Recent L3 results (and questions) on BEC at LEP

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XXXVIII International Symposium on Multiparticle Dynamics
DESY
15–20 September 2008

BEC Introduction

$$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} = \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x \, e^{iQx} S(x)$ – Fourier transform of $S(x)$

$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius $r \Longrightarrow$

$$R_2(Q) = 1 + \lambda \, \mathrm{e}^{-Q^2 r^2}$$

Elongation Results

Results in LCMS frame:

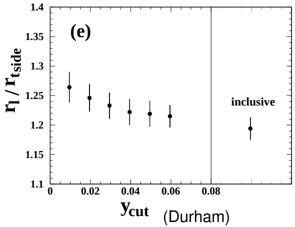
	r_L / r_{side}
L3	$1.25 \pm 0.03^{+0.36}_{-0.05}$
OPAL	$1.19 \pm 0.03^{+0.08}_{-0.01}$

(ZEUS finds similar results in ep)

$\sim$25% elongation along thrust axis

OPAL:

Elongation larger for narrower jets

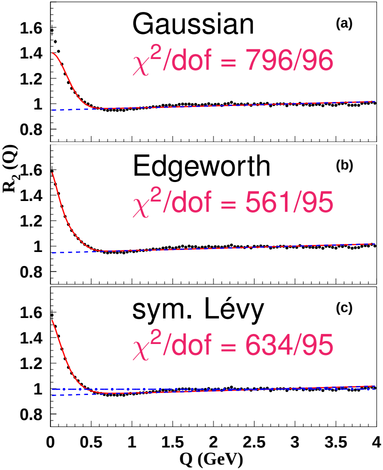


- Conclusion: Elongation, but it is relatively small.
- So: Ignore it. — Assume spherical.

Results on Q from L3 Z decay

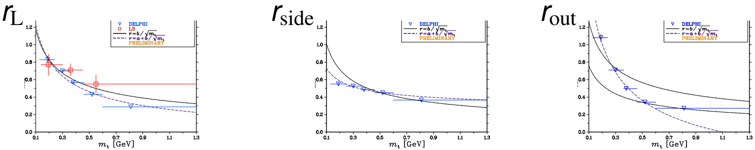
$$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \delta Q)$$

- Gaussian
  $$G = \exp\left(-(rQ)^2\right)$$
- Edgeworth expansion
  $$G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$$
  Gaussian if $\kappa = 0$
  $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $$G = \exp\left(-|rQ|^{\alpha}\right)$$
  $0 < \alpha \leq 2$
  $\alpha = 1.34 \pm 0.04$



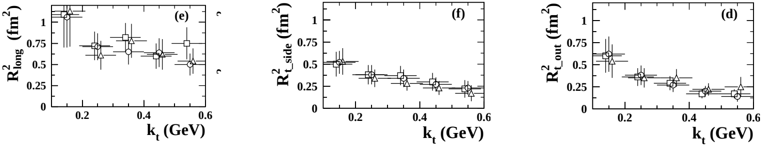
Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

Transverse Mass dependence of *r*



Smirnova&Lörstad,7th Int.Workshop on Correlations and Fluctuations (1996)

Van Dalen,8th Int.Workshop on Correlations and Fluctuations (1998)



OPAL, Eur. Phys. J **C52** (2007) 787

r decreases with m_t (or k_t) for all directions

Conclusions

- BEC depend (approximately) only on Q, not its components.
- BEC depend on m_{t}.

Turn now to a model providing such dependence.

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume momentum is related to avg. production point:

  $$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}$$

  where for 2-jet events, $a = 1/m_{\mathrm{t}}$

  $\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the “longitudinal” proper time

  and $m_{\mathrm{t}} = \sqrt{E^2 - p_z^2}$ is the “transverse” mass

- Let $\delta_{\Delta}(x^{\mu} - \overline{x}^{\mu})$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is

  $$S(x,p) = \int_0^{\infty} \mathrm{d}\tau H(\tau) \delta_{\Delta}(x - a\tau p) \rho_1(p)$$

- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.

$$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 S(x_1,p_1) S(x_2,p_2) \left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$

- Assume $\delta_{\Delta}(x - a\tau p)$ is very narrow — a δ-function. Then

$$R_2(p_1,p_2) = 1 + \lambda \,\mathrm{Re} \widetilde{H}\left(\frac{a_1 Q^2}{2}\right) \widetilde{H}\left(\frac{a_2 Q^2}{2}\right), \qquad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau) \exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction,
  $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\tfrac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\tfrac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right], \quad \alpha \neq 1$$
  where
  - α is the index of stability
  - τ_0 is the proper time of the onset of particle production
  - $\Delta\tau$ is a measure of the width of the dist.
- Assume $a_1 \approx a_2 \approx \bar{a}$
- Then,

$$R_2(Q,\bar{a}) = \gamma\left[1 + \lambda\cos\left(\bar{a}\tau_0 Q^2 + \tan\left(\tfrac{\alpha\pi}{2}\right)\left(\tfrac{\bar{a}\Delta\tau Q^2}{2}\right)^{\alpha}\right)\exp\left(-\left(\tfrac{\bar{a}\Delta\tau Q^2}{2}\right)^{\alpha}\right)\right] \cdot (1 + \delta Q)$$

BEC in the τ-model

$$R_2(Q,\overline{a}) = \gamma\left[1+\lambda\cos\left(\overline{a}\tau_0 Q^2 + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\overline{a}\Delta\tau Q^2}{2}\right)^{\alpha}\right)\exp\left(-\left(\frac{\overline{a}\Delta\tau Q^2}{2}\right)^{\alpha}\right)\right] \cdot (1+\delta Q)$$

Simplification:

- Particle production begins immediately, $\tau_0 = 0$
- effective radius, $R = \sqrt{\overline{a}\Delta\tau/2}$

$$R_2(Q) = \gamma\left[1+\lambda\cos\left[(R_a Q)^{2\alpha}\right]\exp\left[-(RQ)^{2\alpha}\right]\right](1+\delta Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

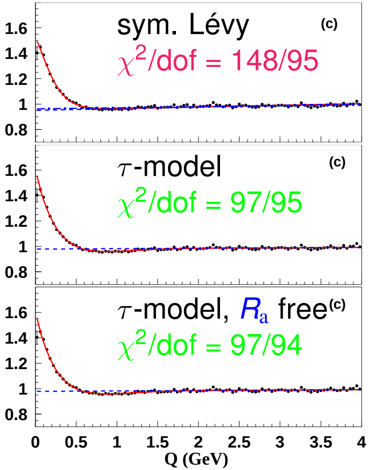
Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma\left[1+\lambda \qquad \exp\left[-|rQ|^{\alpha}\right]\right](1+\delta Q)$$

2-jet Results on Simplified τ-model from L3 Z decay

2-jet events Durham $y_{\rm cut} = 0.006$

- symmetric Lévy
  does not describe dip or large Q
- $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$
  good description
- R_a free
  good description
  Effective R works well

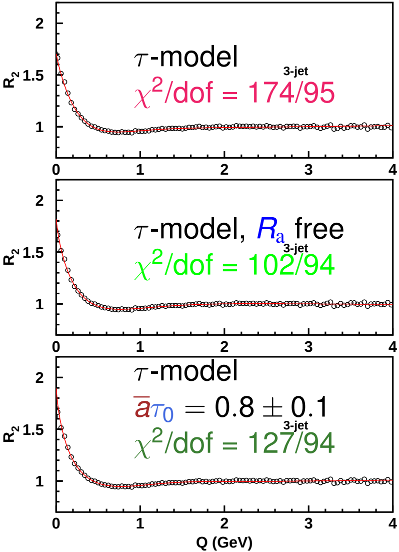


Simplified τ-model works better than sym. dists.

3-jet Results on Simplified τ-model from L3 Z decay

3-jet events Durham $y_{\rm cut} = 0.006$

- $R_{\rm a}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$
  poor description
- $R_{\rm a}$ free
  good description
- $R_{\rm a}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$
  pretty good description
  (CL=1%)
  $\bar{a}\tau_0 > 0$
  (VERY PRELIMINARY)



Summary of Simplified τ-Model.

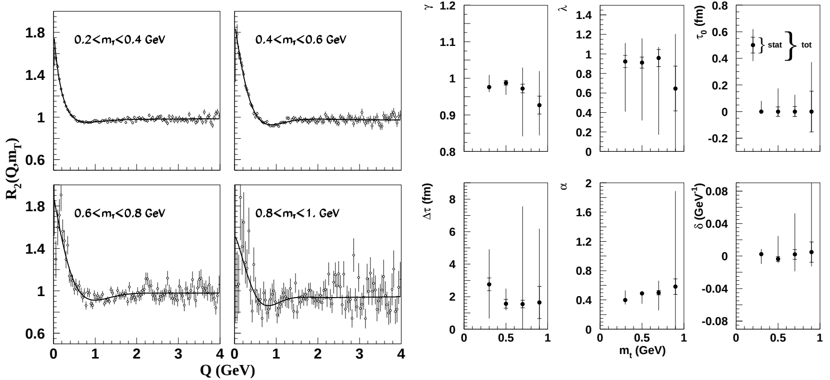
- Simplified τ-model works well
  - For 2-jet events including $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$
  - For 3-jet events
    - only if R_a a free parameter.
      Possibly the assumption $a_1 \approx a_2 \approx \bar{a}$ is less valid.
    - or if $\tau_0 > 0$

Limit analysis to 2-jet events (Durham, $y_{\text{cut}} = 0.006$)

For 2-jet events, $a = 1/m_t$

Full τ-model for 2-jet events

$$R_2(Q,\overline{m}_t) = \gamma \left[1 + \lambda \cos\left(\frac{\tau_0 Q^2}{\overline{m}_t} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right) \exp\left(-\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right)\right] \cdot (1+\delta Q)$$

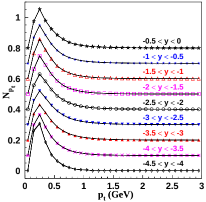


- Parameters α, $\Delta\tau$, τ_0 are $\sim$ independent of m_t
- Note: $\Delta\tau$ indep. of m_t equiv. to radius $r \propto 1/\sqrt{\overline{m}_t}$

Emission Function of 2-jet Events.

In the τ-model, the emission function in configuration space is

$$S(x) = \frac{\mathrm{d}^4 n}{\mathrm{d}\tau \mathrm{d}^3 r} = \left(\frac{m_\mathrm{t}}{\tau}\right)^3 H(\tau)\rho_1\left(p = \frac{r m_\mathrm{t}}{\tau}\right)$$



Factorization OK

For simplicity, assume $S(r, z, t) = G(\eta) I(r) H(\tau)$

(η = space-time rapidity)

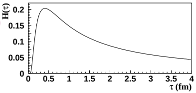
Strongly correlated $x, p \Longrightarrow$

$$\eta = y \qquad r = p_\mathrm{t}\tau/m_\mathrm{t}$$

$$G(\eta) = N_y(\eta) \qquad I(r) = \left(\frac{m_\mathrm{t}}{\tau}\right)^3 N_{p_\mathrm{t}}(r m_\mathrm{t}/\tau)$$

(N_y, N_{p_t} are inclusive single-particle distributions)

$H(\tau)$



$\alpha = 0.43$

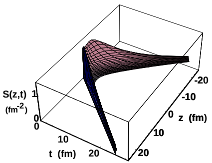
$\Delta\tau = 1.8$ fm

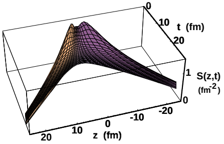
$\tau_0 = 0$

So, using experimental N_y, N_{p_t} dists.
and $H(\tau)$ from BEC fits,
we can reconstruct S.

Emission Function of 2-jet Events.

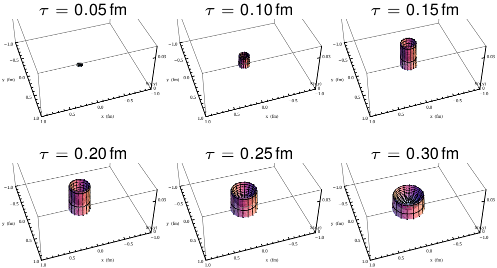
Integrating over r,

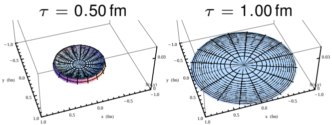




"Boomerang" shape

Integrating over z,





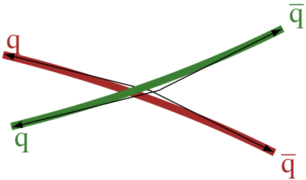
Expanding ring

Particle production is close to the light-cone

Inter-string BEC?

Recall $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$

- 2 strings, inter-string BEC?
- large syst. uncertainty on M_W
- No inter-string BEC found. BUT
  - low statistics
  - small overlap in $\vec{p}$ exasperated by expt. sel. of 4 well-separated jets



Also 2 strings in $e^+e^- \to Z \to q\bar{q}g$

- high statistics
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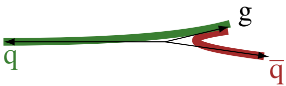

Nick van Remortel, Ph.D. thesis, Univ. Antwerpen, 2003
Qin Wang, Ph.D. thesis, Radboud Univ., 4 Sep 2008

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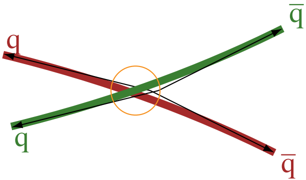

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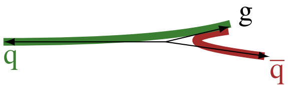

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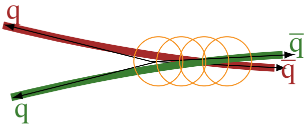

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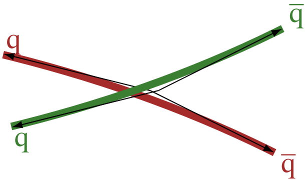

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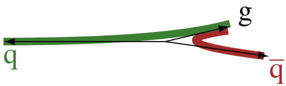

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Signal of Inter-string BEC

Assuming $\lambda_{\text{inter-string}} = \lambda_{\text{intra-string}}$,

- momentum overlap, $f_{\text{str-1}}(\vec{p}) = f_{\text{str-2}}(\vec{p})$
- If no p overlap, can not detect inter-string BEC
  no p overlap $\Longrightarrow \lambda_2 = \lambda_1$ and $r_2 = r_1$
- spatial overlap, $f_{\text{str-1}}(\vec{r}) = f_{\text{str-2}}(\vec{r})$
- Assuming full p overlap and $\lambda_{\text{inter-string}} = \lambda_{\text{intra-string}}$,

	no spatial overlap	full spatial overlap
Inter-string BEC	$\lambda_2 = \lambda_1$ $r_2 > r_1$	$\lambda_2 = \lambda_1$ $r_2 = r_1$ (HBT) $r_2 > r_1$ (Lund)
No inter-str. BEC	$\lambda_2 < \lambda_1$ $r_2 = r_1$	$\lambda_2 < \lambda_1$ $r_2 = r_1$

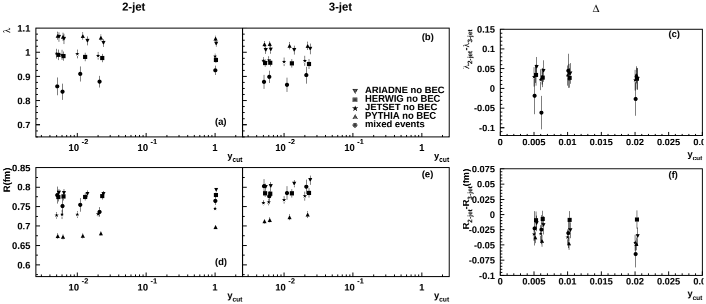
2-jet / 3-jet

fit with Edgeworth expansion parametrization:

$$R_2(Q) = \gamma(1 + \delta Q + \varepsilon Q^2)\left[1 + \lambda e^{-r^2 Q^2}\left(1 + \frac{\kappa}{3!} H_3(rQ)\right)\right]$$

λ

r



- very weak dependence of λ, r on y_{cut}
- $\lambda_{\text{2-jet}} \gtrsim \lambda_{\text{3-jet}} \Longrightarrow \lambda_{\text{2-str}} \lesssim \lambda_{\text{1-str}} \Longrightarrow$ no inter-str. BEC ?
- $r_{\text{2-jet}} \lesssim r_{\text{3-jet}} \Longrightarrow r_{\text{2-str}} \gtrsim r_{\text{1-str}} \Longrightarrow$ inter-str. BEC ?

q/g

fit with Edgeworth expansion parametrization:

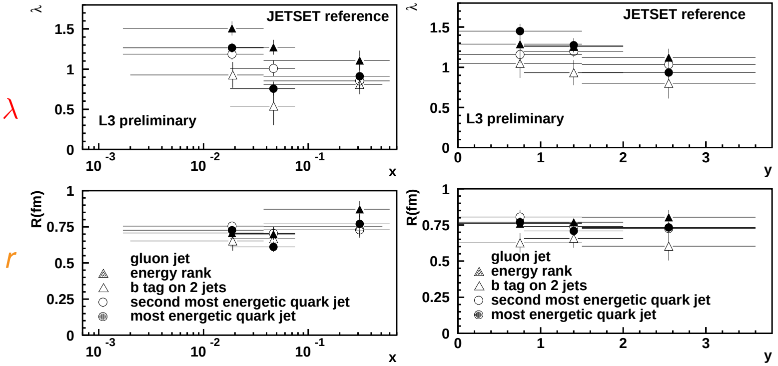
$$R_2(Q) = \gamma \left[1 + \lambda e^{-r^2 Q^2} \left(1 + \frac{\kappa}{3!} H_3(rQ)\right)\right]$$

Sample	g fract. (%)	$\langle E \rangle$ (GeV)	λ	r (fm)
dusc 2-jet	0	43	0.93±0.05	0.75±0.03
b-tag Yg g	32	40	0.96±0.24	0.56±0.10
dusc Merc.	33	29	1.25±0.11	0.78±0.05
dusc Yq	50	24	1.13±0.09	0.78±0.04
b-tag Merc. g	74	25	1.31±0.50	0.96±0.21
b-tag Yq g	75	19	0.83±0.18	0.76±0.09
average			1.01±0.04	0.76±0.02
			CL = 6%	CL = 37%

No evidence for q/g differences – Inter-String BEC (HBT) ?

q/g: *x*, *y* dependence

expect more spatial overlap in tip of g jet



- $\lambda \searrow$, not const. with increasing *x* or *y*, but equally for q, g.
  λ_g of *E*-tag, b-tag inconsistent – systematics?
- *r* const., not decreasing with *x* and *y* – no inter-str. BEC ?

Summary 2

- 2-jet/3-jet: inconclusive.
- No evidence for different BEC in q, g.
  - No inter-string BEC?
  - or Inter-string BEC, but HBT rather than Lund?
  - or no sensitivity? - difficult to assess without a model

Inter-string BEC remains an open question.

Acknowl.: Qin Wang, Ph.D. thesis, Radboud Univ., 4 Sep 2008

Summary 1

- $R_2(Q)$, not $R_2(\vec{Q})$ is a reasonably good approximation
- But sym. Gaussian, Edgeworth, Lévy $R_2(Q)$ do not fit well
- τ-model with a one-sided Lévy proper-time distribution
  - Simplified, it provides a new parametrization of R_2:
    - Works well with eff. R, R_a for all events;
    - with only eff. R for 2-jet events.
  - $R_2(Q, m_t)$ successfully fits R_2 for 2-jet events
    - both Q- and m_t-dependence described correctly
    - Note: we found $\Delta\tau$ to be independent of m_t
      $\Delta\tau$ enters R_2 as $\Delta\tau Q^2/m_t$
      In Gaussian parametrization, r enters R_2 as r^2Q^2
      Thus $\Delta\tau$ independent of m_t corresponds to $r \propto 1/\sqrt{m_t}$
- Emission function shaped like a boomerang in z-t
  and an expanding ring in x-y
  Particle production is close to the light-cone

Acknowl.: Tamás Novák, Ph.D. thesis, Radboud Univ., 4 Sep 2008

L3 Data

- $e^+e^- \longrightarrow \text{hadrons}$ at $\sqrt{s} \approx M_Z$
- about 10^6 events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\text{cut}} = 0.006$
- use mixed events for reference sample in τ-model studies
  use mixed events or MC for reference sample in inter-string BEC studies