this to OPAL $R_{\pm\pm}$, OPAL $R_{00} \approx R_{\pm\pm}$ and $\lambda_{00} \approx \lambda_{\pm\pm}$
- L3 and OPAL $\pi^0\pi^0$ results disagree by 2σ
- But L3: $R_{\pm\pm}(\text{all } \pi) > R_{\pm\pm}(< 6\,\text{GeV})$, $\lambda_{\pm\pm}(\text{all } \pi) > \lambda_{\pm\pm}(< 6\,\text{GeV})$
- Is the L3-OPAL $\pi^0\pi^0$ difference due to $E_\pi > 1$ GeV and/or 2-jet ???
  Will OPAL do a charged track analysis with these cuts and with MC ref. sample?
- OPAL: MC shows that few of selected π^0's are direct from string

Summary

- Comparison between experiments is difficult.
  - reference samples
  - MC corrections
- No evidence for $\sqrt{s}$ dependence of r
  Multiplicity dependence is mainly due to number of jets.
- $r(\text{mesons}) > r(\text{baryons})$
- $\sim$20% elongation along thrust axis — consistent with string model
- genuine 3-π BEC, consistent with 2-π BEC
  consistent with complete incoherence — problem for string model?
- $R_{00} < R_{\pm\pm}$??