Bose-Einstein Correlations and Jet Structure in e^+e^- annihilation

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Introduction - BEC

$$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \Longrightarrow \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda |\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$

$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a symmetric Lévy stable distribution
with radius r, index of stability α ($\alpha = 2$ for a Gaussian) $\Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-(Qr)^{\alpha}}$$

The L3 Data

- $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign pairs of well measured charged tracks from about $0.8 \cdot 10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\mathrm{cut}} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, "3-jet events"
- use mixed events for reference sample, ρ_0

The Simplified τ-model

We use the parametrization of the Simplified τ-model.

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588
L3 Collab., Eur. Phys. J **C71** (2011) 1648

2 parameters:

- effective radius R
- index of stability α of the Lévy distribution

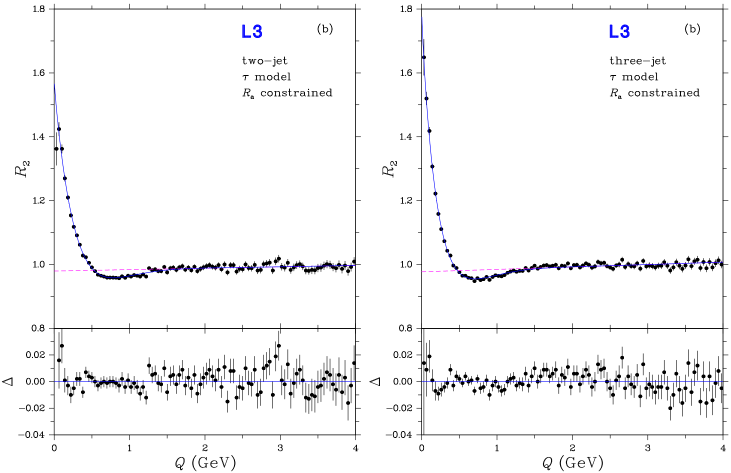
$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right] \cdot (1 + \epsilon Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left[-|rQ|^{\alpha}\right]\right] (1 + \epsilon Q)$$

Results on Simplified τ-model from L3 Z decay



L3, Eur. Phys. J **C71** (2011) 1648

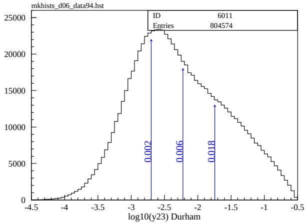
Simplified τ-model works well – also for 3-jet

sym. dists. do not because of anti-correlation region

Jets

Jets — JADE and Durham algorithms

- force event to have 3 jets:
  - normally stop combining when all ‘distances’ between jets are $> y_{\rm cut}$
  - instead, stop combining when there are only 3 jets left
  - y_{23} is the smallest ‘distance’ between any 2 of the 3 jets



- y_{23} is value of $y_{\rm cut}$ where number of jets changes from 2 to 3

define regions of $y_{23}^{\rm D}$ (Durham):

$y_{23}^{\rm D} < 0.002$	narrow two-jet	or	
$0.002 < y_{23}^{\rm D} < 0.006$	less narrow two-jet	$y_{23}^{\rm D} < 0.006$	two-jet
$0.006 < y_{23}^{\rm D} < 0.018$	narrow three-jet	$0.006 < y_{23}^{\rm D}$	three-jet
$0.018 < y_{23}^{\rm D}$	wide three-jet		

and similarly for $y_{23}^{\rm J}$ (JADE): 0.009, 0.023, 0.056

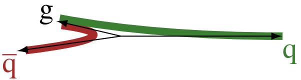
Jets

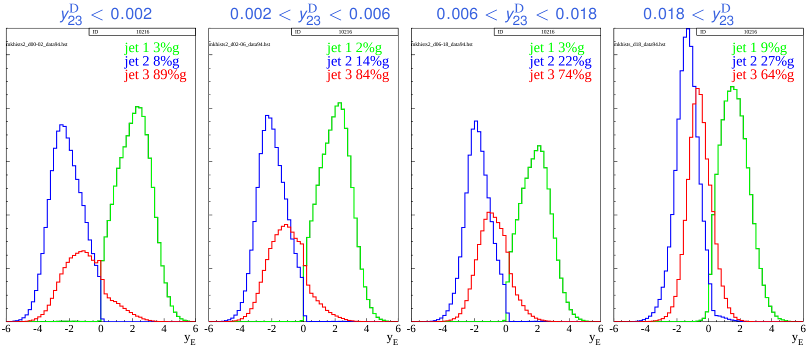
order jets by energy: $E_1 > E_2 > E_3$
Coordinate system: $\mathrm{Z} \longrightarrow \mathrm{q}\bar{\mathrm{q}}(\mathrm{g})$

- estimate $\mathrm{q}\bar{\mathrm{q}}$ axis by thrust axis, *i*.e., axis $\vec{n}_\mathrm{T}$ for which $T = \frac{\sum |\vec{p}_i \cdot \vec{n}_\mathrm{T}|}{\sum |\vec{p}_i|}$ is maximal
- 3-jet events are planar.
  Estimate event plane by thrust, major axes.
  Major is analogous to thrust, but in plane perpendicular to $\vec{n}_\mathrm{T}$.
- Note: thrust only defines axis $|\vec{n}_\mathrm{T}|$, not its direction.
  Choose positive thrust direction such that jet 1 is in positive thrust hemisphere
- Similarly, choose positive major direction such that jet 3 is in positive major hemisphere

Jets

rapidity, y_E, of particles from
jet 1, jet 2, jet 3:



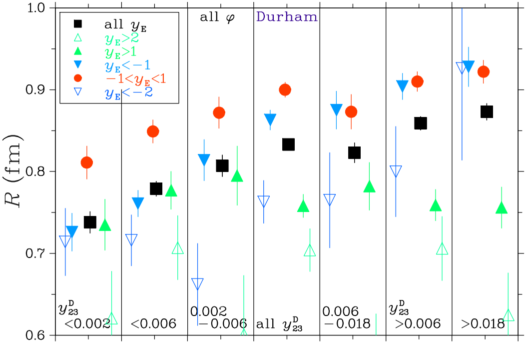


- $y_E > 1$ almost all jet 1 — almost all quark
- $y_E < -1$ mostly jet 2, some jet 3 — mostly quark
- $-1 < y_E < 1$ jet-3 enriched — largely gluon

Fits of simplified τ-model – L3 preliminary

To stabilize fits against large correlation of α, R, fix $\alpha = 0.44$

Select particle pairs by rapidity of pair



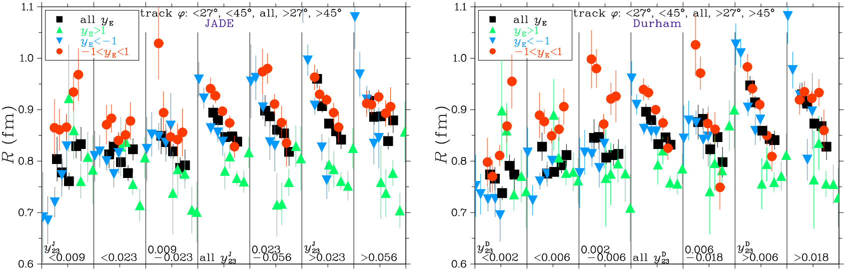
With y_{23}^D,

- all y: R increases
- 'pure' q jet, $y_E > 1$, or $y_E < -1$ & y_{23}^D small, or $y_E < -2$: R const.
- $R_{-1<y_E<1} > R_{\text{'pure' } q}$
- $R_{y_E<-1}$ increases
- at large y_{23}^D $R_{-1<y_E<1} = R_{y_E<-1}$

Conclusion (JADE agrees):
Increase in R with y_{23}^D is due to appearance of gluon jet

Fits of simplified τ-model – L3 preliminary

ϕ track-event plane: $< 27°$, $< 45°$, all, $> 27°$, $> 45°$



2-jet, all y_{E} selections and 3-jet, $y_{\mathrm{E}} > 1$:

no significant differences between $R_{\text{in plane}}$, $R_{\text{out of plane}}$

3-jet, $-1 < y_{\mathrm{E}} < 1$ or $y_{\mathrm{E}} < -1$: $R_{\text{in plane}} > R_{\text{out of plane}}$

LCMS and the Simplified τ-model

Consider 2 frames:

1. LCMS: $$Q^2 = Q_L^2 + Q_{side}^2 + Q_{out}^2 - (\Delta E)^2 = Q_L^2 + Q_{side}^2 + Q_{out}^2\left(1-\beta^2\right), \quad \beta = \frac{p_{1out}+p_{2out}}{E_1+E_2}$$
2. LCMS-rest: $$Q^2 = Q_L^2 + Q_{side}^2 + q_{out}^2, \qquad q_{out}^2 = Q_{out}^2\left(1-\beta^2\right)$$

q_{out} is Q_{out} boosted (β) along out direction to rest frame of pair

In simplified τ-model, replace R^2Q^2 by

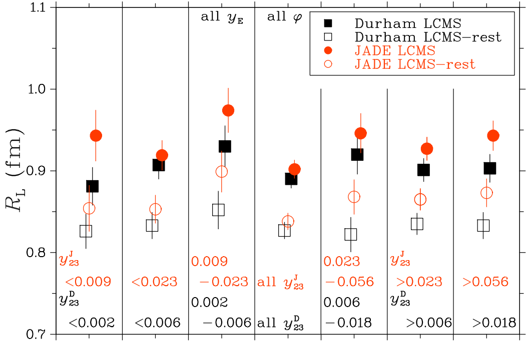
1. $A^2 = R_L^2 Q_L^2 + R_{side}^2 Q_{side}^2 + \rho_{out}^2 Q_{out}^2$
2. $B^2 = R_L^2 Q_L^2 + R_{side}^2 Q_{side}^2 + r_{out}^2 q_{out}^2$

Then in τ-model, for case 1:

$$R_2(Q_L, Q_{side}, Q_{out}) = \gamma\left[1 + \lambda\cos\left(\tan\left(\frac{\alpha\pi}{2}\right)A^{2\alpha}\right)\exp\left(-A^{2\alpha}\right)\right] \cdot \left(1 + \epsilon_L Q_L + \epsilon_{side} Q_{side} + \epsilon_{out} Q_{out}\right)$$

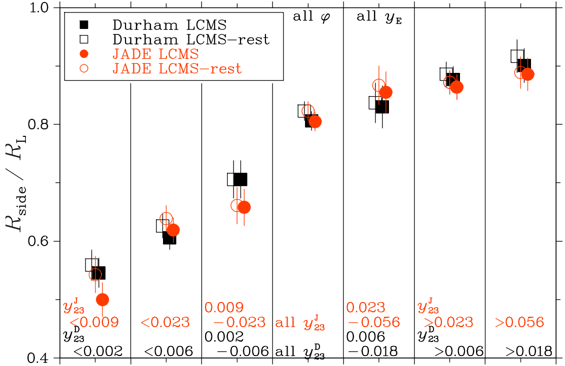
and comparable expression for case 2, $R_2(Q_L, Q_{side}, q_{out})$

3-d Fits R_L – L3 preliminary



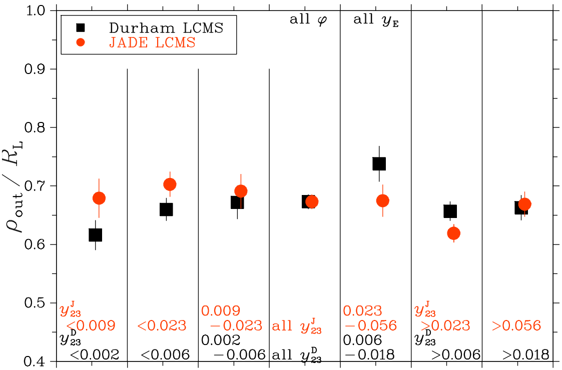
- Durham, JADE agree
- systematic difference LCMS, LCMS-rest
- R_L constant with y_{23}

3-d Fits $R_{\rm side}$ – L3 preliminary



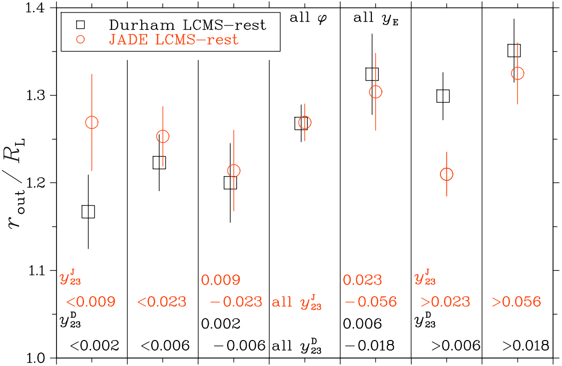
- LCMS, LCMS-rest agree
- Durham, JADE agree
- $R_{\rm side}$ increases with y_{23}, approx. 0.5–0.9 R_L

3-d Fits ρ_{out} – L3 preliminary



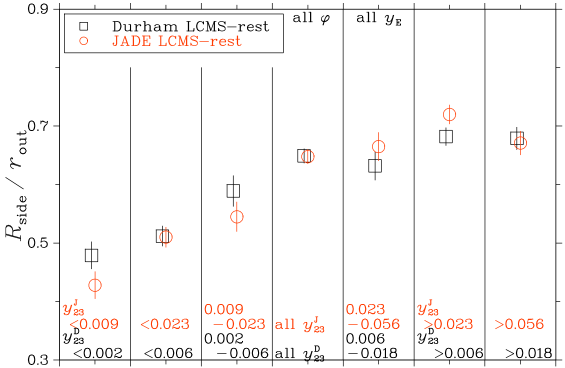
- Durham, JADE agree
- ρ_{out} constant with y_{23}

3-d Fits $r_{\rm out}$ – L3 preliminary



- Durham, JADE roughly agree
- $y_{23}^{\rm J}$: $r_{\rm out}$ approx. constant with y_{23}, approx. 1.27 $R_{\rm L}$
  or $y_{23}^{\rm D}$: slightly increasing with y_{23}, approx. 1.15–1.35 $R_{\rm L}$

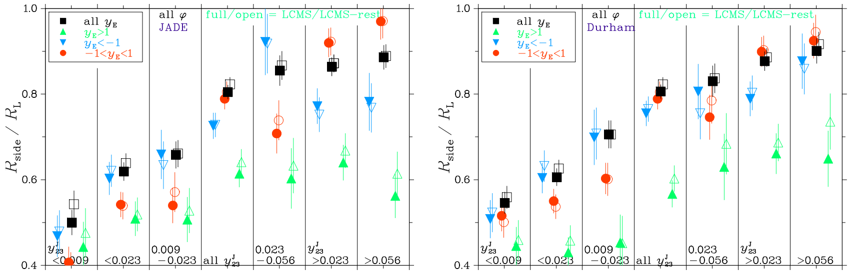
3-d Fits $R_{\rm side}/r_{\rm out}$ – L3 preliminary



- Durham, JADE agree
- $R_{\rm side} < r_{\rm out}$ for all y_{23}
  $R_{\rm side}/r_{\rm out}$ smallest for 2-jet
  Not azimuthally symmetric; not even for narrow 2-jet !!!

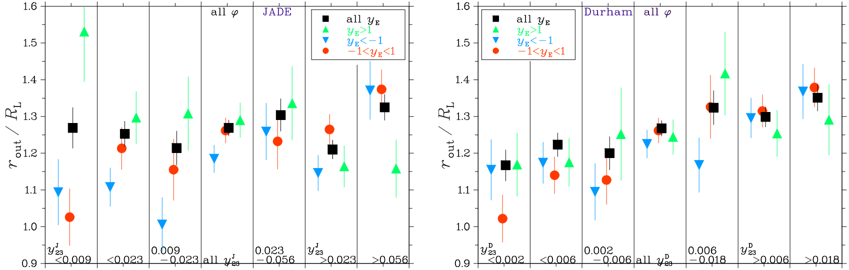
3-d Fits, y_E dependence $R_{\rm side}$ – L3 preliminary

In each y_E interval, R_L, $\rho_{\rm out}$ $\approx$ constant with y_{23} (not shown)



$R_{\rm side}$ increases, less for $y_E > 1$ than for other y_E regions

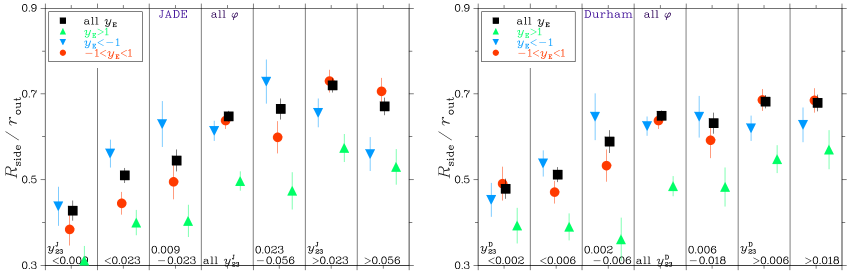
3-d Fits, $y_{\rm E}$ dependence $r_{\rm out}$ – L3 preliminary



$r_{\rm out}$ independent of $y_{\rm E}$ for $y_{\rm E} > 1$

$r_{\rm out}$ perhaps increases slightly for $y_{\rm E} < -1$ and $-1 < y_{\rm E} < 1$

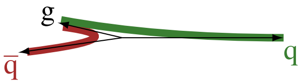
3-d Fits, y_{E} dependence $R_{\mathrm{side}}/r_{\mathrm{out}}$ – L3 preliminary



- $R_{\mathrm{side}} < r_{\mathrm{out}} < 1$ for all y_{23}
- $R_{\mathrm{side}}/r_{\mathrm{out}}$ smaller for $y_{\mathrm{E}} > 1$ – 'pure q jet'
- $R_{\mathrm{side}}/r_{\mathrm{out}}$ smallest for 2-jet
- Not azimuthally symmetric;
  not even for narrow 2-jet !!!

Choice of Longitudinal direction – L3 preliminary

2-jet event: Long = thrust axis
3-jet event: Long = thrust axis,
or $\vec{p}_{\text{jet }1}$, $\vec{p}_{\text{jet }2}$, $\vec{p}_{\text{jet }2} - 0.5\vec{p}_{\text{jet }3}$, $\vec{p}_{\text{jet }3}$



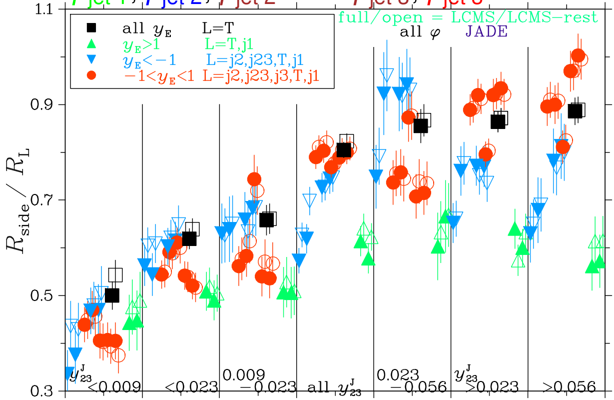
No clear dependence of R_{L} or r_{out} on L (not shown)

Choice of Longitudinal direction R_{side} – L3 preliminary

2-jet event: Long = thrust axis

3-jet event: Long = thrust axis,

or $\vec{p}_{\mathrm{jet}\,1}$, $\vec{p}_{\mathrm{jet}\,2}$, $\vec{p}_{\mathrm{jet}\,2} - 0.5\vec{p}_{\mathrm{jet}\,3}$, $\vec{p}_{\mathrm{jet}\,3}$



Durham is similar

JADE:

No clear dependence of R_{side} on L

except perhaps

3-jet, $y_E < -1$:

R_{side} smaller for L=j2

$R_{\mathrm{side}}(y_E<-1)$

$\approx R_{\mathrm{side}}(y_E>1)$

3-jet, $-1 < y_E < 1$:

R_{side} smaller for L=j3

3-jet:

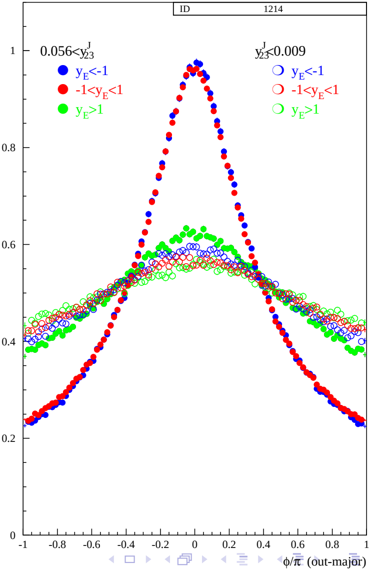
$R_{\mathrm{side}}(-1<y_E<1)$

$> R_{\mathrm{side}}(y_E<-1)$

$\approx R_{\mathrm{side}}(y_E>1)$

out-event plane

- 2-jet: for all y_E, small preference for out direction to be in event plane
- 3-jet: for $y_E > 1$, like 2-jet
- 3-jet: for $y_E < -1$ and $-1 < y_E < 1$, large preference for out direction to be in event plane



Summary

- 1-d
  - R increases with y_{23} ~ 0.7–$0.9\,\mathrm{fm}$
  - but not in 'pure quark' regions,
    ($y_E > 1$ or $y_E < -2$, all y_{23}) and ($y_E < -1$ for narrow 2-jet)
  - $R(-1 < y_E < 1) > R(\text{all } y_E) > y_E > 1)$
  - for 3-jet events R is larger in the event plane
- 3-d
  - R_L, $\rho_{out} \sim$ constant with y_{23}
    $R_L \approx 0.9\,\mathrm{fm}$ (LCMS) $\approx 0.85\,\mathrm{fm}$ (LCMS-rest)
    $\rho_{out}/R_L \approx 0.65$
  - R_{side} increases with y_{23} $R_{side}/R_L \approx 0.5$–0.9
    increase is less for $y_E > 1$
  - r_{out} perhaps increases slightly $r_{out}/R_L \approx 1.2$–1.3
  - $R_{side}/r_{out} < 1$ for all y_{23}
    R_{side}/r_{out} smaller for $y_E > 1$ – 'pure q jet'
    R_{side}/r_{out} smallest for 2-jet
    Not azimuthally symmetric;
    least symmetric for narrow 2-jet events !!!

Summary

- 3-d dependence on Longitudinal axis
  - 3-jet: $R_{\rm side}$ is perhaps smaller using
    L=j2 for $y_{\rm E} < -1$ and L=j3 for $-1 < y_{\rm E} < 1$
    Then $R_{\rm side}(-1 < y_{\rm E} < 1) > R_{\rm side}(y_{\rm E} < -1) \approx R_{\rm side}(y_{\rm E} > 1)$
- 3-d: out direction is preferentially in the event plane
  slight preference for 2-jet and for 3-jet, $y_{\rm E} > 1$
  strong preference for $-1 < y_{\rm E} < 1$ and $R_{\rm side}(y_{\rm E} < -1)$

Qualitative Conclusions

- R larger in event plane for 3-jet events agrees with $r_{\mathrm{out}} > R_{\mathrm{side}}$ and preference of out to lie in event plane.
- For 3-jet,
  R_{L} and $r_{\mathrm{out}} \approx 1.25 R_{\mathrm{L}}$ are insensitive to choice of L.
  R_{side} does vary with L, increasing to 0.8–0.9R_{L} for 3-jet
  This may explain why τ-model works for 3-jet.
- Behavior of R and R_{side} in different y_{E} regions suggests $R_{\mathrm{gluon}} > R_{\mathrm{quark}}$.
  R and R_{side} are larger in gluon regions;
  they increase as gluon energy (and hence number of particles from gluon) increases.

Qualitative Conclusions/Speculations

- Picture of 'region of homogeneity' seems to be:
  - squashed ellipsoid
    r_{out} slightly larger than R_{L}
    R_{side} considerably smaller
  - in 'pure' quark jets (2-jet or 3-jet with $y_{\mathrm{E}} > 1$)
    ellipsoid oriented approx. isotropically about thrust axis
  - in other cases (3-jet with $y_{\mathrm{E}} < 1$ – gluon contribution)
    r_{out} tends to be in event plane
- But why is $R_{\mathrm{side}} \neq r_{\mathrm{out}}$, *i*.e., No azimuthal symmetry; not even for narrow 2-jet events?
  local p_{t} compensation defining a plane?

> There is something fascinating about science.
> One gets such wholesale returns of conjecture
> out of such a trifling investment of fact.
>
> – Mark Twain

Speculation

- CMS has observed the anti-correlation region as predicted in the τ-model and observed by L3.
  This suggests strings – like in $\mathrm{e}^+\mathrm{e}^-$.
- In pp, can the onset of hard jet production be seen in the BEC radii?
  like the third jet $\mathrm{e}^+\mathrm{e}^-$.
- Therefore, I suggest studying BEC as a function of p_t of highest p_t particle.