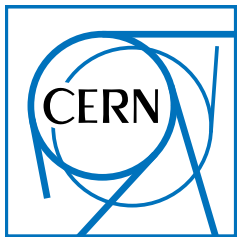
Elongation of the Pion Source

in

Z $\Longrightarrow$ hadrons

from **at** 

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Outline

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- Event and Charged Particle Selection
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The BE Effect

Since 1960, "Bose-Einstein" correlations have been used to measure the space-time volume of pion emission.
G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. **120**(1960)300

Assume pion j emitted at space-time point, x_j,
with 4-momentum, p_j, has a plane-wave wave function:

$$\psi_j(x) \propto \exp\left[\imath p(x_j - x) + \imath\phi_j\right]$$

The wave function of 2 pions is (after symmetrization), neglecting phases ϕ_1, ϕ_2,

$$\Psi_2(x) = \frac{1}{\sqrt{2}}\{\exp\left[\imath p_1(x_1 - x) + \imath p_2(x_2 - x)\right] + \exp\left[ip_2(x_1 - x) + \imath p_1(x_2 - x)\right]\}$$

We measure p_1, p_2. Therefore, average over x_1, x_2.

Assume pion production is incoherent
$\Longrightarrow$ average probability, $= |\psi_2|^2$

$$|\psi_2|^2 = 1 + \cos\left[(p_1 - p_2)(x_1 - x_2)\right]$$

If pion production is coherent, $|\psi_2|^2 = 1$
Hence, correlation function we can measure is

$$R(p_1, p_2) = 1 + \lambda\langle\cos\left[(p_1 - p_2)(x_1 - x_2)\right]\rangle_{x_1,x_2}$$

coherent $0 \leq \lambda \leq 1$ incoherent

The BE Effect

$$R(p_1, p_2) = 1 + \lambda \langle \cos\left[(p_1 - p_2)(x_1 - x_2)\right] \rangle_{x_1, x_2}$$

- for $p_1 \approx p_2$, $\cos\left[...\right] \approx 1 \Longrightarrow R \approx 1 + \lambda$
- for $|p_1 - p_2|$ large, $\cos$ is rapidly oscillating function of $(x_1 - x_2)$ and will average to $\approx 0 \Longrightarrow R \approx 1$
- Therefore, expect

To average, we need the distribution of $(x_1 - x_2)$, $f(x_1, x_2)$
Assuming $f(x_1, x_2) = f(x_1) f(x_2)$,

$$R(p_1, p_2) = 1 + \lambda \left| \int f(x) \, \exp\left[\imath (p_1 - p_2) x\right] \, \mathrm{d}x \right|^2$$

which is just the Fourier transform of $f(x)$

Simplest assumption: $f(x) =$ 4-dimensional Gaussian $\Longrightarrow$

$$R(p_1, p_2) = 1 + \lambda \cdot \text{Gaussian in } (p_1 - p_2)$$

Event Selection

Data from 1994 run of L3 at LEP, $\sqrt{s} \approx 91.2$ GeV $\approx M_Z$

Select about 1 million hadronic events
with well-measured tracks.

Since we are mainly interested in small $|p_1 - p_2|$,
and hence tracks which are close together,
we impose severe track quality cuts.
$\sim$40% of tracks are rejected.

Final data sample: $\sim$36 million like-sign pairs

To Measure the BE correlation function

The 2-particle correlation function R_2
of 2 particles with 4-momenta p_1, p_2 is defined

$$R_2 = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_2(p_2)}$$

$\rho_2(p_1, p_2)$	2-particle number density
$\rho_i(p_i)$ $(i = 1, 2)$	1-particle number density

But there are many correlations besides BE,
$e.g.$, resonances and, $E, \vec{p}$ conservation

So, remove these other correlations by measuring

$$R(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)}$$

$\rho_0(p_1, p_2)$ = 2-particle density *if* no BE

But $\rho_0(p_1, p_2)$ does NOT exist in nature.
So how do we determine it?

Determining ρ_0

Common methods:

1. Use pairs of tracks of opposite sign.
   But resonances different

2. Use a Monte Carlo which describes data
   but does not simulate BE.
   But fragmentation and BE are correlated.
   Does this remove too much?

3. Mix tracks from different events.
   But this removes ALL correlations

We use method 3,
adding back the non-BE correlations by Monte Carlo.
This should be less dependent
on the details of the MC than method 2.

R

Recall, simplest assumption:
Space-time distribution of pion emission
is a 4-dimensional Gaussian $\Longrightarrow$

$$R(p_1, p_2) = 1 + \lambda \cdot \text{Gaussian in } (p_1 - p_2)$$

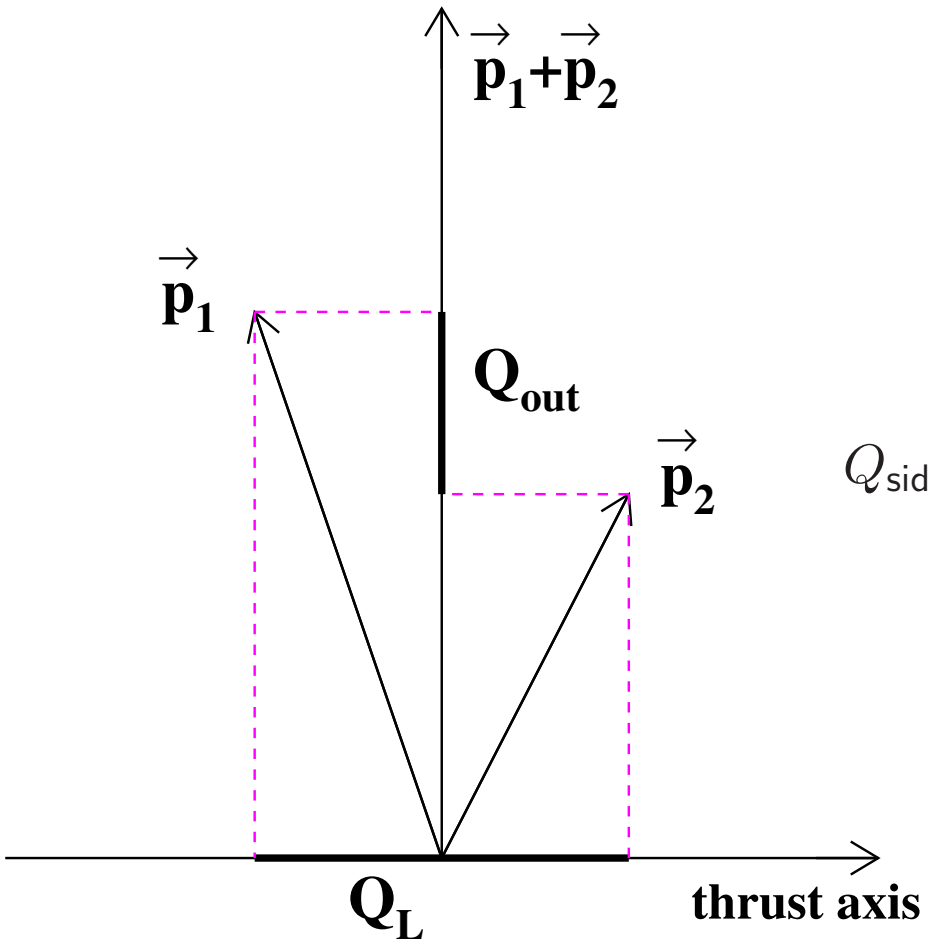
i.e., in the 4-momentum difference, $Q = |p_1 - p_2|$

Since BE effect occurs predominantly at small Q,
often parametrize R as 1-dimensional Gaussian in Q.

This assumes that the 4-dimensional Gaussian
is simply a product of 4 identical 1-dim. Gaussians.
But,
there is no reason to expect this symmetry in $e^+e^- \longrightarrow q\bar{q}$.
Therefore, we do a 3-dim. analysis.

LCMS

Boost each π-pair along thrust axis to its
Longitudinal Center of Mass System



$Q_{\text{side}} \perp (Q_{\text{L}}, Q_{\text{out}})$

Advantages of LCMS:

$$\begin{aligned} Q^2 &= Q_{\text{L}}^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 - (\Delta E)^2 \\ &= Q_{\text{L}}^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2(1-\beta^2) \end{aligned}$$

where $\beta \equiv \frac{p_{\text{out}\,1}+p_{\text{out}\,2}}{E_1+E_2}$
Thus, the energy difference,
and therefore the difference in emission time of the pions
couples only to the out-component, Q_{out}.

R in the LCMS

Thus,
Q_{L} and Q_{side} reflect only spatial dimensions of the source
Q_{out} reflects a mixture of spatial and temporal dimensions.

Writing R in terms of $\vec{Q} = (Q_{\mathrm{L}}, Q_{\mathrm{side}}, Q_{\mathrm{out}})$

$$R(\vec{Q}) = \frac{\rho_2(\vec{Q})}{\rho_0(\vec{Q})}$$

We measure $\rho_2(\vec{Q})$ from the data.

We measure $\rho_0(\vec{Q})$ from a sample obtained
by mixing tracks from different events

$\rho_0(\vec{Q})$

Determine $\rho_{\text{mix}}(\vec{Q})$ from a sample of mixed data events:
Form a "pool" of data events
rotated so that z-axis is along thrust axis

- Randomly pick an event from the pool.
  Call this event E_0.
  Denote its charged track multiplicity by N.

- Randomly pick N other events from the pool
  having approx. same multiplicity. Call them E_i

- Replace each track i in E_0 by a track of the same charge
  randomly chosen from event E_i

- Remove used events, E_0 and E_i, from the pool

But this procedure removes *all* correlations.
Correct $\rho_{\text{mix}}(\vec{Q})$ by applying same procedure to MC events
having no simulation of BE (JETSET or HERWIG)

$$\rho_0(\vec{Q}) = \rho_{\text{mix}}(\vec{Q}) \cdot C_{\text{mix}}(\vec{Q}) \ , \quad C_{\text{mix}}(\vec{Q}) = \left[\frac{\rho_2(\vec{Q})}{\rho_{\text{mix}}(\vec{Q})} \right]_{\text{MC, noBE}}$$

Also mix without restrictions of
same multiplicity and same charge
Differences $\Longrightarrow$ systematic errors

$\rho_2(\vec{Q})$

$\rho_2(\vec{Q})$ is determined directly from the data

However, the ratio $\frac{\rho_2(\vec{Q})}{\rho_0(\vec{Q})}$ must still be corrected for detector acceptance, efficiency, and resolution to obtain $R(\vec{Q})$

Use multiplicative factor obtained from MC:

$$C_{\text{det}}(\vec{Q}) = \left(\frac{\rho_2(\vec{Q})}{\rho_{\text{mix}}(\vec{Q})}\right)_{\text{gen, pions}} \Bigg/ \left(\frac{\rho_2(\vec{Q})}{\rho_{\text{mix}}(\vec{Q})}\right)_{\text{det, all}}$$

at generator level, using only pions
after detector simulation, reconstruction,
event and track selection;
using all tracks, since L3 cannot identify pions
We use 3 MCs: JETSET with BE, JETSET without BE, HERWIG

Finally,

$$R(\vec{Q}) = \frac{\rho_2(\vec{Q})}{\rho_{\text{mix}}(\vec{Q})} \cdot \frac{1}{C_{\text{mix}}(\vec{Q})} \cdot C_{\text{det}}(\vec{Q})$$

In terms of numbers of events in 3-dimensional bins klm,

$$R_{klm} = \left[\frac{N_{klm}}{N^{\text{mix}}_{klm}}\right]_{\text{data}} \cdot \left[\frac{N^{\text{mix}}_{klm}}{N_{klm}}\right]_{\text{MC, noBE}} \cdot \left[\frac{N^{\text{gen}}_{klm}}{N^{\text{gen, mix}}_{klm}} \cdot \frac{N^{\text{det, mix}}_{klm}}{N^{\text{det}}_{klm}}\right]_{\text{MC}}$$

Parametrization of R

We parametrize $R(\vec{Q})$ by a 3-dimensional Gaussian

$$R(Q_{\mathsf{L}}, Q_{\mathsf{out}}, Q_{\mathsf{side}}) = \gamma \cdot B \cdot (1 + \lambda G)$$

where

- $\gamma =$ normalization (≈ 1)
- $B = (1 + \delta Q_{\mathsf{L}} + \varepsilon Q_{\mathsf{out}} + \xi Q_{\mathsf{side}})$
  describes large Q (long-range correlations)
- $\lambda =$ "incoherence", or strength of BE effect
- $G =$ azimuthally symmetric Gaussian

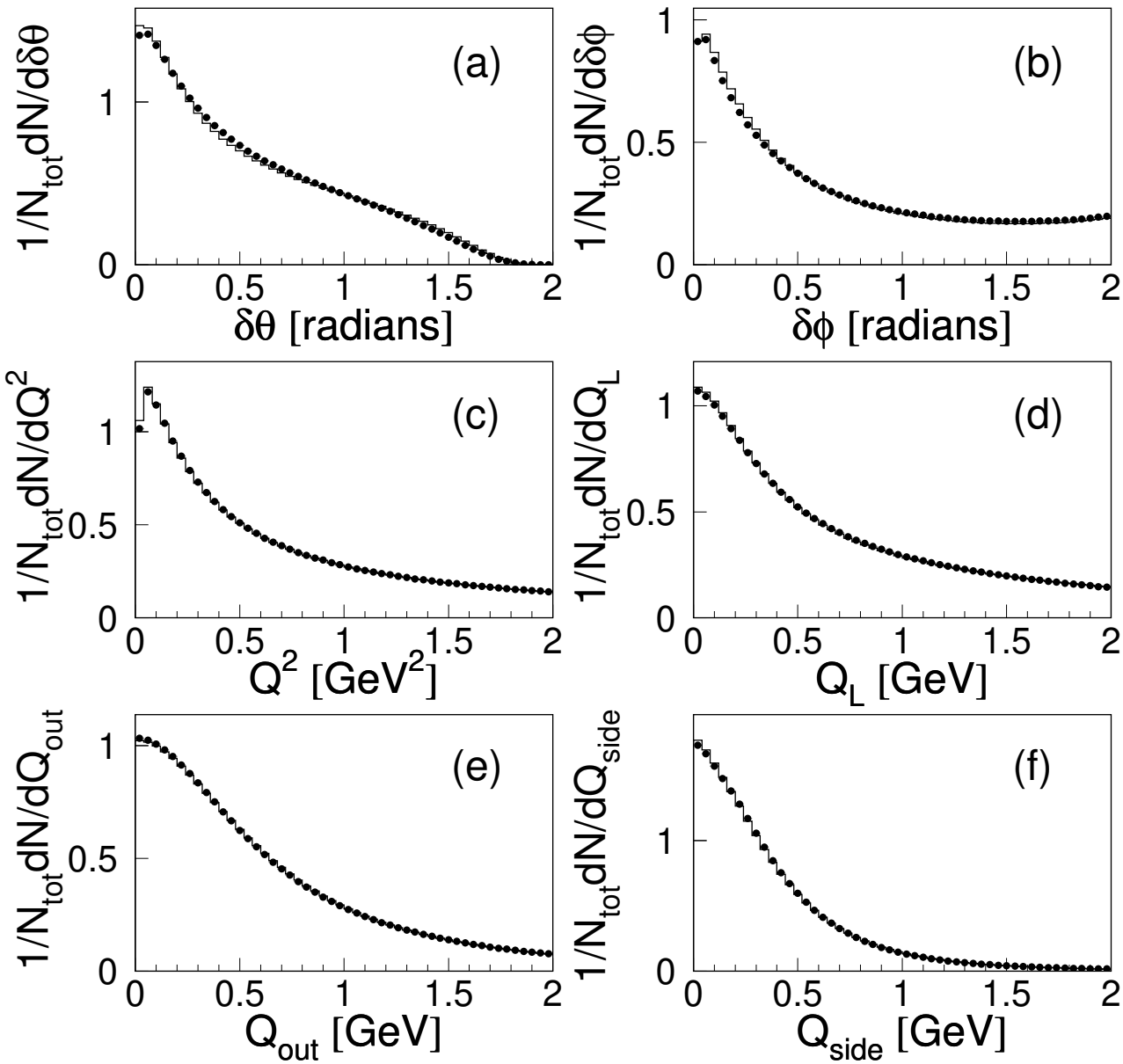
$$G = \exp\left(-R_{\mathsf{L}}^2 Q_{\mathsf{L}}^2 - R_{\mathsf{out}}^2 Q_{\mathsf{out}}^2 - R_{\mathsf{side}}^2 Q_{\mathsf{side}}^2 + 2\rho_{\mathsf{L,out}} R_{\mathsf{L}} R_{\mathsf{out}} Q_{\mathsf{L}} Q_{\mathsf{out}}\right)$$

Fit $R(Q_{\mathsf{L}}, Q_{\mathsf{out}}, Q_{\mathsf{side}})$ to the measured 3-dimensional histogram R_{klm}

Checks on the procedure

Correction procedures are very dependent on MC

- Does MC agree with raw data? YES



- Use different MCs: Differences $\Longrightarrow$ systematic errors

  $C_{\text{mix}}(\vec{Q})$ JETSET without BE, HERWIG

  $C_{\text{det}}(\vec{Q})$ JETSET with BE, JETSET without BE, HERWIG

Checks on the procedure

Analyze MC without BE (JETSET without BE, HERWIG) as though it were data:

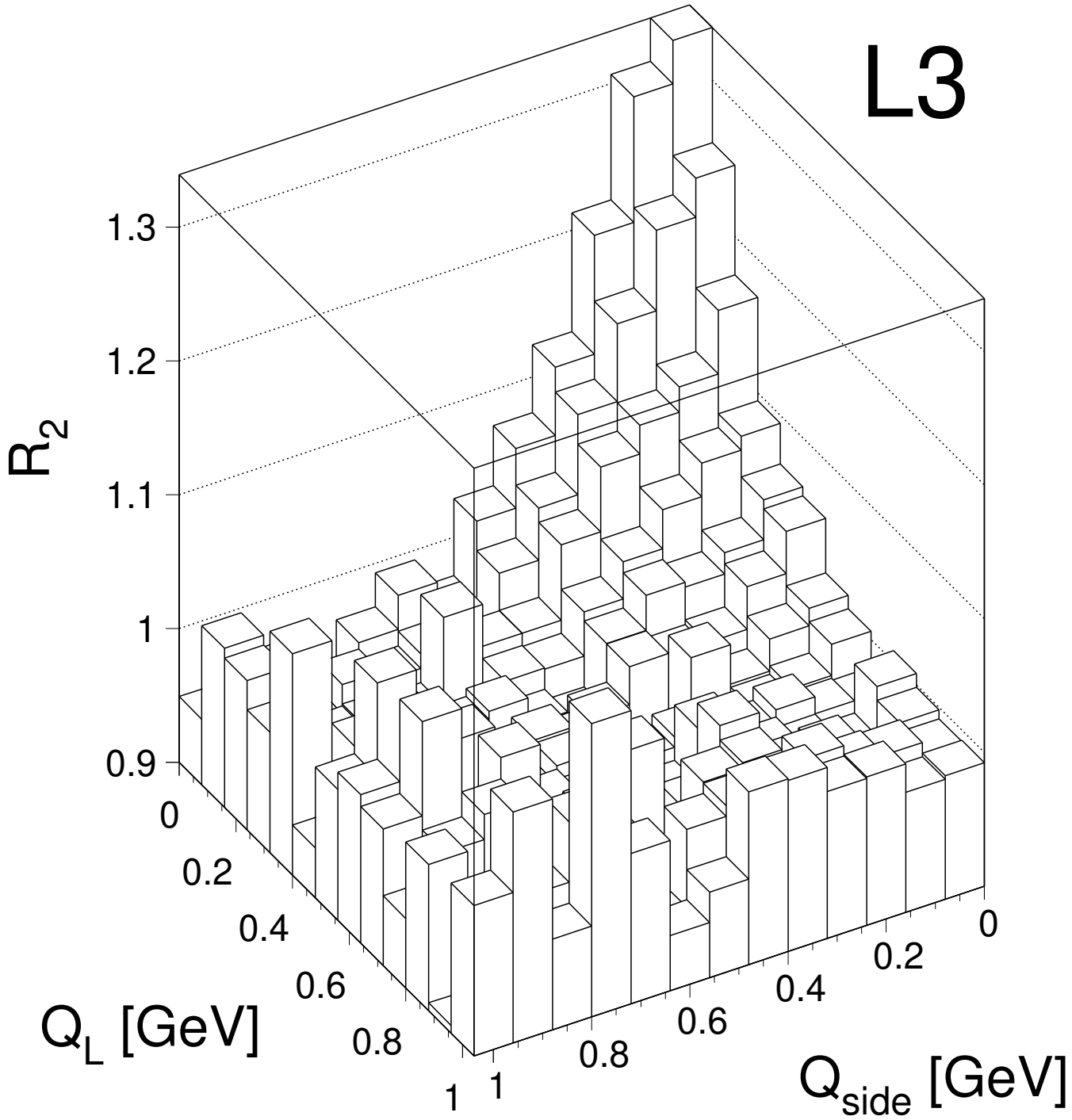
"Data"	C_{mix}	C_{det}
JETSET (without BE)	JETSET without BE	JETSET with BE
	JETSET without BE	HERWIG
	HERWIG	JETSET with BE
	HERWIG	HERWIG
HERWIG	JETSET without BE	JETSET with BE
	JETSET without BE	JETSET without BE
	HERWIG	JETSET with BE
	HERWIG	JETSET without BE

In all 8 cases $\chi^2/\text{DoF} \approx 1$, consistent with $\lambda = 0$ as expected for no BE

Vary track selection criteria,
and upper limit of fit range in Q
Differences $\Longrightarrow$ systematic errors

Results

Difficult to show 3-dim. histogram.
So a projection for $Q_{\text{out}} < 240\,\text{MeV}$:



Results of Gaussian fit to data

Fit $R(Q_{\mathrm{L}}, Q_{\mathrm{out}}, Q_{\mathrm{side}})$ to the measured 3-dimensional histogram R_{klm} in the range $Q_i < 1.04\,\mathrm{GeV}$

Off-diagonal term in Gaussian is consistent with 0
($\rho_{\mathrm{L,out}} = -0.008 \pm 0.057$)
So fix $\rho_{\mathrm{L,out}}$ to 0

Fit has $\chi^2/\mathrm{DoF} = 2314/2189$ – CL $= 3\%$ — not too good

Exponential is often used instead of Gaussian – CL is worse

An approximately Gaussian function can be expanded in terms of derivatives of the Gaussian (Edgeworth expansion)
Keeping only lowest-order non-Gaussian terms,
the Gaussian G in

$$R(Q_{\mathrm{L}}, Q_{\mathrm{out}}, Q_{\mathrm{side}}) = \gamma \cdot B \cdot (1 + \lambda G)$$

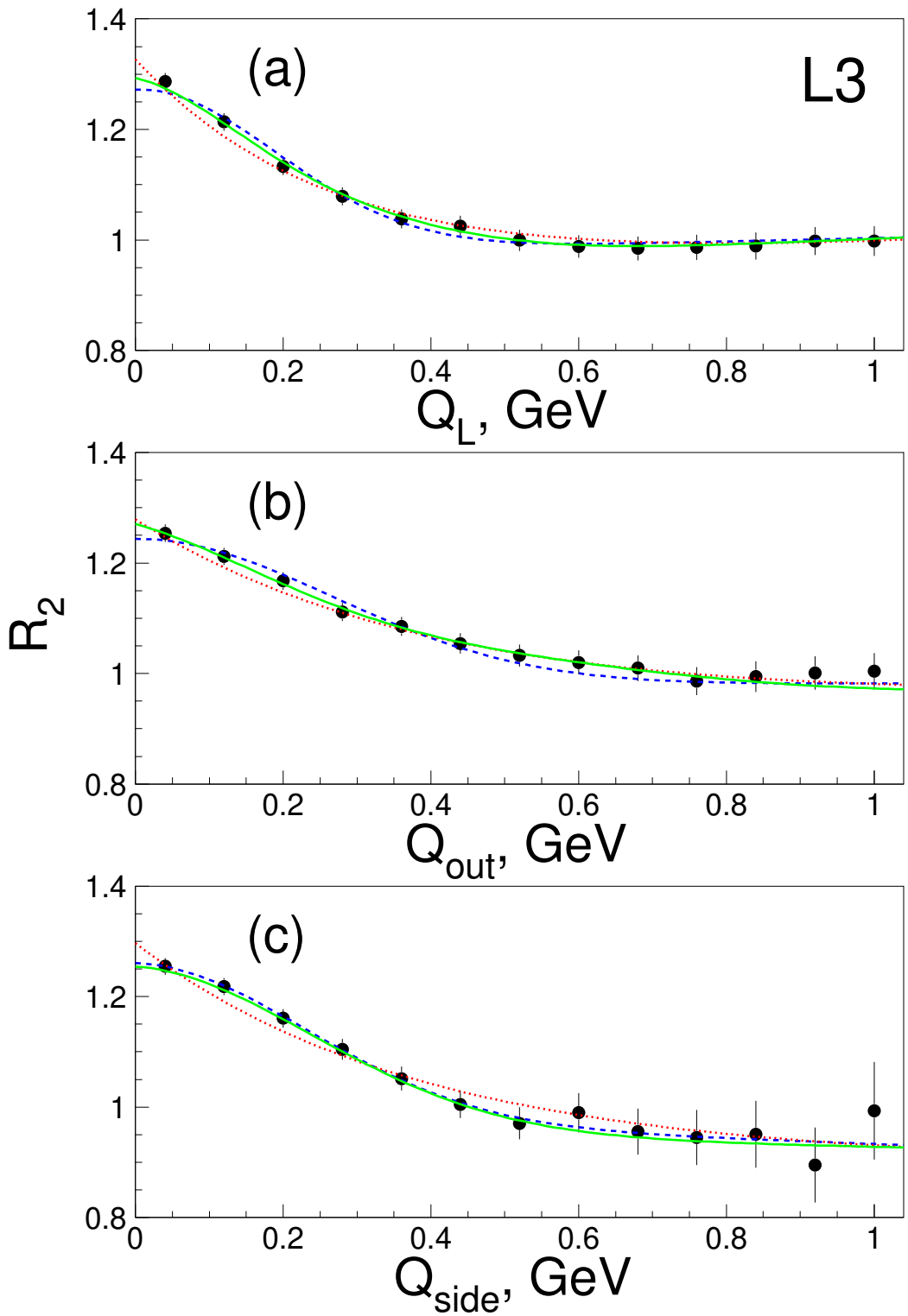
becomes (H_3 is third-order Hermite polynomial)

$$\begin{aligned} G \;=\; & \left[\exp\left(-R_{\mathrm{L}}^2 Q_{\mathrm{L}}^2 - R_{\mathrm{out}}^2 Q_{\mathrm{out}}^2 - R_{\mathrm{side}}^2 Q_{\mathrm{side}}^2\right)\right] \\ & \cdot \left[1 + \frac{\kappa_{\mathrm{L}}}{3!} H_3(R_{\mathrm{L}} Q_{\mathrm{L}})\right] \\ & \cdot \left[1 + \frac{\kappa_{\mathrm{out}}}{3!} H_3(R_{\mathrm{out}} Q_{\mathrm{out}})\right] \\ & \cdot \left[1 + \frac{\kappa_{\mathrm{side}}}{3!} H_3(R_{\mathrm{side}} Q_{\mathrm{side}})\right] \end{aligned}$$

Using this, we get CL $= 30\%$

Results

Projections, non-projected $Q_i < 240\,\mathrm{MeV}$



· · · exponential, - - - Gaussian, — Edgeworth

Results

parameter	Gaussian	Edgeworth
λ	$0.41 \pm 0.01^{+0.02}_{-0.19}$	$0.54 \pm 0.02^{+0.04}_{-0.26}$
R_{L} (fm)	$0.74 \pm 0.02^{+0.04}_{-0.03}$	$0.69 \pm 0.02^{+0.04}_{-0.03}$
R_{out} (fm)	$0.53 \pm 0.02^{+0.05}_{-0.06}$	$0.44 \pm 0.02^{+0.05}_{-0.06}$
R_{side} (fm)	$0.59 \pm 0.01^{+0.03}_{-0.13}$	$0.56 \pm 0.02^{+0.03}_{-0.12}$
$R_{\mathrm{out}}/R_{\mathrm{L}}$	$0.71 \pm 0.02^{+0.05}_{-0.08}$	$0.65 \pm 0.03^{+0.06}_{-0.09}$
$R_{\mathrm{side}}/R_{\mathrm{L}}$	$0.80 \pm 0.02^{+0.03}_{-0.18}$	$0.81 \pm 0.02^{+0.03}_{-0.19}$
κ_{L}	–	$0.5 \pm 0.1^{+0.1}_{-0.2}$
κ_{out}	–	$0.8 \pm 0.1 \pm 0.3$
κ_{side}	–	$0.1 \pm 0.1 \pm 0.3$
δ	$0.025 \pm 0.005^{+0.014}_{-0.015}$	$0.036 \pm 0.007^{+0.012}_{-0.023}$
ϵ	$0.005 \pm 0.005^{+0.034}_{-0.012}$	$0.011 \pm 0.005^{+0.037}_{-0.012}$
ξ	$-0.035 \pm 0.005^{+0.031}_{-0.024}$	$-0.022 \pm 0.006^{+0.020}_{-0.025}$
χ^2/DoF	2314/2189	2220/2186
C.L. (%)	3.1	30

- $\lambda > 0$ — the BE effect
- $R_{\mathrm{side}}/R_{\mathrm{L}} < 0$ — more than 5 std. dev. Elongation along thrust axis

Conclusion

- Edgeworth fit significantly better than Gaussian
- Elongation of pion source along thrust axis
  $R_{\text{side}}/R_{\text{L}} = 0.81 \pm 0.02^{+0.03}_{-0.19}$

Models which assume a spherical source are too simple.