Bose-Einstein Correlations and Jet Structure in e^+e^- annihilation

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Introduction - BEC

$$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \Longrightarrow \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$
$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a symmetric Lévy stable distribution
with radius r, index of stability α ($\alpha = 2$ for a Gaussian) $\Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-(Qr)^\alpha}$$

The L3 Data

- $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign pairs of well measured charged tracks from about $0.8 \cdot 10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\rm cut} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, “3-jet events”
- use mixed events for reference sample, ρ_0

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is proportional to momentum:
$$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}$$
For 2-jet events, $a = 1/m_{\rm t}$, $\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$, $m_{\rm t} = \sqrt{E^2 - p_z^2}$
- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is
$$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$
- Assume $\delta_\Delta(x - a\tau p)$ is very narrow — a δ-function. Then
$$R_2(p_1,p_2) = 1 + \lambda \,\mathrm{Re}\widetilde{H}\left(\frac{a_1 Q^2}{2}\right)\widetilde{H}\left(\frac{a_2 Q^2}{2}\right), \qquad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$
- Assume a one-sided Lévy distribution for $H(\tau)$
Three parameters:
  - α is the index of stability;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2:

BEC in the τ-model

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then

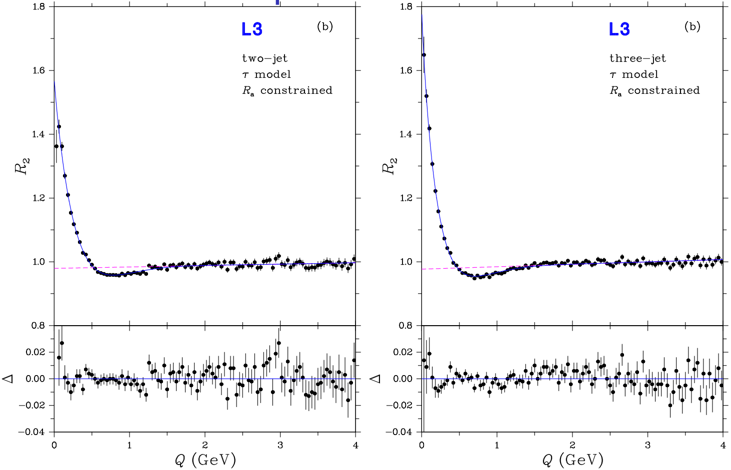
$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \exp\left[-|rQ|^{\alpha} \right] \right] (1 + \epsilon Q)$$

Results on Simplified τ-model from L3 Z decay



Simplified τ-model works well – also for 3-jet L3, Eur. Phys. J **C71** (2011) 1648

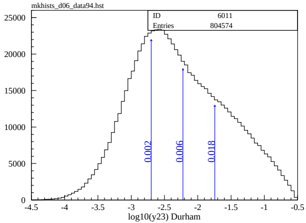
sym. dists. do not because of anti-correlation region

So we use the simplified τ-model parametrization.

Jets

Jets — JADE and Durham algorithms

- force event to have 3 jets:
  - normally stop combining when all 'distances' between jets are $> y_{\rm cut}$
  - instead, stop combining when there are only 3 jets left
  - y_{23} is the smallest 'distance' between any 2 of the 3 jets
- y_{23} is value of $y_{\rm cut}$ where number of jets changes from 2 to 3



define regions of $y_{23}^{\rm D}$ (Durham):

$y_{23}^{\rm D} < 0.002$	narrow two-jet	or		
$0.002 < y_{23}^{\rm D} < 0.006$	less narrow two-jet		$y_{23}^{\rm D} < 0.006$	two-jet
$0.006 < y_{23}^{\rm D} < 0.018$	narrow three-jet		$0.006 < y_{23}^{\rm D}$	three-jet
$0.018 < y_{23}^{\rm D}$	wide three-jet			

and similarly for $y_{23}^{\rm J}$ (JADE): 0.009, 0.023, 0.056

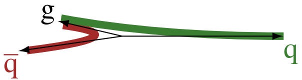
Jets

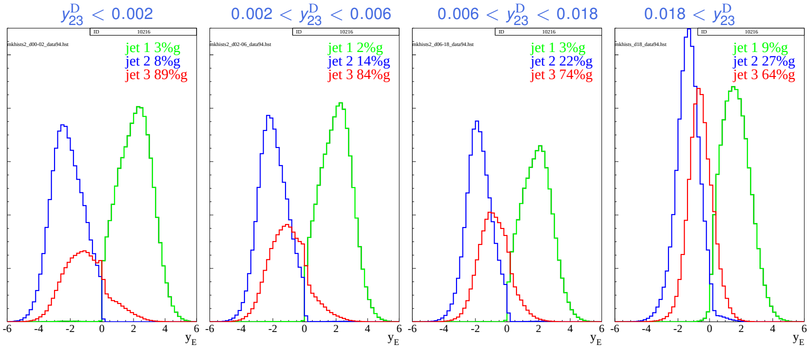
order jets by energy: $E_1 > E_2 > E_3$
Coordinate system: $\mathrm{Z} \longrightarrow \mathrm{q\overline{q}(g)}$

- estimate $\mathrm{q\overline{q}}$ axis by thrust axis, *i*.e., axis $\vec{n}_\mathrm{T}$ for which $T = \frac{\sum |\vec{p}_i \cdot \vec{n}_\mathrm{T}|}{\sum |\vec{p}_i|}$ is maximal
- 3-jet events are planar.
  Estimate event plane by thrust, major axes.
  Major is analogous to thrust, but in plane perpendicular to $\vec{n}_\mathrm{T}$.
- Note: thrust only defines axis $|\vec{n}_\mathrm{T}|$, not its direction.
  Choose positive thrust direction such that jet 1 is in positive thrust hemisphere
- Similarly, choose positive major direction such that jet 3 is in positive major hemisphere

Jets

rapidity, y_E, of particles from
jet 1, jet 2, jet 3:



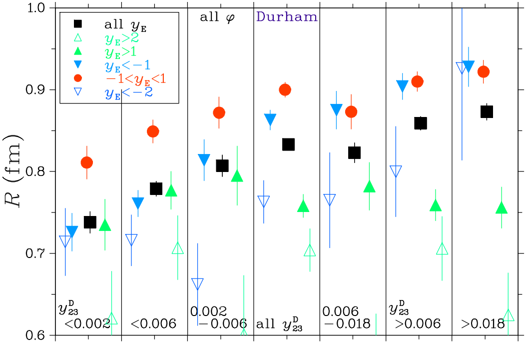


- $y_E > 1$ almost all jet 1 — almost all quark
- $y_E < -1$ mostly jet 2, some jet 3 — mostly quark
- $-1 < y_E < 1$ jet-3 enriched — largely gluon

Fits of simplified τ-model – L3 preliminary

To stabilize fits against large correlation of α, *R*, fix $\alpha = 0.44$

Select particle pairs by rapidity of pair



With y_{23}^D,

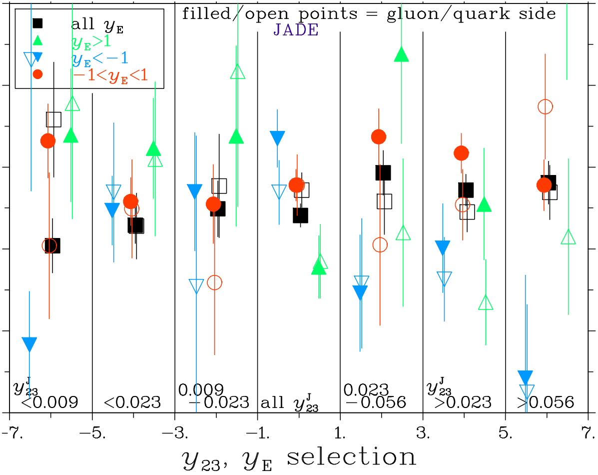
- all *y*: *R* increases
- 'pure' q jet, $y_E > 1$, or $y_E < -1$ & y_{23}^D small, or $y_E < -2$: *R* const.
- $R_{-1<y_E<1} > R_{\text{'pure' q}}$
- $R_{y_E<-1}$ increases
- at large y_{23}^D $R_{-1<y_E<1} = R_{y_E<-1}$

Conclusion (JADE agrees):
Increase in *R* with y_{23}^D is due to appearance of gluon jet

Fits of simplified τ-model – L3 preliminary

filled points = ϕ(tracks) on gluon side (in pos. major hemisphere)

open points = ϕ(tracks) on quark side (in neg. major hemisphere)

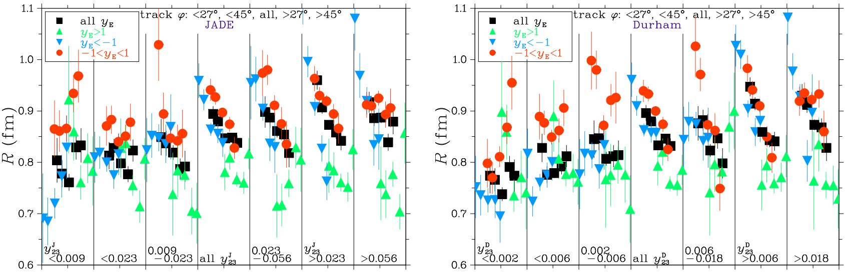


No significant differences q/g sides

Durham agrees

Fits of simplified τ-model – L3 preliminary

ϕ track-event plane: $< 27°$, $< 45°$, all, $> 27°$, $> 45°$



2-jet, all y_E selections and 3-jet, $y_E > 1$:

no significant differences between $R_{\text{in plane}}$, $R_{\text{out of plane}}$

3-jet, $-1 < y_E < 1$ or $y_E < -1$: $R_{\text{in plane}} > R_{\text{out of plane}}$

LCMS and the Simplified τ-model

Consider 2 frames:

1. LCMS: $$Q^2 = Q_{\rm L}^2 + Q_{\rm side}^2 + Q_{\rm out}^2 - (\Delta E)^2 = Q_{\rm L}^2 + Q_{\rm side}^2 + Q_{\rm out}^2\left(1-\beta^2\right), \quad \beta = \frac{p_{1\rm out}+p_{2\rm out}}{E_1+E_2}$$
2. LCMS-rest: $$Q^2 = Q_{\rm L}^2 + Q_{\rm side}^2 + q_{\rm out}^2, \quad q_{\rm out}^2 = Q_{\rm out}^2\left(1-\beta^2\right)$$

$q_{\rm out}$ is $Q_{\rm out}$ boosted (β) along out direction to rest frame of pair

In simplified τ-model, replace $R^2 Q^2$ by

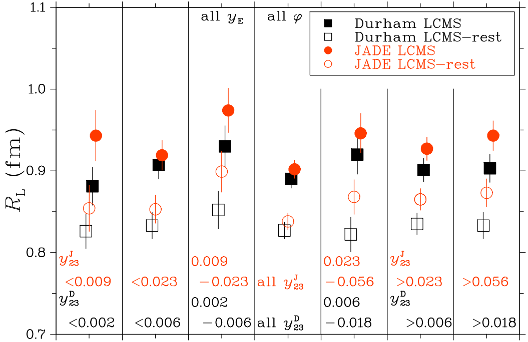
1. $A^2 = R_{\rm L}^2 Q_{\rm L}^2 + R_{\rm side}^2 Q_{\rm side}^2 + \rho_{\rm out}^2 Q_{\rm out}^2$
2. $B^2 = R_{\rm L}^2 Q_{\rm L}^2 + R_{\rm side}^2 Q_{\rm side}^2 + r_{\rm out}^2 q_{\rm out}^2$

Then in τ-model, for case 1:

$$R_2(Q_{\rm L}, Q_{\rm side}, Q_{\rm out}) = \gamma\left[1 + \lambda\cos\left(\tan\left(\frac{\alpha\pi}{2}\right)A^{2\alpha}\right)\exp\left(-A^{2\alpha}\right)\right] \cdot \left(1 + \epsilon_{\rm L} Q_{\rm L} + \epsilon_{\rm side} Q_{\rm side} + \epsilon_{\rm out} Q_{\rm out}\right)$$

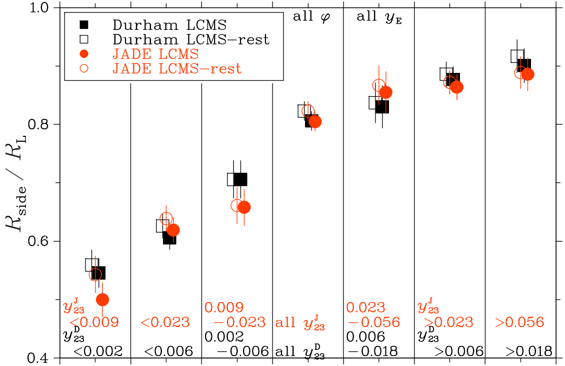
and comparable expression for case 2, $R_2(Q_{\rm L}, Q_{\rm side}, q_{\rm out})$

3-d Fits R_L – L3 preliminary



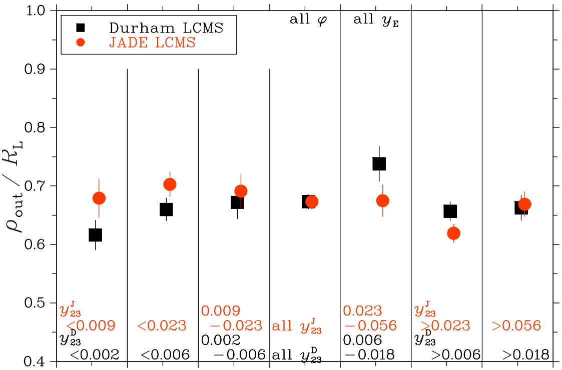
- Durham, JADE agree
- systematic difference LCMS, LCMS-rest
- R_L constant with y_{23}

3-d Fits R_{side} – L3 preliminary



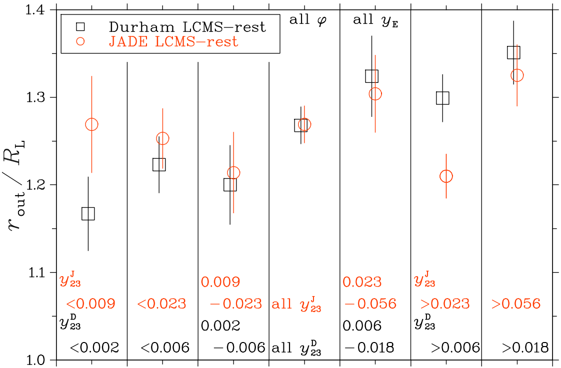
- LCMS, LCMS-rest agree
- Durham, JADE agree
- R_{side} increases with y_{23}, approx. 0.5–0.9 R_{L}

3-d Fits ρ_{out} – L3 preliminary



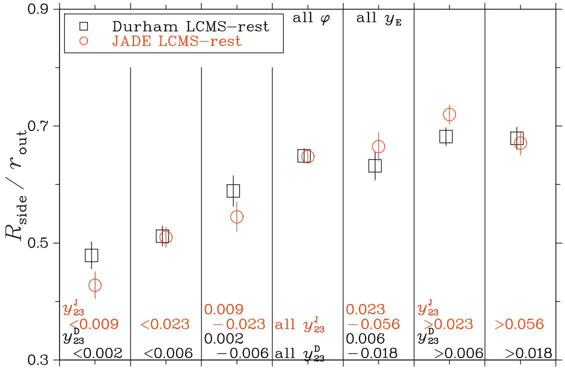
- Durham, JADE agree
- ρ_{out} constant with y_{23}

3-d Fits $r_{\rm out}$ – L3 preliminary



- Durham, JADE roughly agree
- $y_{23}^{\rm J}$: $r_{\rm out}$ approx. constant with y_{23}, approx. 1.27 $R_{\rm L}$
  or $y_{23}^{\rm D}$: slightly increasing with y_{23}, approx. 1.15–1.35 $R_{\rm L}$

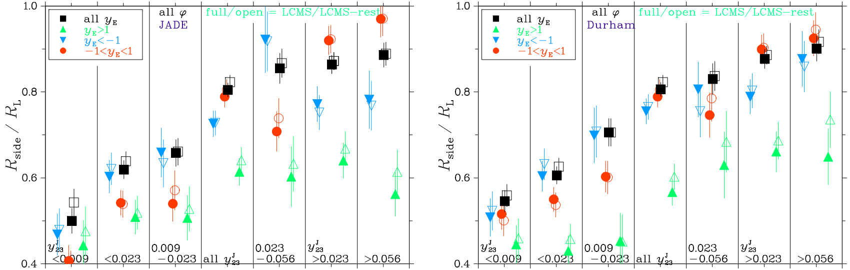
3-d Fits R_{side}/r_{out} – L3 preliminary



- Durham, JADE agree
- $R_{side} < r_{out}$ for all y_{23}
  R_{side}/r_{out} smallest for 2-jet
  Not azimuthally symmetric; not even for narrow 2-jet !!!

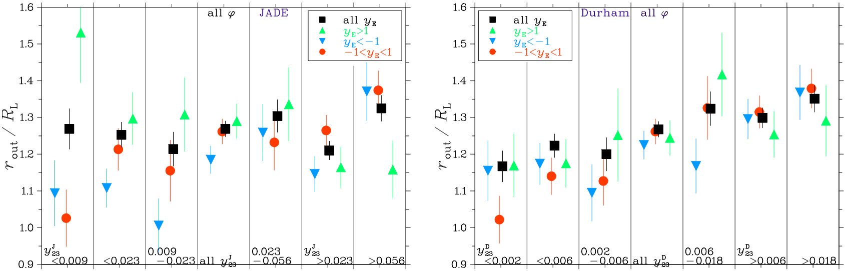
3-d Fits, y_E dependence R_{side} – L3 preliminary

In each y_E interval, R_L, ρ_{out} ≈ constant with y_{23} (not shown)



R_{side} increases, less for $y_E > 1$ than for other y_E regions

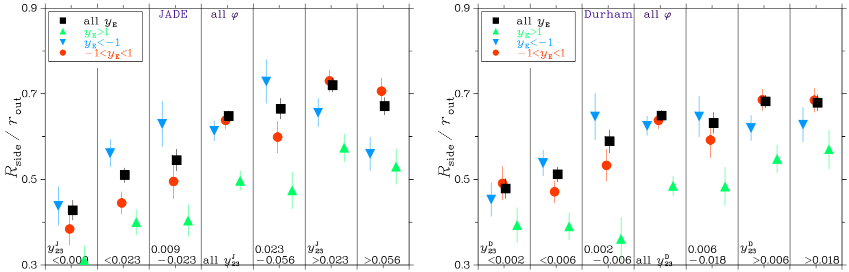
3-d Fits, y_E dependence r_{out} – L3 preliminary



r_{out} independent of y_E for $y_E > 1$

r_{out} perhaps increases slightly for $y_E < -1$ and $-1 < y_E < 1$

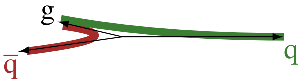
3-d Fits, y_E dependence R_{side}/r_{out} – L3 preliminary

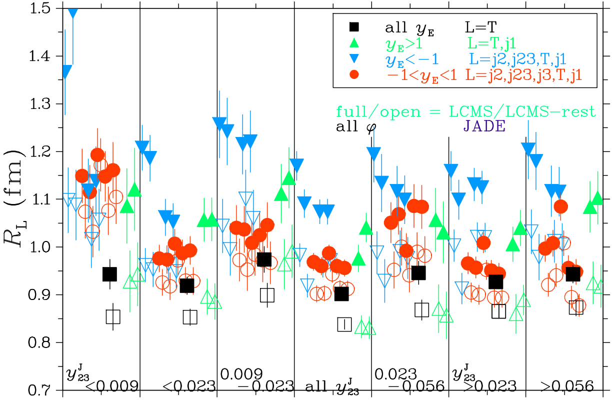


- $R_{side} < r_{out} < 1$ for all y_{23}
- R_{side}/r_{out} smaller for $y_E > 1$ – 'pure q jet'
- R_{side}/r_{out} smallest for 2-jet
- Not azimuthally symmetric; not even for narrow 2-jet !!!

Choice of Longitudinal direction R_L – L3 preliminary

2-jet event: Long = thrust axis
3-jet event: Long = thrust axis,
or $\vec{p}_{\mathrm{jet}\,1}$, $\vec{p}_{\mathrm{jet}\,2}$, $\vec{p}_{\mathrm{jet}\,2} - 0.5\vec{p}_{\mathrm{jet}\,3}$, $\vec{p}_{\mathrm{jet}\,3}$





JADE:
No clear dependence of R_L on L

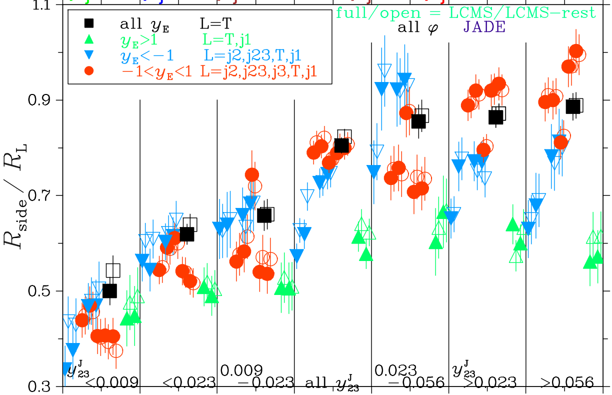
Durham agrees

Choice of Longitudinal direction R_{side} – L3 preliminary

2-jet event: Long = thrust axis

3-jet event: Long = thrust axis,

or $\vec{p}_{\mathrm{jet}\,1}$, $\vec{p}_{\mathrm{jet}\,2}$, $\vec{p}_{\mathrm{jet}\,2} - 0.5\vec{p}_{\mathrm{jet}\,3}$, $\vec{p}_{\mathrm{jet}\,3}$



Durham is similar

JADE:

No clear dependence of R_{side} on L except perhaps

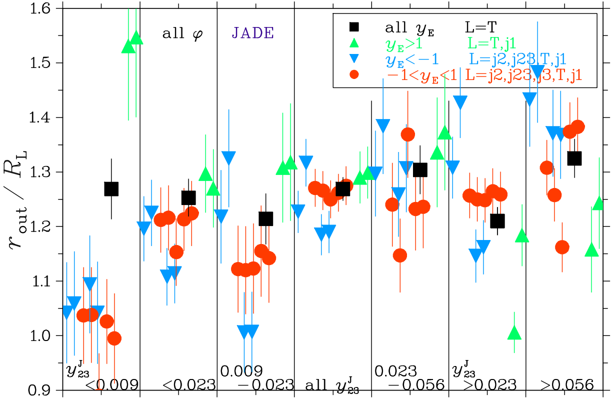
3-jet, $y_E < -1$:
R_{side} smaller for L=j2
$R_{\mathrm{side}}(y_E<-1) \approx R_{\mathrm{side}}(y_E>1)$

3-jet, $-1 < y_E < 1$:
R_{side} smaller for L=j3

3-jet:
$R_{\mathrm{side}}(-1<y_E<1) > R_{\mathrm{side}}(y_E<-1) \approx R_{\mathrm{side}}(y_E>1)$

Choice of Longitudinal direction r_{out} – L3 preliminary

2-jet event: Long = thrust axis
3-jet event: Long = thrust axis,
or $\vec{p}_{\text{jet 1}}$, $\vec{p}_{\text{jet 2}}$, $\vec{p}_{\text{jet 2}} - 0.5\vec{p}_{\text{jet 3}}$, $\vec{p}_{\text{jet 3}}$

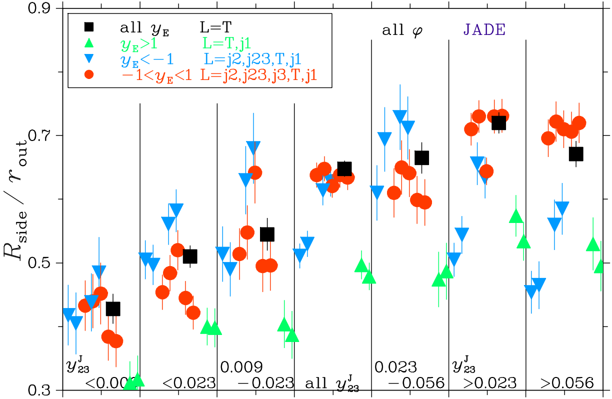


JADE

No clear dependence of R_{out} on L

Choice of Longitudinal direction $R_{\mathrm{side}}/r_{\mathrm{out}}$ – L3 preliminary

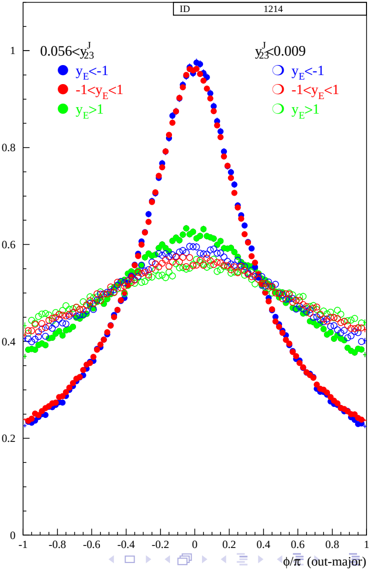
2-jet event: Long = thrust axis
3-jet event: Long = thrust axis,
or $\vec{p}_{\text{jet 1}}$, $\vec{p}_{\text{jet 2}}$, $\vec{p}_{\text{jet 2}} - 0.5\vec{p}_{\text{jet 3}}$, $\vec{p}_{\text{jet 3}}$



JADE:
2-jet, all L ≈ same
y_E>1,
L=T,j1≈same
3-jet,
y_E<−1, (L=j2)
≈ y_E>1, (L=T,j1)

out-event plane

- 2-jet: for all y_E, small preference for out direction to be in event plane
- 3-jet: for $y_\mathrm{E} > 1$, like 2-jet
- 3-jet: for $y_\mathrm{E} < -1$ and $-1 < y_\mathrm{E} < 1$, large preference for out direction to be in event plane



Summary

- 1-d
  - R increases with y_{23} ~ 0.7–$0.9\,\mathrm{fm}$
  - but not in ‘pure quark’ regions,
    ($y_E > 1$ or $y_E < -2$, all y_{23}) and ($y_E < -1$ for narrow 2-jet)
  - $R(-1 < y_E < 1) > R(\text{all } y_E) > y_E > 1)$
  - for 3-jet events R is larger in the event plane
- 3-d
  - R_L, $\rho_{out} \sim$ constant with y_{23}
    $R_L \approx 0.9\,\mathrm{fm}$ (LCMS) $\approx 0.85\,\mathrm{fm}$ (LCMS-rest)
    $\rho_{out}/R_L \approx 0.65$
  - R_{side} increases with y_{23} $R_{side}/R_L \approx 0.5$–0.9
    increase is less for $y_E > 1$
  - r_{out} perhaps increases slightly $r_{out}/R_L \approx 1.2$–1.3
  - $R_{side}/r_{out} < 1$ for all y_{23}
    R_{side}/r_{out} smaller for $y_E > 1$ – ‘pure q jet’
    R_{side}/r_{out} smallest for 2-jet
    Not azimuthally symmetric;
    least symmetric for narrow 2-jet events !!!

Summary

- 3-d dependence on Longitudinal axis
  - 3-jet: R_{side} is perhaps smaller using
    L=j2 for $y_{\text{E}} < -1$ and L=j3 for $-1 < y_{\text{E}} < 1$
    Then $R_{\text{side}}(-1 < y_{\text{E}} < 1) > R_{\text{side}}(y_{\text{E}} < -1) \approx R_{\text{side}}(y_{\text{E}} > 1)$
- 3-d: out direction is preferentially in the event plane
  slight preference for 2-jet and for 3-jet, $y_{\text{E}} > 1$
  strong preference for $-1 < y_{\text{E}} < 1$ and $R_{\text{side}}(y_{\text{E}} < -1)$

Qualitative Conclusions

- R larger in event plane for 3-jet events agrees with $r_{\mathrm{out}} > R_{\mathrm{side}}$ and preference of out to lie in event plane.
- For 3-jet, R_{L} and r_{out} are insensitive to choice of L.
  R_{side} does vary with L, but R_{side} is small.
  This may explain why τ-model works for 3-jet.
- Behavior of R and R_{side} in different y_{E} regions suggests $R_{\text{gluon}} > R_{\text{quark}}$.
  R and R_{side} are larger in gluon regions;
  they increase as gluon energy (and hence number of particles from gluon) increases.

Qualitative Conclusions/Speculations

- Picture of 'region of homogeneity' seems to be:
  - squashed ellipsoid
    r_{out} slightly larger than R_{L}
    R_{side} considerably smaller
  - in 'pure' quark jets (2-jet or 3-jet with $y_{\mathrm{E}} > 1$)
    ellipsoid oriented approx. isotropically about thrust axis
  - in other cases (3-jet with $y_{\mathrm{E}} < 1$ – gluon contribution)
    r_{out} tends to be in event plane
- But why is $R_{\mathrm{side}} \neq r_{\mathrm{out}}$, *i*.e., No azimuthal symmetry; not even for narrow 2-jet events?
  local p_{t} compensation defining a plane?

There is something fascinating about science.
One gets such wholesale returns of conjecture
out of such a trifling investment of fact.

– Mark Twain

Speculation

- CMS has observed the anti-correlation region as predicted in the τ-model and observed by L3.
  This suggests strings – like in e^+e^-.
- In pp, can the onset of hard jet production be seen in the BEC radii?
  like the third jet e^+e^-.
- Therefore, I suggest studying BEC as a function of p_t of highest p_t particle.