Bose-Einstein Correlations in e^+e^- annihilation

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X Workshop on Particle Correlations and Femtoscopy
Gyöngyös
25–29 August 2014

BEC Introduction

$$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \Rightarrow \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

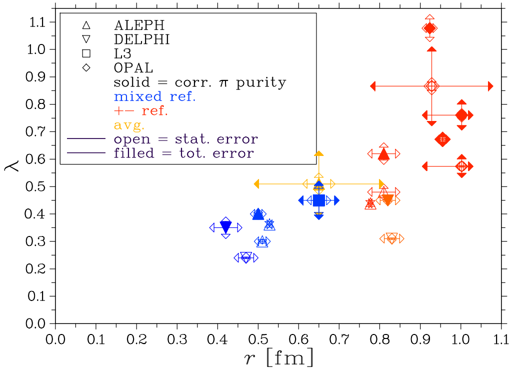
$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$
$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius $r \Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-Q^2 r^2}$$

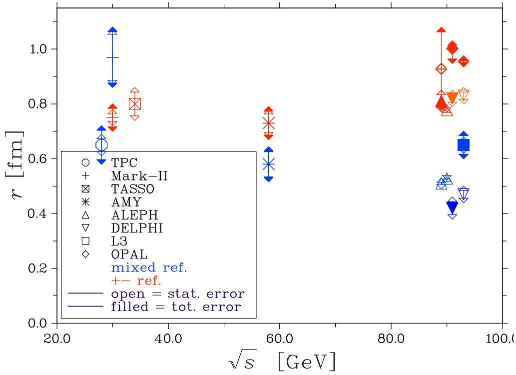
Results from R_2, $\sqrt{s} = M_Z$



– correction for π purity increases λ

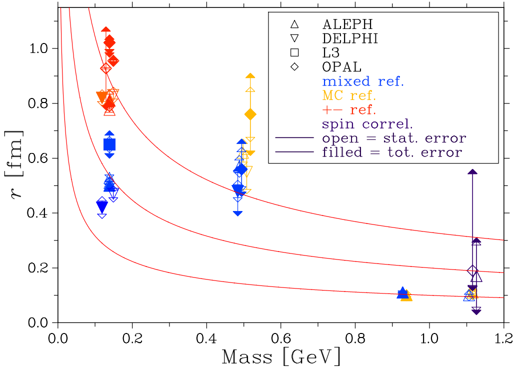
– mixed ref. gives smaller λ, r than $+-$ ref. – Average means little

$\sqrt{s}$ dependence of r



No evidence for $\sqrt{s}$ dependence

Mass dependence of *r* — BEC and FDC



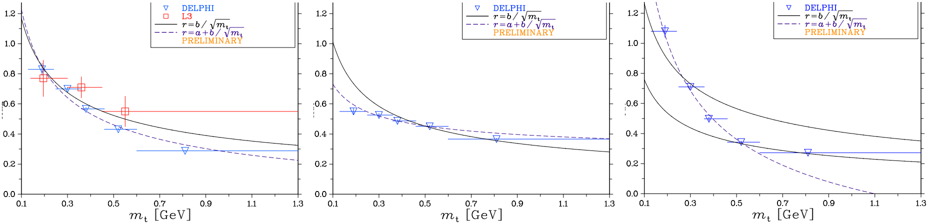
No evidence for $r \sim 1/\sqrt{m}$

$r_{\pi-\pi} \approx r_{\text{K-K}}$

$r(\text{mesons}) > r(\text{baryons})$

Transverse Mass dependence of *r* in LCMS

longitudinal side out



r decreases with m_t

but not equally fast in all components

The L3 Data

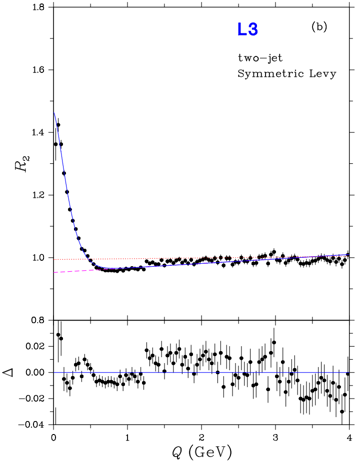
- $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign pairs of well measured charged tracks from about $0.8 \cdot 10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{cut} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, "3-jet events"
- use mixed events for reference sample, ρ_0
  corrected by MC (no BEC) for kinematics, resonances, etc.

$$\rho_0 \Longrightarrow \rho_0 \cdot \frac{\rho^{\mathrm{MC}}}{\rho_0^{\mathrm{MC}}}$$

Results – 'Classic' Parametrizations

$$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$$

- Gaussian
  $$G = \exp\left(-(rQ)^2\right)$$
- Edgeworth expansion
  $$G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$$
  Gaussian if $\kappa = 0$ $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $$G = \exp\left(-|rQ|^{\alpha}\right)$$
  $0 < \alpha \leq 2$
  $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:
  $$\overline{x}^\mu(p^\mu) = a\tau p^\mu$$
  where for 2-jet events, $a = 1/m_t$
  $$\tau = \sqrt{\bar{t}^2 - \bar{r}_z^2} \text{ is the “longitudinal” proper time}$$
  $$\text{and } m_t = \sqrt{E^2 - p_z^2} \text{ is the “transverse” mass}$$
- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is
  $$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$
- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.
  $$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$
- Assume $\delta_\Delta(x^\mu - \overline{x}^\mu)$ is very narrow — a δ-function. Then
  $$R_2(p_1,p_2) = 1 + \lambda \operatorname{Re}\widetilde{H}\left(\frac{a_1Q^2}{2}\right)\widetilde{H}\left(\frac{a_2Q^2}{2}\right), \quad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction,
  $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right], \qquad \alpha \neq 1$$
  where
  - α is the index of stability;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$R_2(Q, a_1, a_2) = \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right\} \cdot (1 + \epsilon Q)$$

BEC in the τ-model

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$

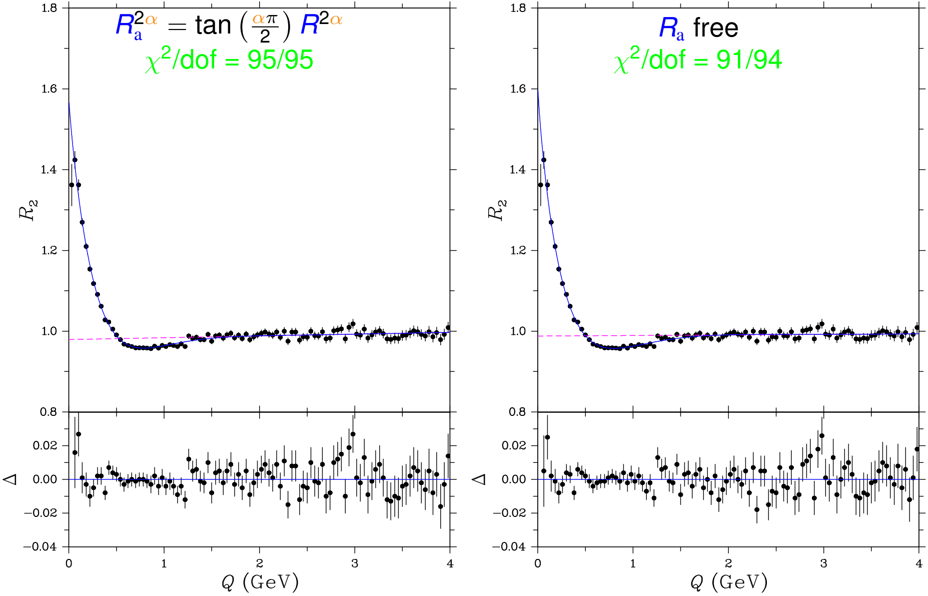
where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

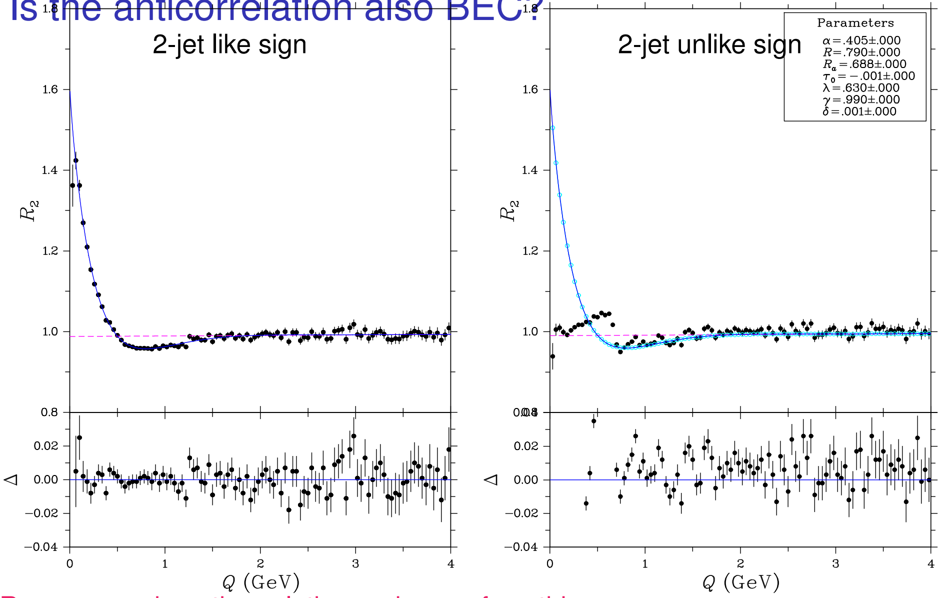
$$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left[-|rQ|^{\alpha} \right] \right] (1 + \epsilon Q)$$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to ‘width’ $\Delta\tau$ of $H(\tau)$

2-jet Results on Simplified τ-model from L3 Z decay



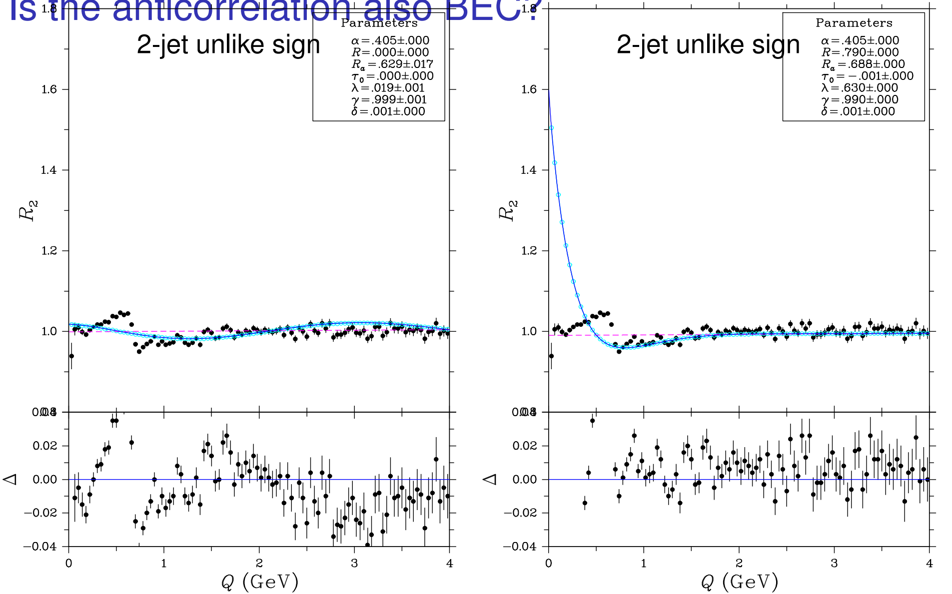
Is the anticorrelation also BEC?



Resonances in anticorrelation region confuse things

But anticorrelation may be present in unlike sign

Is the anticorrelation also BEC?



If anticorrelation is present in unlike sign,

it requires the damping of the exp of the BEC peak

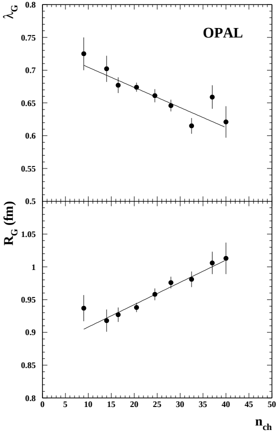
Multiplicity/Jet dependence – OPAL

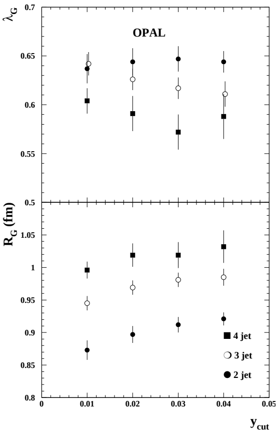
$$R_2(Q) = \gamma(1 + \lambda\, e^{-Q^2 r^2})(1 + \delta Q + \epsilon Q^2)$$

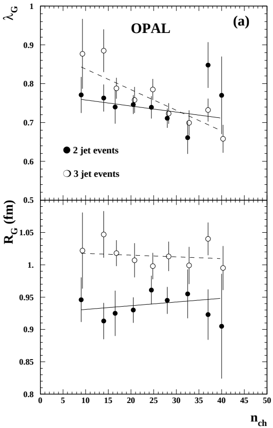
OPAL,Z.Phys.C72(1996)389

λ

r







$\lambda \searrow$ with n_{ch}
$r \nearrow$ with n_{ch}

$\lambda \searrow$ with n_{jet}
$r \nearrow$ with n_{jet}

$\lambda_{\text{n-jet}} \approx$ indep. of n_{ch}
$r_{\text{n-jet}}$ indep. of n_{ch}

Multiplicity dependence appears to be largely due to number of jets.

Multiplicity/Jet dependence in τ-model

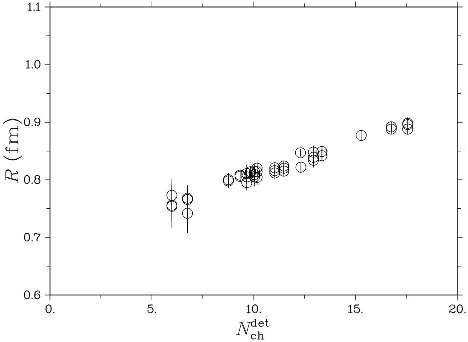
Use simplified τ-model, $\tau_0 = 0$
to investigate multiplicity and jet dependence

To stabilize fits against large correlation of parameters α and R fix $\alpha = 0.44$

Multiplicity dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

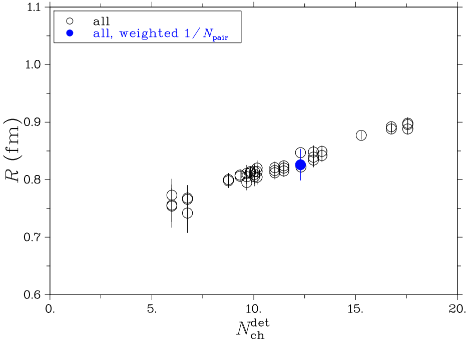
L3 PRELIMINARY



R increases with multiplicity

Multiplicity dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ L3 PRELIMINARY



R not constant

$\Longrightarrow$ R from fit is an average

But maybe not the average we want

To get R at avg. multiplicity of sample, should weight pairs by $1/N_{\text{pairs in event}}$

or calculate average multiplicity as

$$\frac{\sum_{\text{events}} N_{\text{event}} N_{\text{pairs in event}}}{N_{\text{pairs}}}$$

But the difference is small

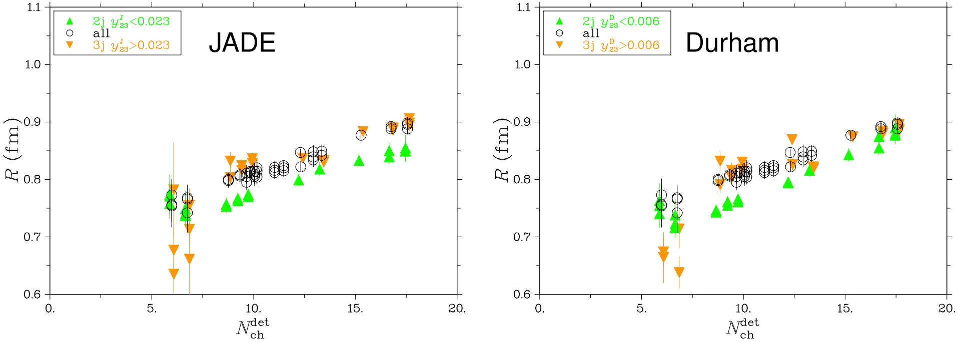
So I ignore it.

R increases with multiplicity

Multiplicity/Jet dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

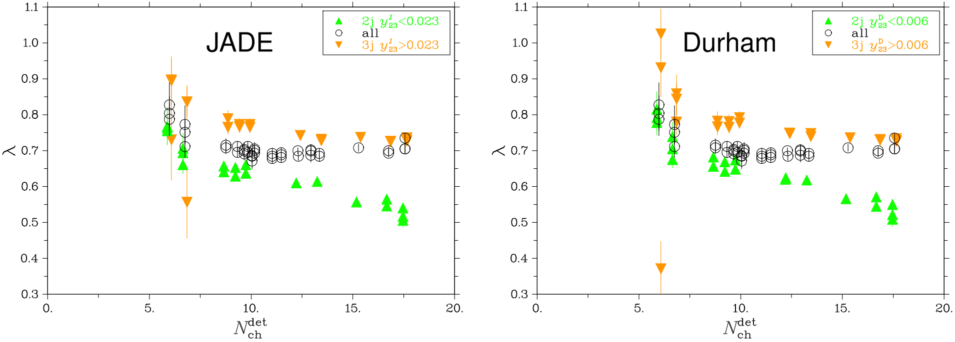


- R increases with N_{ch} and with number of jets
  whereas OPAL found $r_{\text{n-jet}}$ approx. indep. of N_{ch}
- Increase of R with N_{ch} similar for 2- and 3-jet events
- However, $R_{\text{3-jet}} \approx R_{\text{all}}$

Multiplicity/Jet dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY



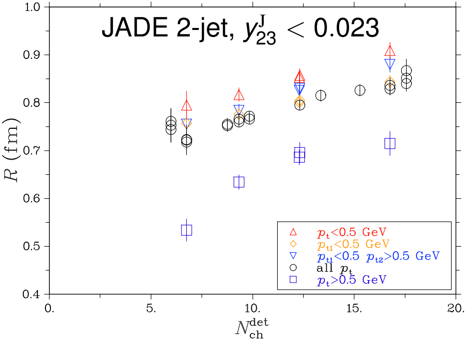
- $\lambda_{\text{3-jet}} > \lambda_{\text{2-jet}}$ opposite of OPAL
- λ initially decreases with N_{ch}
- then λ_{all} and $\lambda_{\text{3-jet}}$ approx. constant
  while $\lambda_{\text{2-jet}}$ continues to decrease, but more slowly
- whereas OPAL found λ_{all} decreasing approx. linearly with N_{ch}

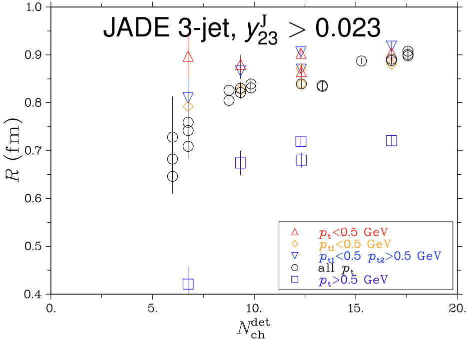
m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)





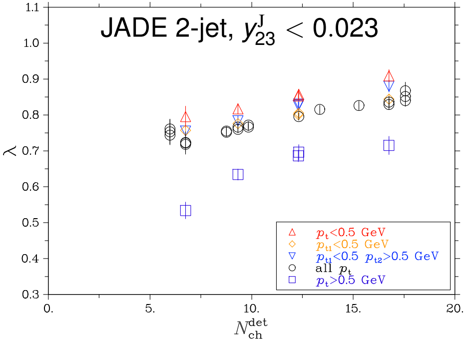
- R decreases with m_t for all N_{ch}
  smallest when both particles at high p_t

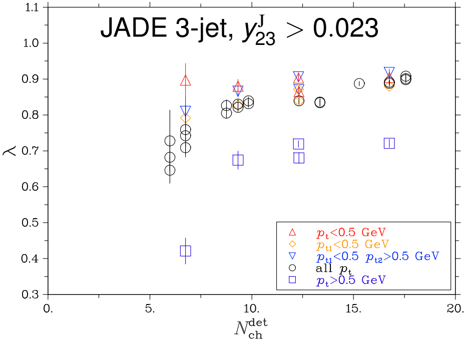
m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)





- λ decreases with m_t
  smallest when both particles at high p_t

On what do r, R, λ depend?

- r, R increase with N_{ch}
- r, R increase with N_{jets}
- for fixed number of jets, R increases with N_{ch}
  but r constant with N_{ch} (OPAL)
- r, R decrease with m_{t}
- Although m_{t}, N_{ch}, N_{jets} are correlated, each contributes to the increase/decrease of R
  but only m_{t}, N_{jets} contribute to the increase/decrease of r
- λ decreases with N_{ch}, N_{jets}
  though somewhat differently for τ-model, Gaussian (OPAL)
- λ decreases with m_{t}