

Bose-Einstein Results from L3 and the Tau Model

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BEC Introduction

$$R_2 = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} = \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda |\tilde{S}(Q)|^2$$

where $\tilde{S}(Q) = \int dx e^{iQx} S(x)$ — Fourier transform of $S(x)$

$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius $r \Rightarrow$

$$R_2(Q) = 1 + \lambda e^{-Q^2 r^2}$$

► intro

The L3 Data

- $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign **pairs** of well measured charged tracks from about $0.8 \cdot 10^6$ **events**
- about $0.5 \cdot 10^6$ **2-jet** events — Durham $y_{\text{cut}} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, “**3-jet** events”
- **use mixed events for reference sample, ρ_0**



Previous Results: Elongation

Results in **LCMS frame**: Longitudinal = Thrust axis

► LCMS

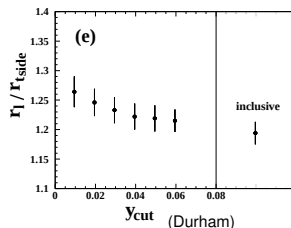
$$R_L / R_{\text{side}}$$

L3	$1.25 \pm 0.03^{+0.36}_{-0.05}$
OPAL	$1.19 \pm 0.03^{+0.08}_{-0.01}$

(ZEUS finds similar results in ep)
 $\sim 25\%$ elongation along thrust axis

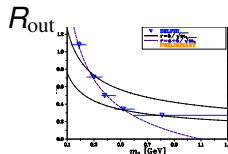
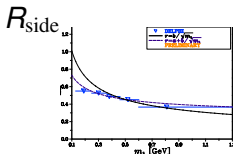
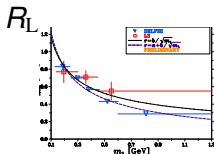
OPAL:

Elongation larger for narrower jets



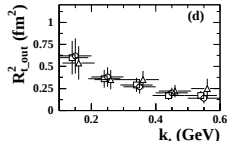
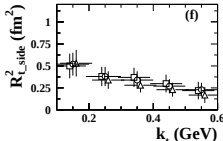
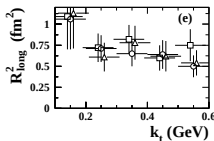
- Conclusion: Elongation, but it is relatively small.
- So: Ignore it. — Assume spherical.

Transverse Mass dependence of r



Smirnova&Lörstad, 7th Int. Workshop on Correlations and Fluctuations (1996)

Van Dalen, 8th Int. Workshop on Correlations and Fluctuations (1998)



OPAL, Eur. Phys. J **C52** (2007) 787

r decreases with m_t (or k_t) for all directions

Results on Q from L_3 Z decay

$$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$$

- Gaussian

$$G = \exp(-(rQ)^2)$$

- Edgeworth expansion

$$G = \exp(-(rQ)^2) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$$

Gaussian if $\kappa = 0$

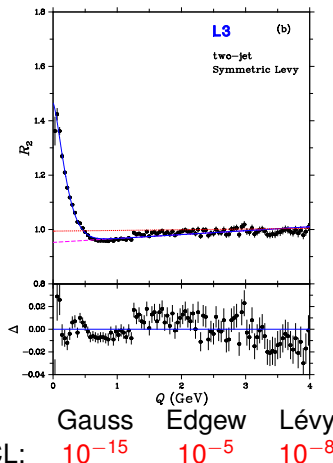
$$\kappa = 0.71 \pm 0.06$$

- symmetric Lévy

$$G = \exp(-|rQ|^\alpha)$$

$$0 < \alpha \leq 2$$

$$\alpha = 1.34 \pm 0.04$$



Poor χ^2 . Edgeworth and Lévy better than Gaussian, but poor.
 Problem is the dip of R_2 in the region $0.6 < Q < 1.5$ GeV



Summary

- BEC depend (approximately) only on Q , not its components.
- BEC depend on m_t .
- Gaussian and similar parametrizations do not fit.

Turn now to a model providing such dependence.

The τ -model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214

T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:

$$\bar{x}^\mu(p^\mu) = a\tau p^\mu$$

where for 2-jet events, $a = 1/m_t$

$\tau = \sqrt{t^2 - \vec{r}_z^2}$ is the “longitudinal” proper time

and $m_t = \sqrt{E^2 - p_z^2}$ is the “transverse” mass

- Let $\delta_\Delta(x^\mu - \bar{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ . Then the emission function is

$$S(x, p) = \int_0^\infty d\tau H(\tau) \delta_\Delta(x - a\tau p) \rho_1(p)$$

- In the plane-wave approx.

F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.

$$\rho_2(p_1, p_2) = \int d^4x_1 d^4x_2 S(x_1, p_1) S(x_2, p_2) (1 + \cos([p_1 - p_2][x_1 - x_2]))$$

- Assume $\delta_\Delta(x - a\tau p)$ is very narrow — a δ -function. Then

$$R_2(p_1, p_2) = 1 + \lambda \operatorname{Re} \tilde{H}\left(\frac{a_1 Q^2}{2}\right) \tilde{H}\left(\frac{a_2 Q^2}{2}\right), \quad \tilde{H}(\omega) = \int d\tau H(\tau) \exp(i\omega\tau)$$

BEC in the τ -model

- Assume a Lévy distribution for $H(\tau)$

Since no particle production before the interaction,

$H(\tau)$ is one-sided.

Characteristic function is

$\tilde{H}(\omega) = \exp \left[-\frac{1}{2} (\Delta\tau |\omega|)^\alpha \left(1 - i \operatorname{sign}(\omega) \tan \left(\frac{\alpha\pi}{2} \right) \right) + i\omega\tau_0 \right]$, $\alpha \neq 1$
where

- α is the index of stability
 - τ_0 is the proper time of the onset of particle production
 - $\Delta\tau$ is a measure of the width of the dist.
- Then, R_2 depends on Q, a_1, a_2

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos \left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan \left(\frac{\alpha\pi}{2} \right) \left(\frac{\Delta\tau Q^2}{2} \right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right. \\ \left. \cdot \exp \left[- \left(\frac{\Delta\tau Q^2}{2} \right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

BEC in the τ -model

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos \left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan \left(\frac{\alpha \pi}{2} \right) \left(\frac{\Delta \tau Q^2}{2} \right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right. \\ \left. \cdot \exp \left[- \left(\frac{\Delta \tau Q^2}{2} \right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

Simplification:

- Particle production begins immediately, $\tau_0 = 0$
- effective radius, R , defined by $R^{2\alpha} = \left(\frac{\Delta \tau}{2} \right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2}$

- Then

$$R_2(Q) = \gamma \left[1 + \lambda \cos \left((R_a Q)^{2\alpha} \right) \exp \left(- (R Q)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$

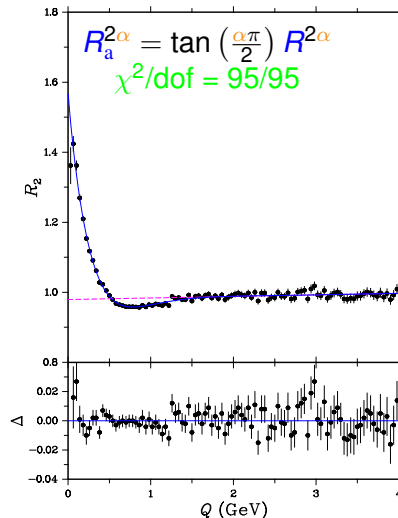
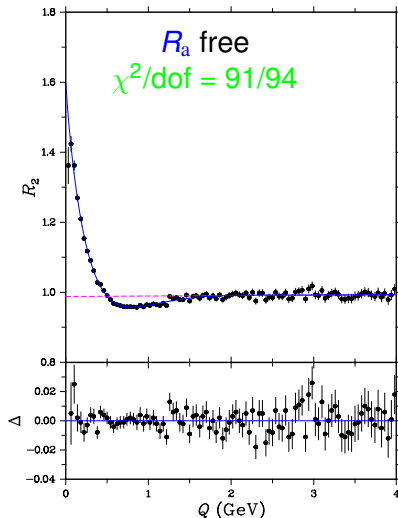
where $R_a^{2\alpha} = \tan \left(\frac{\alpha \pi}{2} \right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \exp \left[- |r Q|^\alpha \right] \right] (1 + \epsilon Q)$$

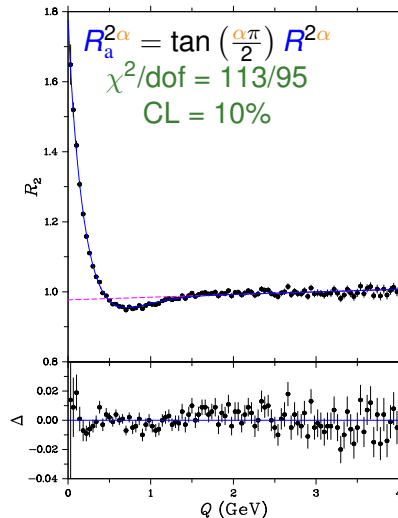
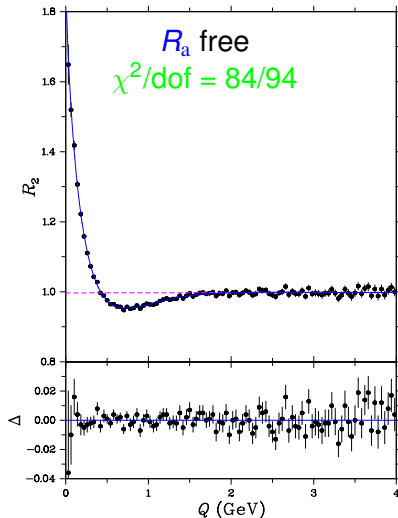


2-jet Results on Simplified τ -model from L3 $\mathbb{Z}$ decay





3-jet Results on Simplified τ -model from L3 Z decay



Summary of Simplified τ -model

	α	R (fm)	R_a (fm)	CL
2-jet	$0.41 \pm 0.02^{+0.04}_{-0.06}$	$0.79 \pm 0.04^{+0.09}_{-0.19}$	$0.69 \pm 0.04^{+0.21}_{-0.09}$	57%
3-jet	$0.35 \pm 0.01^{+0.03}_{-0.04}$	$1.06 \pm 0.05^{+0.59}_{-0.31}$	$0.85 \pm 0.04^{+0.15}_{-0.05}$	76%
3-jet	$0.41 \pm \text{fixed}$	0.93 ± 0.03	0.76 ± 0.01	38%
2-jet	$0.44 \pm 0.01^{+0.05}_{-0.02}$	$0.78 \pm 0.04^{+0.09}_{-0.16}$	—	49%
3-jet	$0.42 \pm 0.01^{+0.02}_{-0.04}$	$0.98 \pm 0.04^{+0.55}_{-0.14}$	—	10%
3-jet	$0.44 \pm \text{fixed}$	0.87 ± 0.01	—	3%

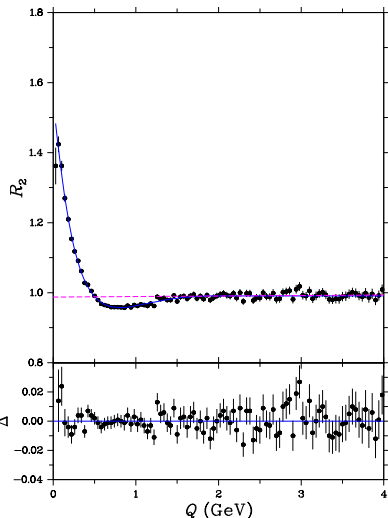
- consistent values of α
- $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ to 0.5σ for 2-jet and to 1.5σ for 3-jet
- Simplified τ -model works well
- R seems to be larger for 3-jet than for 2-jet events

► fitR

► fitRR

Full τ -model for 2-jet events — $a = 1/m_t$

$$R_2(Q, m_{t1}, m_{t2}) = \gamma \left\{ 1 + \lambda \cos \left[\frac{\tau_0 Q^2 (m_{t1} + m_{t2})}{2(m_{t1} m_{t2})} + \tan \left(\frac{\alpha \pi}{2} \right) \left(\frac{\Delta \tau Q^2}{2} \right)^\alpha \frac{m_{t1}^\alpha + m_{t2}^\alpha}{2(m_{t1} m_{t2})^\alpha} \right] \right. \\ \left. \cdot \exp \left[- \left(\frac{\Delta \tau Q^2}{2} \right)^\alpha \frac{m_{t1}^\alpha + m_{t2}^\alpha}{2(m_{t1} m_{t2})^\alpha} \right] \right\} \cdot (1 + \epsilon Q)$$



- Fit $R_2(Q)$ using
avg m_{t1}, m_{t2} in each Q
bin, $m_{t1} > m_{t2}$
- $\tau_0 = 0.00 \pm 0.02$
so fix to 0
- $\chi^2/\text{dof} = 90/95$

► fit

Summary of τ -model

- τ -model with a one-sided Lévy proper-time distribution describes BEC well
 - in simplified form it provides a new parametrization of $R_2(Q)$ for both 2- and 3-jet events,
 - in full form for 2-jet events, $R_2(Q, m_{t1}, m_{t2})$
 - both Q - and m_t -dependence described correctly
 - Note: we found $\Delta\tau$ to be independent of m_t
 $\Delta\tau$ enters R_2 as $\Delta\tau Q^2 / m_t$
 In Gaussian parametrization, r enters R_2 as $r^2 Q^2$
 Thus $\Delta\tau$ independent of m_t corresponds to $r \propto 1/\sqrt{m_t}$
- BUT, what about elongation?



Elongation?

- Previous elongation results used fits of Gaussian or Edgeworth
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ -model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function?
or is the τ -model in need of modification?
- So, we modify *ad hoc* the τ -model description to allow elongation and see what we get

Elongation in the Simplified τ -model?

$$\begin{aligned}\text{LCMS: } Q^2 &= Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 - (\Delta E)^2 \\ &= Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 (1 - \beta^2), \quad \beta = \frac{p_{1\text{out}} + p_{2\text{out}}}{E_1 + E_2}\end{aligned}$$

Replace $R^2 Q^2 \Rightarrow A^2 = R_L^2 Q_L^2 + R_{\text{side}}^2 Q_{\text{side}}^2 + R_{\text{out}}^2 Q_{\text{out}}^2$

Then in τ -model,

$$R_2(Q_L, Q_{\text{side}}, Q_{\text{out}}) = \gamma \left[1 + \lambda \cos \left(\tan \left(\frac{\alpha\pi}{2} \right) A^{2\alpha} \right) \exp \left(-A^{2\alpha} \right) \right] \cdot (1 + \epsilon_L Q_L + \epsilon_{\text{side}} Q_{\text{side}} + \epsilon_{\text{out}} Q_{\text{out}})$$

for 2-jet events:

	R_{side}/R_L (fm)	χ^2/dof	CL
τ -model	0.61 ± 0.02	14847/14921	66%
Edgeworth	0.64 ± 0.02	14891/14919	56%
	consistent		

► fit

Elongation is real

Direct Test of Q^2 -only Dependence

1. $Q^2 = Q_{LE}^2 + Q_{side}^2 + Q_{out}^2$ where $Q_{LE}^2 = Q_L^2 - (\Delta E)^2$
inv. boosts along thrust axis
2. $Q^2 = Q_L^2 + Q_{side}^2 + q_{out}^2$ where $q_{out} = Q_{out}$ boosted (β) along
out direction to rest frame of pair

In τ -model, for case 1

$$R_2(Q_{LE}, Q_{side}, Q_{out}) = \gamma \left[1 + \lambda \cos \left(\tan \left(\frac{\alpha\pi}{2} \right) B^{2\alpha} \right) \exp \left(-B^{2\alpha} \right) \right] b$$

$$\text{where } B^2 = R_{LE}^2 Q_{LE}^2 + R_{side}^2 Q_{side}^2 + R_{out}^2 Q_{out}^2$$

$$b = 1 + \epsilon_{LE} Q_{LE} + \epsilon_{side} Q_{side} + \epsilon_{out} Q_{out}$$

and comparable expression for case 2, $R_2(Q_L, Q_{side}, q_{out})$

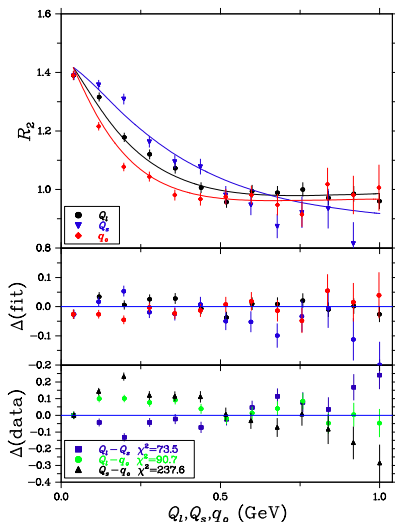
Direct Test of Q^2 -only Dependence

Compare fits with all 'radii' **free**
to fits with all 'radii' constrained to be **equal**

case 1	α	0.46 ± 0.01	0.46 ± 0.01	► fits1 difference $\Delta\chi^2 = 296/2$ ≈ 0
	R_{LE} (fm)	0.84 ± 0.04	0.71 ± 0.04	
	R_{side}/R_{LE}	0.60 ± 0.02	1	
	R_{out}/R_{LE}	0.986 ± 0.003	1	
	χ^2/DoF	14590/14538	14886/14540	
	CL	38%	2%	
case 2	α	0.41 ± 0.01	0.44 ± 0.01	► fits2 difference $\Delta\chi^2 = 464/2$ ≈ 0
	R_L (fm)	0.96 ± 0.05	0.82 ± 0.04	
	R_{side}/R_L	0.62 ± 0.02	1	
	r_{out}/R_L	1.23 ± 0.03	1	
	χ^2/DoF	10966/10647	11430/10649	
	CL	2%	10^{-7}	

Dependence on components of Q is strongly preferred.

Q Dependence



case 2, $R_2(Q_L, Q_{\text{side}}, q_{\text{out}})$ vs.

Q_L for $Q_{\text{side}}, q_{\text{out}} < 0.08 \text{ GeV}$

Q_{side} for $Q_L, q_{\text{out}} < 0.08 \text{ GeV}$

q_{out} for $Q_L, Q_{\text{side}} < 0.08 \text{ GeV}$

Dependence on components of Q is preferred.

Summary

- R_2 depends, to some degree, separately on components of Q , i.e., on $\vec{Q}$
- contradicts τ -model, where dependence is on Q , not on $\vec{Q}$
- Nevertheless, τ -model with a one-sided Lévy proper-time distribution succeeds:
 - Simplified, provides a new parametrization of $R_2(Q)$ which works well
 - $R_2(Q, m_{t1}, m_{t2})$ successfully fits R_2 for 2-jet events both Q - and m_t -dependence described correctly
- But dependence of R_2 on components of Q implies τ -model is in need of modification

Perhaps, a should be different for transverse/longitudinal

$$\bar{x}^\mu(p^\mu) = a\tau p^\mu, \quad a = 1/m_t \text{ for 2-jet}$$

Emission Function of 2-jet Events.

In the τ -model, the emission function in configuration space is

$$S(\vec{x}, \tau) = \frac{1}{\bar{n}} \frac{d^4 n}{d\tau d\vec{x}} = \frac{1}{\bar{n}} \left(\frac{m_t}{\tau} \right)^3 H(\tau) \rho_1 \left(\vec{p} = \frac{m_t \vec{x}}{\tau} \right)$$

For simplicity, assume $\rho_1(\vec{p}) = \rho_y(y) \rho_{p_t}(p_t) / \bar{n}$

(ρ_1 , ρ_y , ρ_{p_t} are inclusive single-particle distributions)

Then $S(\vec{x}, \tau) = \frac{1}{\bar{n}^2} H(\tau) G(\eta) I(r)$

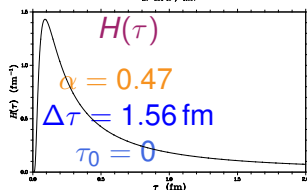
Strongly correlated $x, p \Rightarrow$

$$\eta = y \quad r = p_t \tau / m_t$$

$$G(\eta) = \rho_y(\eta) \quad I(r) = \left(\frac{m_t}{\tau} \right)^3 \rho_{p_t}(r m_t / \tau)$$

So, using experimental $\rho_y(y)$, $\rho_{p_t}(p_t)$ dists.
and $H(\tau)$ from BEC fits,
we can reconstruct S .

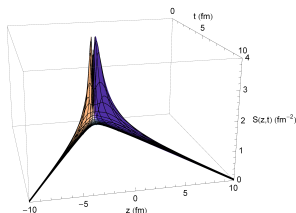
expt. –
Factorization OK



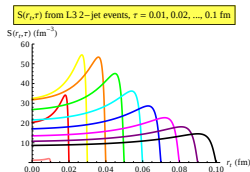


Emission Function of 2-jet Events.

Integrating over r ,



Integrating over z ,



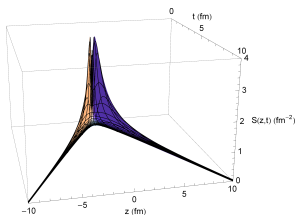
“Boomerang” shape

Particle production is close to the light-cone

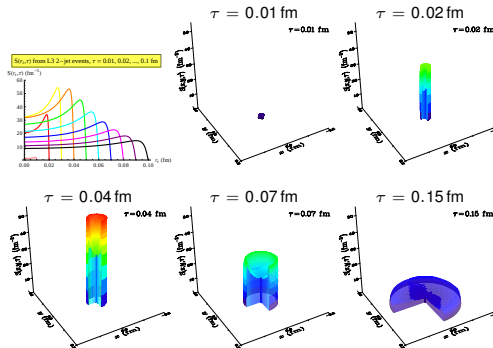
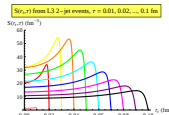
Emission Function of 2-jet Events.

Integrating over z ,

Integrating over r ,



“Boomerang” shape



Expanding ring

Particle production is close to the light-cone

$$\alpha_s$$

- LLA parton shower leads to a fractal in momentum space
fractal dimension is related to α_s Gustafson et al.
- Lévy dist. arises naturally from a fractal, or random walk,
or anomalous diffusion Metzler and Klafter, Phys.Rep.339(2000)1.
- strong momentum-space/configuration space correlation of
 τ -model $\implies$ fractal in configuration space with same α
- generalized LPHD suggests particle dist. has same
properties as gluon dist.
- Putting this all together leads to Csörgő et al.

$$\alpha_s = \frac{2\pi}{3} \alpha^2$$

- Using our value of $\alpha = 0.47 \pm 0.04$ yields $\alpha_s = 0.46 \pm 0.04$
- This value is reasonable for a **scale** of **1–2 GeV**,
where production of hadrons takes place
cf., from τ decays $\alpha_s(m_\tau \approx 1.8 \text{ GeV}) = 0.34 \pm 0.03$ PDG

BEC Introduction

q -particle density $\rho_q(p_1, \dots, p_q) = \frac{1}{\sigma_{\text{tot}}} \frac{d^q \sigma_q(p_1, \dots, p_q)}{dp_1 \dots dp_q}$

2-particle correlation: $\frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)}$

To study only BEC, not all correlations,
let $\rho_0(p_1, p_2)$ be the 2-particle density if no BEC
(= ρ_2 of the 'reference sample') and define

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} \cdot \frac{\rho_1(p_1)\rho_1(p_2)}{\rho_0(p_1, p_2)} = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)}$$

Since $2\text{-}\pi$ BEC only at small Q

$$Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$$

integrate over other variables:

$$R_2(Q) = \frac{\rho(Q)}{\rho_0(Q)}$$

LCMS

Advantages of LCMS:

$$\begin{aligned} Q^2 &= Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 - (\Delta E)^2 \\ &= Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 (1 - \beta^2) \quad \text{where } \beta \equiv \frac{p_{\text{out } 1} + p_{\text{out } 2}}{E_1 + E_2} \end{aligned}$$

Thus, the **energy difference**,
and therefore the **difference in emission time** of the pions
couples only to the **out-component**, Q_{out} .

Thus,

Q_L and Q_{side} reflect only **spatial** dimensions of the source
 Q_{out} reflects a mixture of **spatial and temporal** dimensions.



Fit Results Simplified τ -model

parameter	two-jet	three-jet
λ	$0.63 \pm 0.03^{+0.08}_{-0.35}$	$0.92 \pm 0.05^{+0.06}_{-0.48}$
α	$0.41 \pm 0.02^{+0.04}_{-0.06}$	$0.35 \pm 0.01^{+0.03}_{-0.04}$
R (fm)	$0.79 \pm 0.04^{+0.09}_{-0.19}$	$1.06 \pm 0.05^{+0.59}_{-0.31}$
R_a (fm)	$0.69 \pm 0.04^{+0.21}_{-0.09}$	$0.85 \pm 0.04^{+0.15}_{-0.05}$
ϵ (GeV $^{-1}$)	$0.001 \pm 0.002^{+0.005}_{-0.008}$	$0.000 \pm 0.002^{+0.001}_{-0.007}$
γ	$0.988 \pm 0.005^{+0.026}_{-0.012}$	$0.997 \pm 0.005^{+0.019}_{-0.002}$
χ^2/DoF	91/94	84/94
confidence level	57%	76%



Fit Results Simplified τ -model

parameter	two-jet	three-jet
λ	$0.61 \pm 0.03^{+0.08}_{-0.26}$	$0.84 \pm 0.04^{+0.04}_{-0.37}$
α	$0.44 \pm 0.01^{+0.05}_{-0.02}$	$0.42 \pm 0.01^{+0.02}_{-0.04}$
R (fm)	$0.78 \pm 0.04^{+0.09}_{-0.16}$	$0.98 \pm 0.04^{+0.55}_{-0.14}$
ϵ (GeV $^{-1}$)	$0.005 \pm 0.001 \pm 0.003$	$0.008 \pm 0.001 \pm 0.005$
γ	$0.979 \pm 0.002^{+0.009}_{-0.003}$	$0.977 \pm 0.001^{+0.013}_{-0.008}$
χ^2/DoF	95/95	113/95
confidence level	49%	10%



Fit Results Full τ -model for 2-jet events

m_t regions (GeV)		average m_t (GeV)		confidence level (%)	λ
m_{t1}	m_{t2}	$Q < 0.4$	all		
0.14 – 0.26	0.14 – 0.22	0.19	0.19	10	0.39 ± 0.02
0.14 – 0.34	0.22 – 0.30	0.27	0.27	48	0.76 ± 0.03
0.14 – 0.46	0.30 – 0.42	0.37	0.37	74	0.83 ± 0.03
0.14 – 0.66	0.42 – 4.14	0.52	0.52	13	0.97 ± 0.04
0.26 – 0.42	0.14 – 0.22	0.25	0.26	22	0.53 ± 0.02
0.34 – 0.46	0.22 – 0.30	0.32	0.33	33	0.80 ± 0.03
0.46 – 0.58	0.30 – 0.42	0.43	0.44	34	0.91 ± 0.04
0.66 – 0.86	0.42 – 4.14	0.65	0.65	66	1.01 ± 0.05
0.42 – 0.62	0.14 – 0.22	0.34	0.34	17	0.41 ± 0.03
0.46 – 0.70	0.22 – 0.30	0.41	0.41	55	0.64 ± 0.03
0.58 – 0.82	0.30 – 0.42	0.52	0.52	59	0.70 ± 0.04
0.86 – 1.22	0.42 – 4.14	0.80	0.81	24	0.66 ± 0.05
0.70 – 4.14	0.22 – 0.30	0.59	0.65	4	0.37 ± 0.04
0.82 – 4.14	0.30 – 0.42	0.71	0.76	11	0.56 ± 0.05

Fit Result $R_2(Q, m_{t1}, m_{t2})$

parameter	
λ	$0.58 \pm 0.03^{+0.08}_{-0.24}$
α	$0.47 \pm 0.01^{+0.04}_{-0.02}$
$\Delta\tau$ (fm)	$1.56 \pm 0.12^{+0.32}_{-0.45}$
ϵ (GeV $^{-1}$)	$0.001 \pm 0.001 \pm 0.003$
γ	$0.988 \pm 0.002^{+0.006}_{-0.002}$
χ^2/DoF	90/95
confidence level	62%



Fit Results elongation in τ -model for 2-jet events

λ	0.49 ± 0.02
α	0.46 ± 0.01
R_L (fm)	0.85 ± 0.04
R_{side}/R_L	0.61 ± 0.02
R_{out}/R_L	0.66 ± 0.02
ϵ_L (GeV $^{-1}$)	0.001 ± 0.001
ϵ_{side} (GeV $^{-1}$)	-0.076 ± 0.003
ϵ_{out} (GeV $^{-1}$)	-0.029 ± 0.002
γ	1.011 ± 0.002
χ^2/DoF	14847/14921
CL	66%



Fit Results of direct tests for 2-jet events

case 1	λ	0.51 ± 0.03	0.49 ± 0.03
	α	0.46 ± 0.01	0.46 ± 0.01
	R_{LE} (fm)	0.84 ± 0.04	0.71 ± 0.04
	$R_{\text{side}}/R_{\text{LE}}$	0.60 ± 0.02	1
	$R_{\text{out}}/R_{\text{LE}}$	0.986 ± 0.003	1
	ϵ_{LE} (GeV^{-1})	0.001 ± 0.001	0.000 ± 0.001
	ϵ_{side} (GeV^{-1})	-0.069 ± 0.003	-0.064 ± 0.003
	ϵ_{out} (GeV^{-1})	-0.032 ± 0.002	-0.035 ± 0.002
	γ	1.010 ± 0.002	1.012 ± 0.002
	χ^2/DoF	14590/14538	14886/14540
	CL	38%	2%



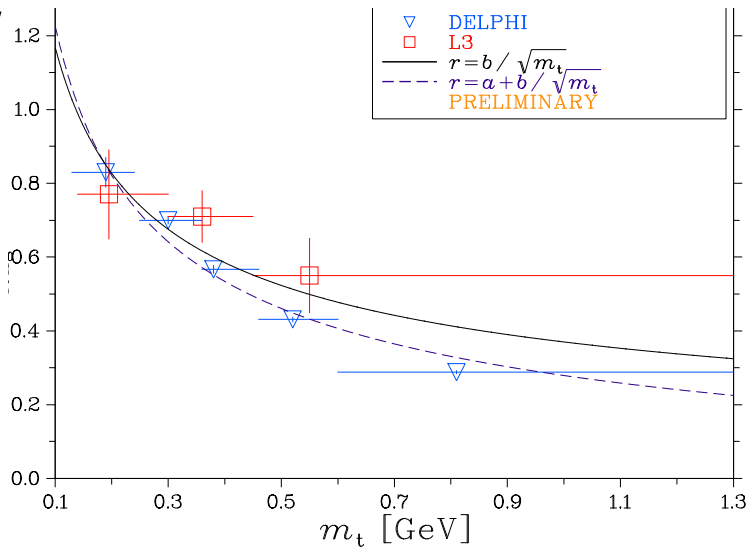
Fit Results of direct tests for 2-jet events

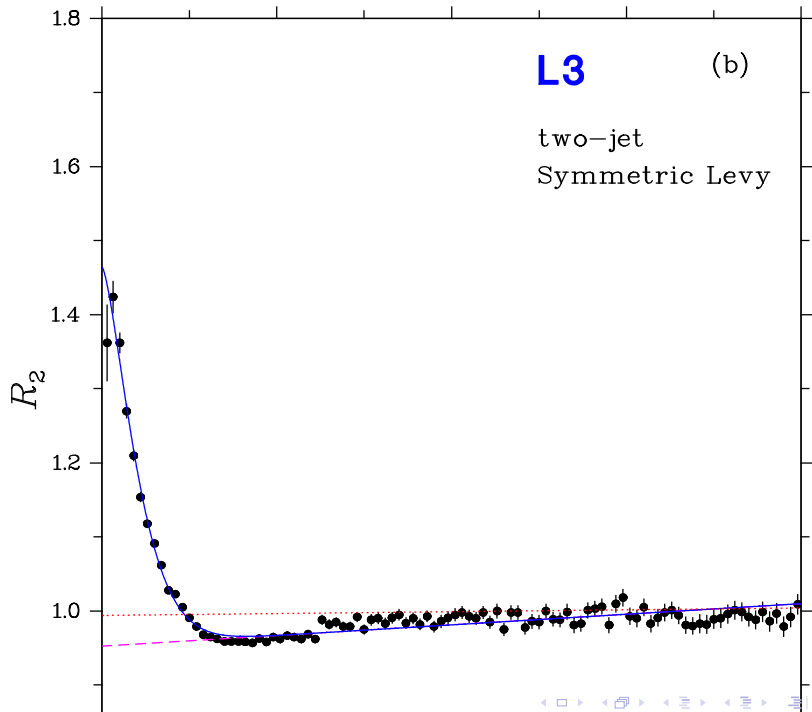
case 2	λ	0.65 ± 0.03	0.57 ± 0.03
	α	0.41 ± 0.01	0.44 ± 0.01
	R_L (fm)	0.96 ± 0.05	0.82 ± 0.04
	R_{side}/R_L	0.62 ± 0.02	1
	r_{out}/R_L	1.23 ± 0.03	1
	ϵ_L (GeV $^{-1}$)	0.004 ± 0.001	0.003 ± 0.001
	ϵ_{side} (GeV $^{-1}$)	-0.067 ± 0.003	-0.059 ± 0.003
	ϵ_{out} (GeV $^{-1}$)	-0.022 ± 0.003	-0.029 ± 0.002
	γ	1.000 ± 0.002	1.003 ± 0.002
	χ^2/DoF	10966/10647	11430/10649
	CL	2%	10^{-7}

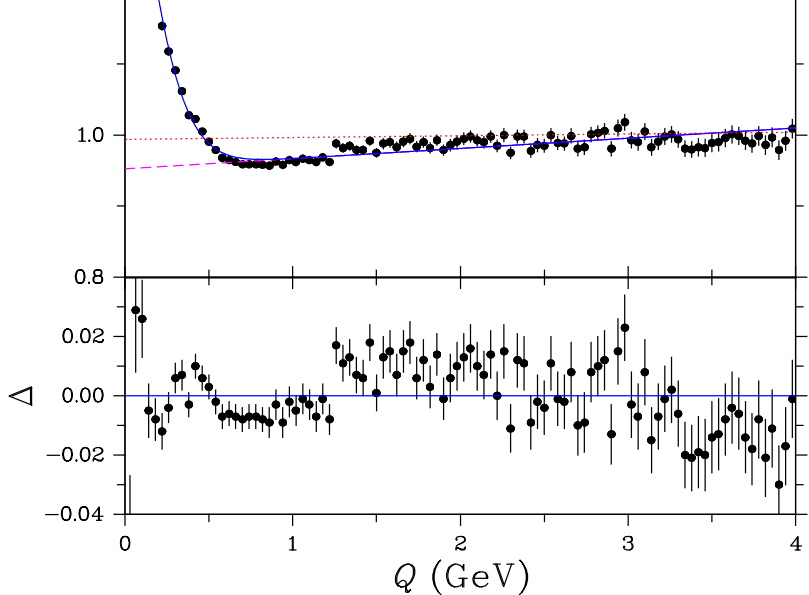


Transverse Mass dependence of r

R_L







Gauss

Edgew

Lévy

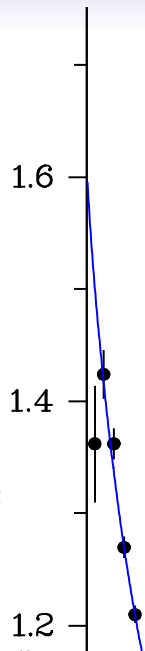
10^{-15}

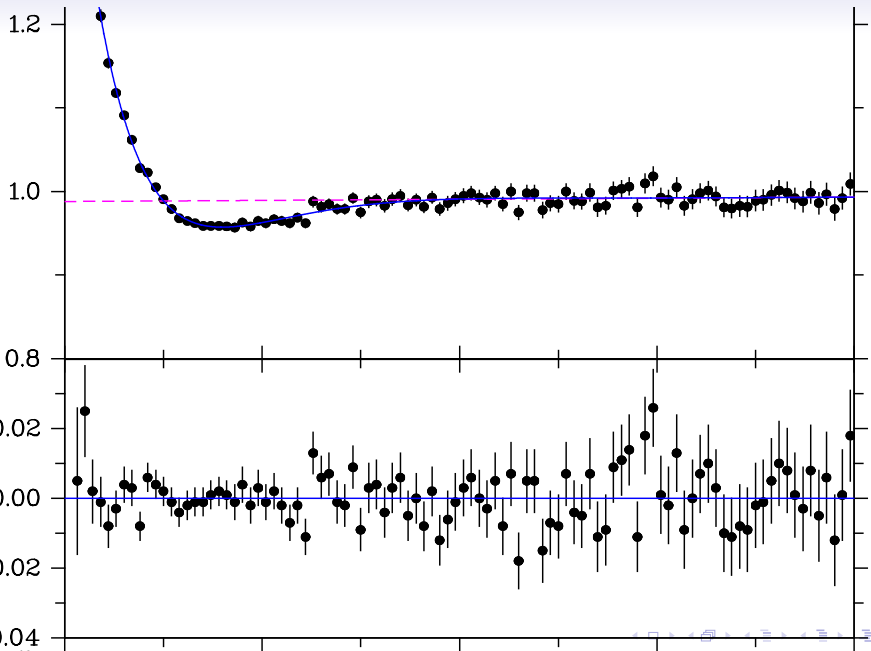
10^{-5}

10^{-8}

R_a free

$$\chi^2/\text{dof} = 91/94$$

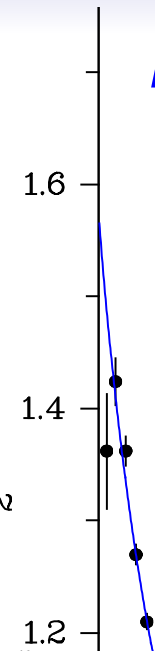


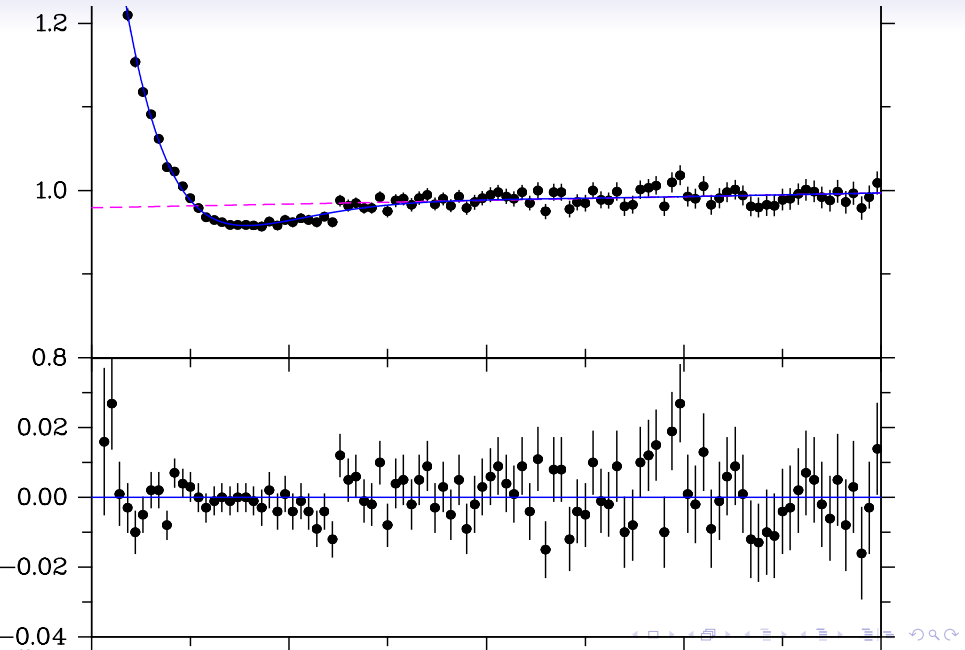


Δ

$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$$

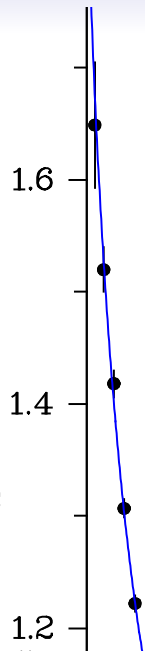
$$\chi^2/\text{dof} = 95/95$$

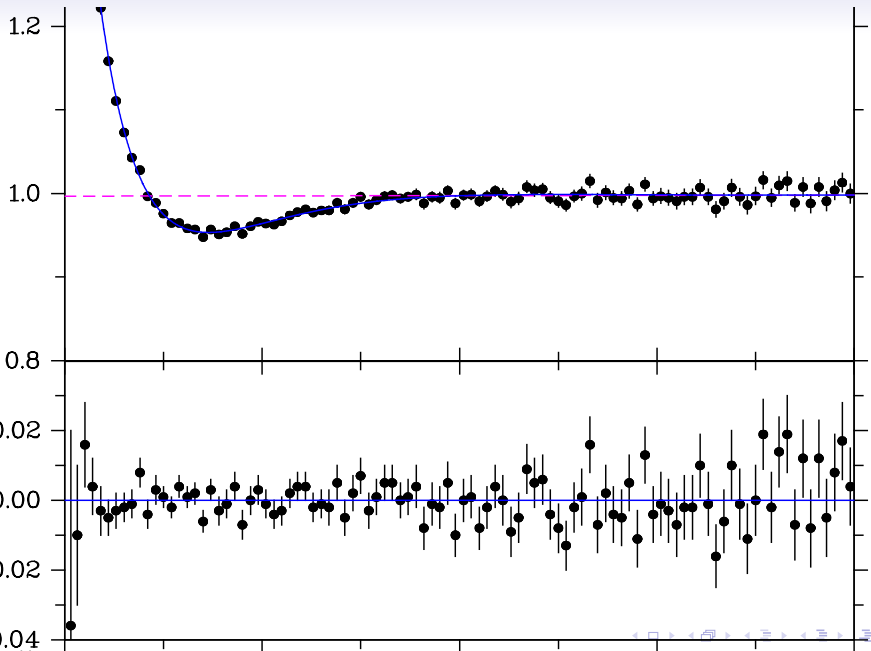




R_a free

$$\chi^2/\text{dof} = 84/94$$



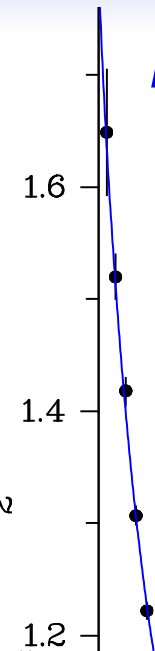


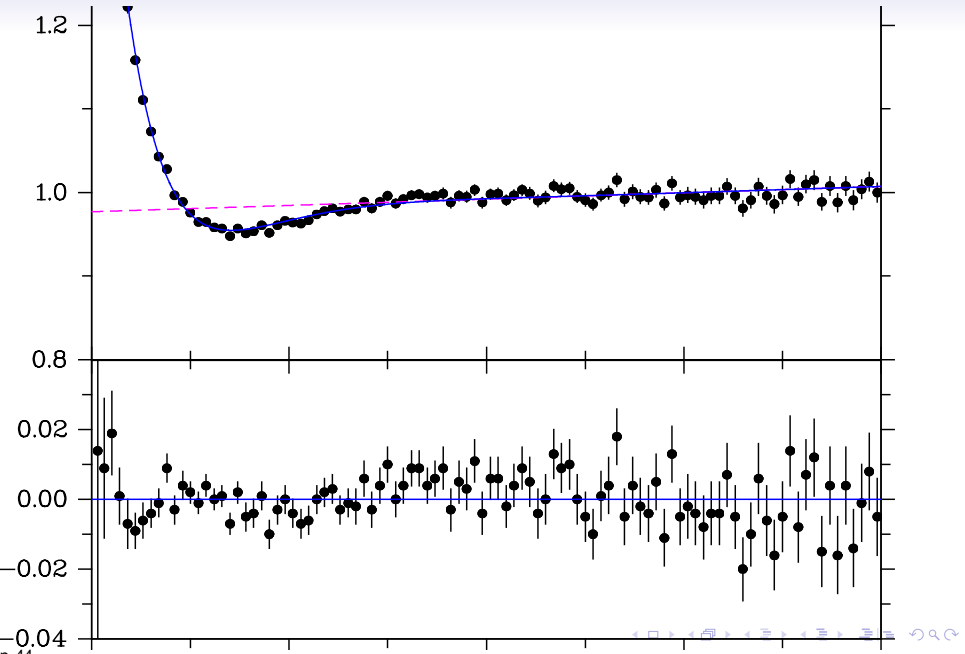
Δ

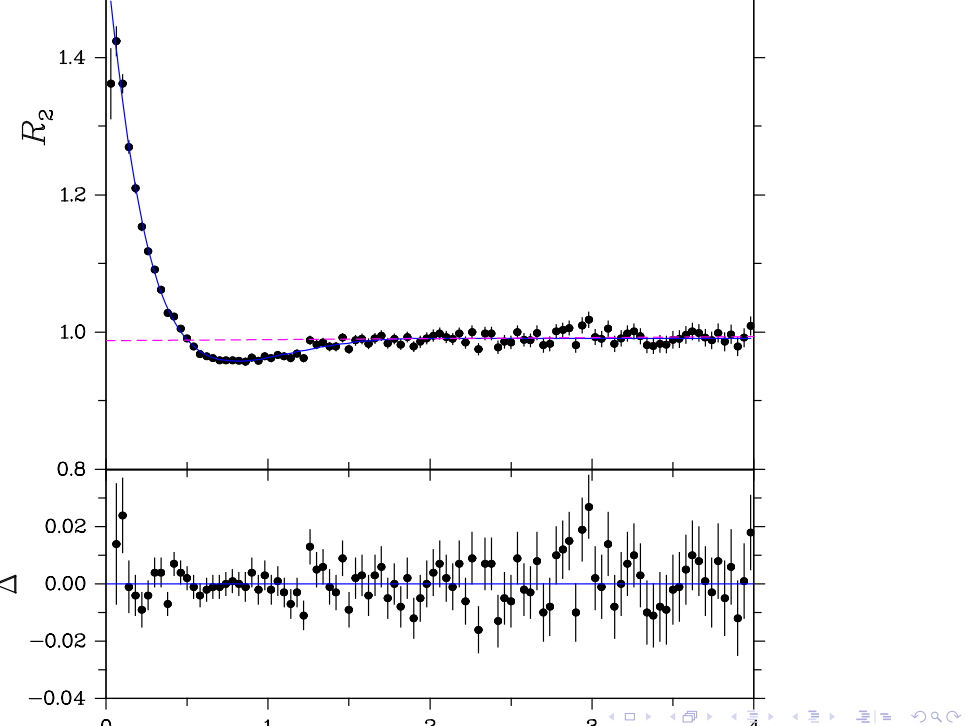
$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$$

$$\chi^2/\text{dof} = 113/95$$

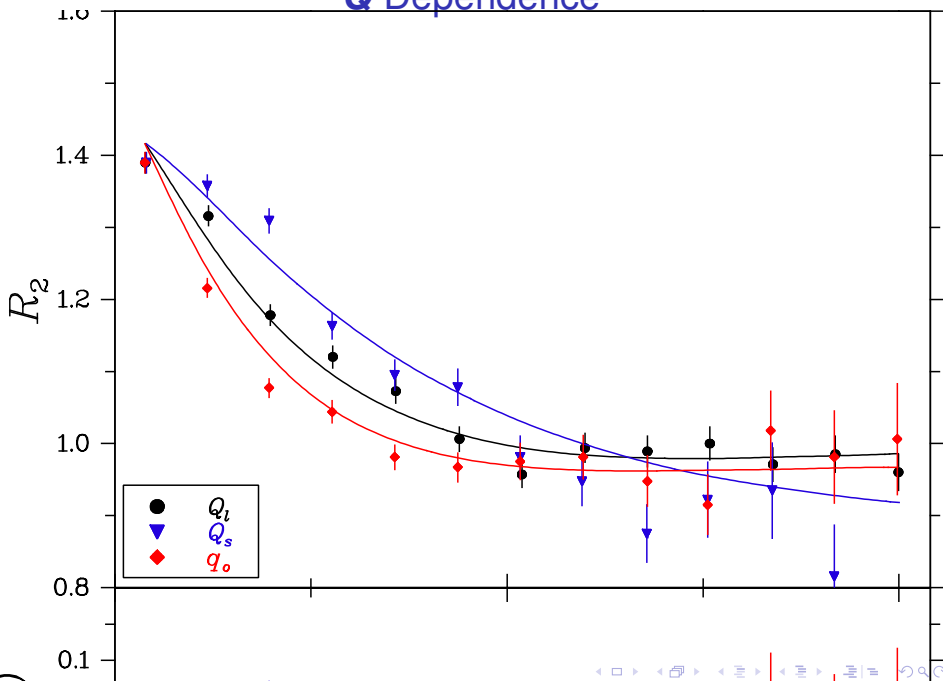
$$\text{CL} = 10\%$$



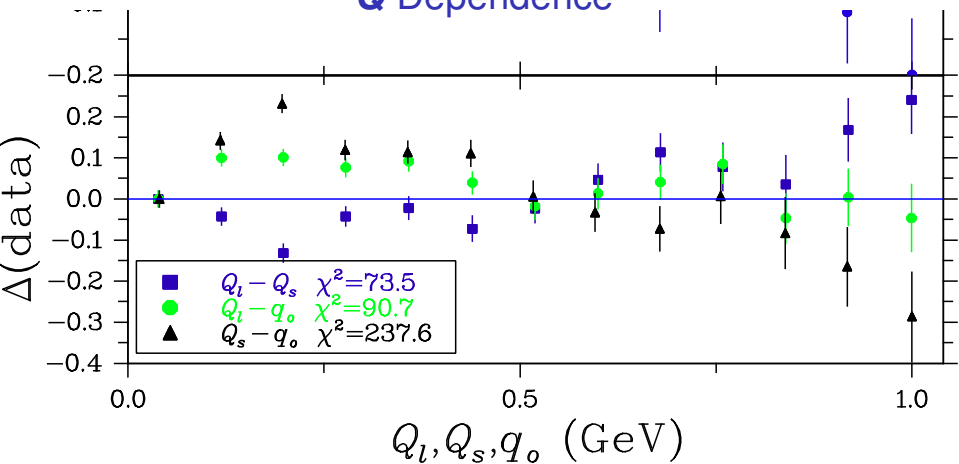




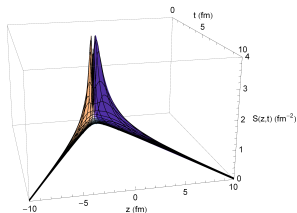
Q Dependence



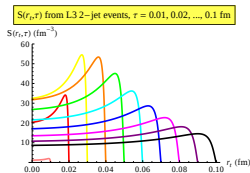
Q Dependence



Integrating over r ,



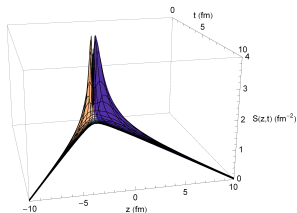
Integrating over z ,



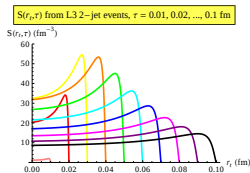
“Boomerang” shape

Particle production is close to the light-cone

Integrating over r ,



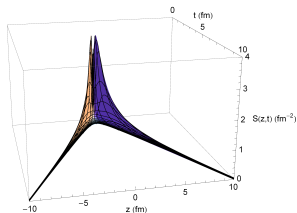
Integrating over z ,



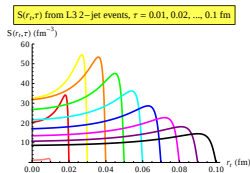
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Particle production is close to the light-cone

Integrating over r ,



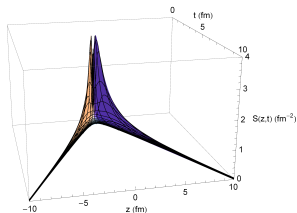
Integrating over z ,



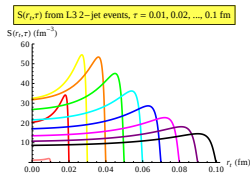
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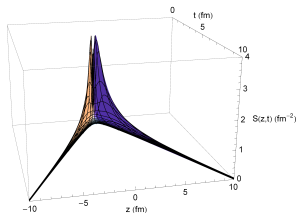
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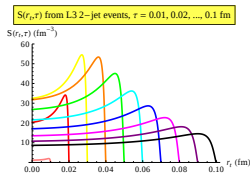
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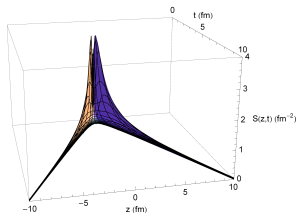
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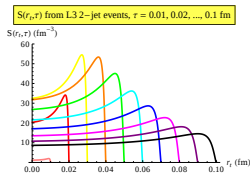
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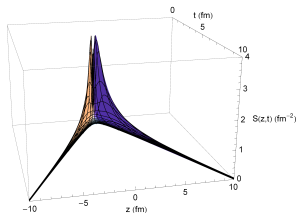
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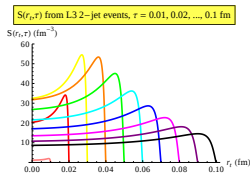
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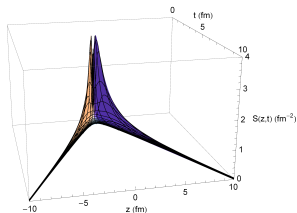
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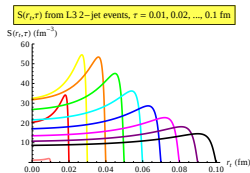
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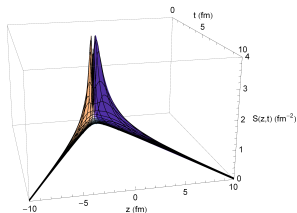
Integrating over z ,



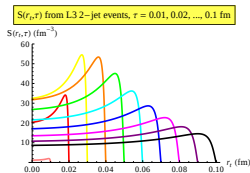
“Boomerang” shape

Particle production is close to the light-cone

Integrating over r ,

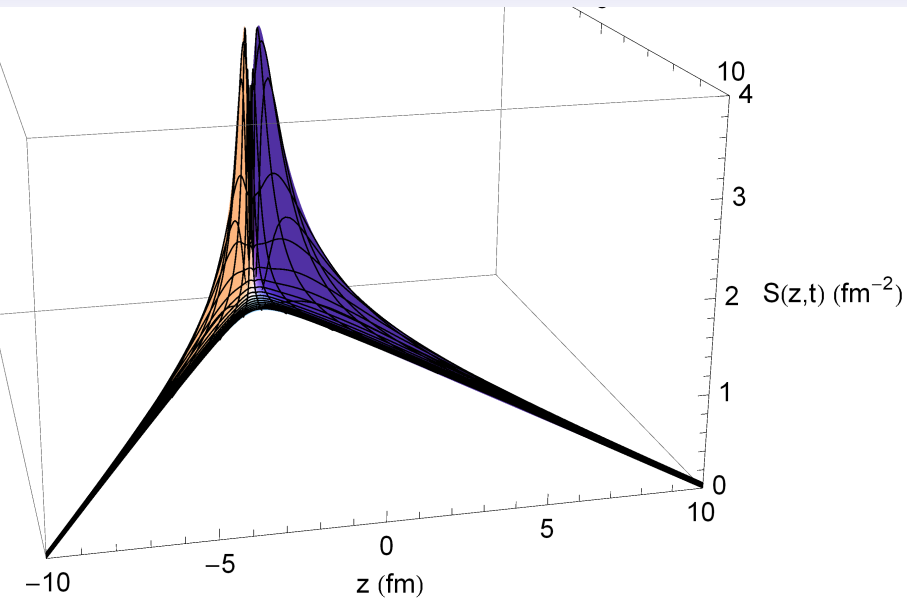


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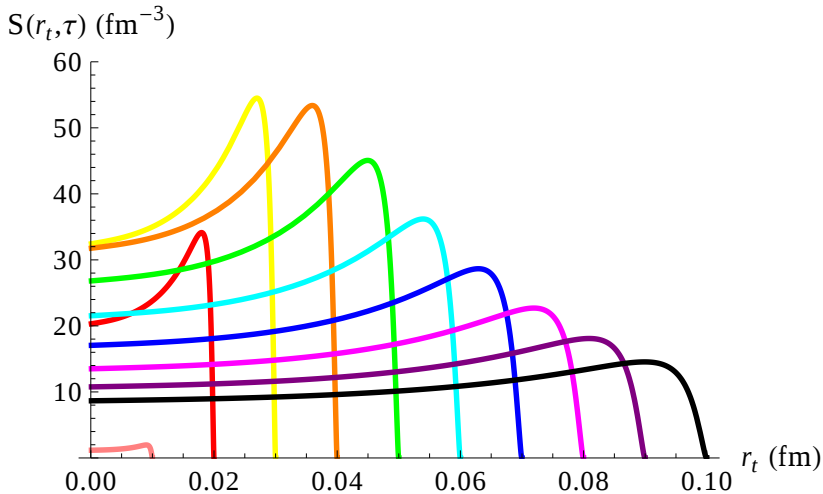


“Boomerang” shape

Particle production is close to the light-cone

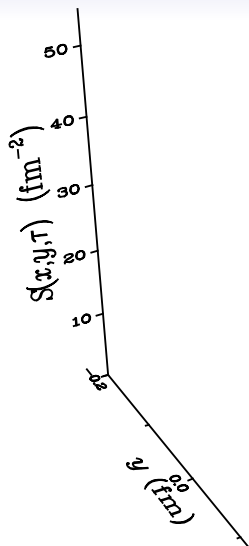
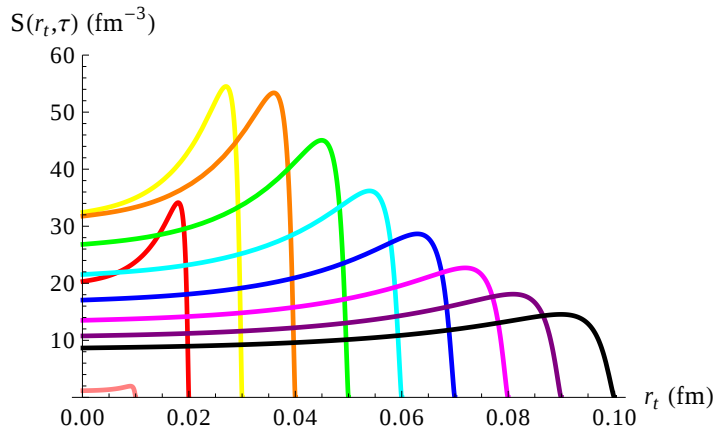


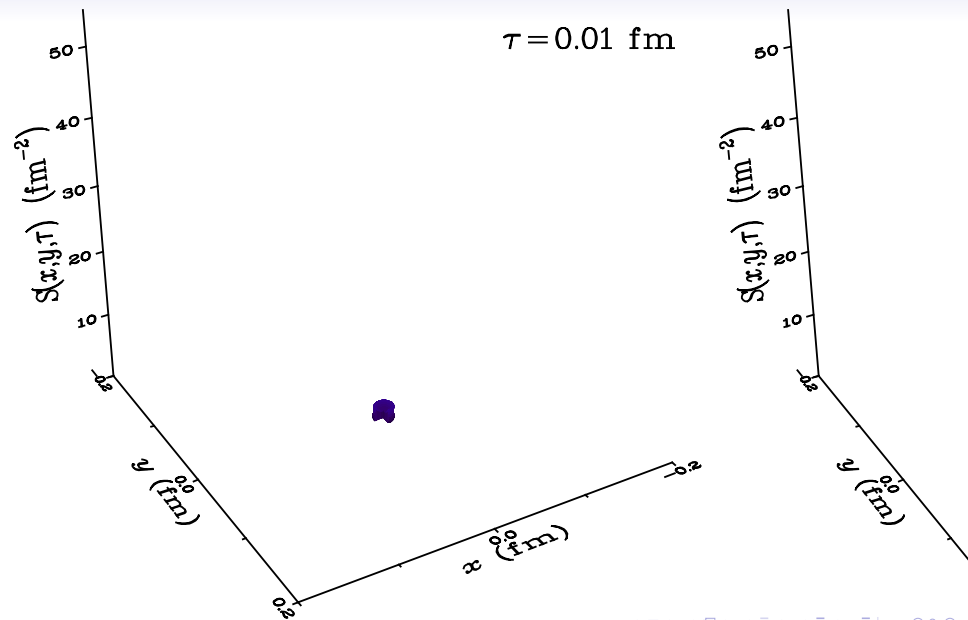
$S(r_t, \tau)$ from L3 2-jet events, $\tau = 0.01, 0.02, \dots, 0.1$ fm

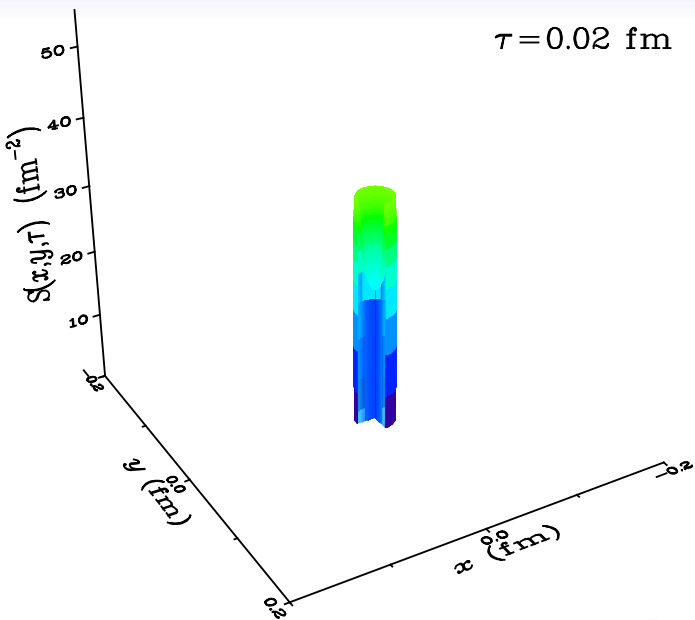


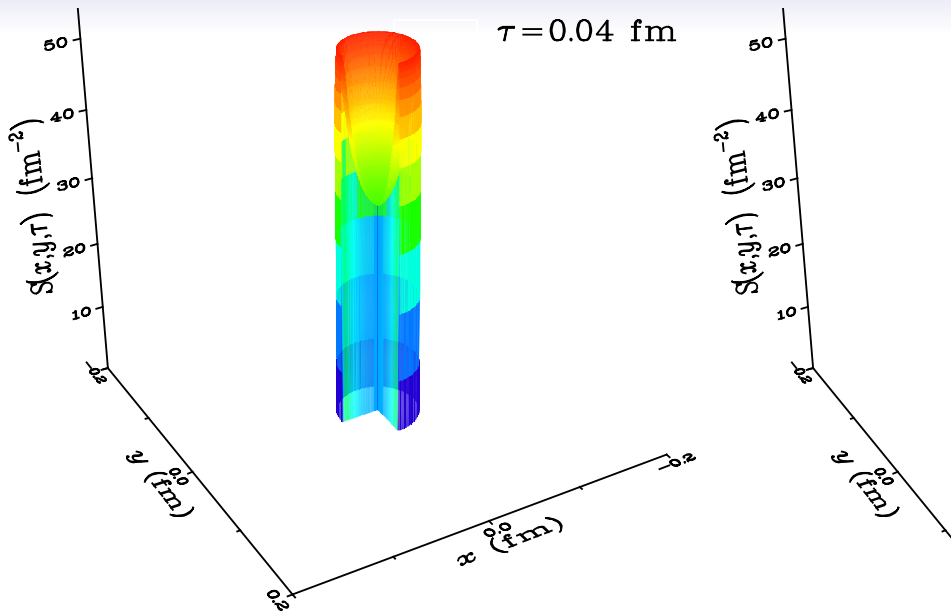
(Loading movie...)

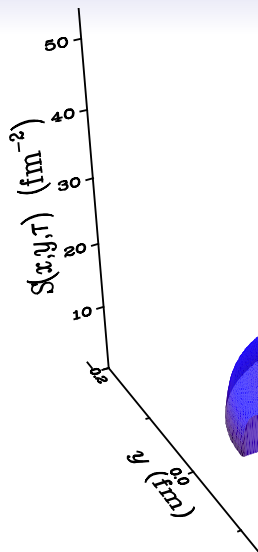
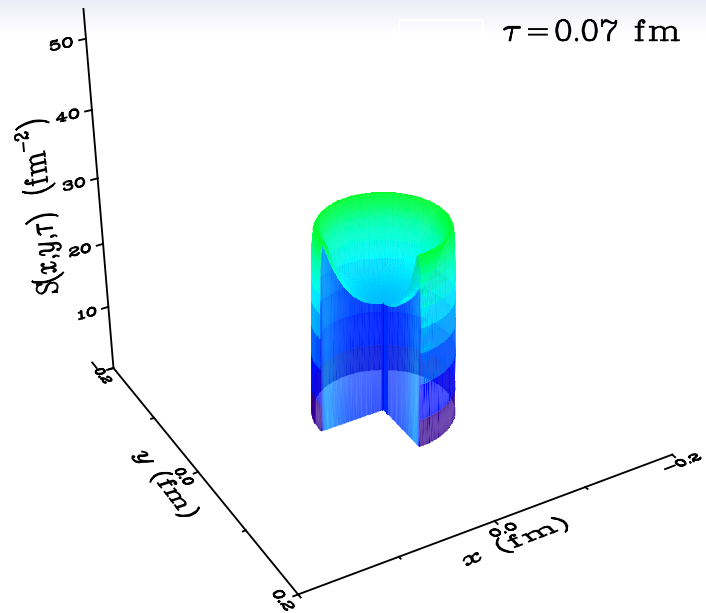
$S(r_t, \tau)$ from L3 2-jet events, $\tau = 0.01, 0.02, \dots, 0.1$ fm

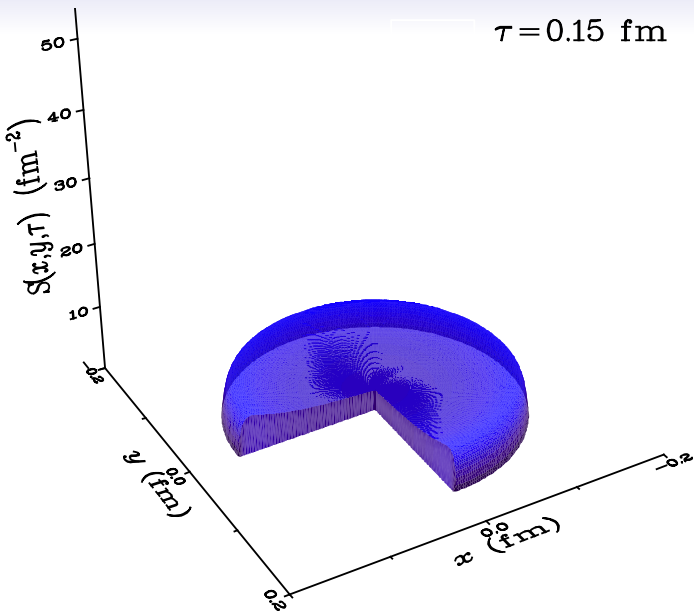






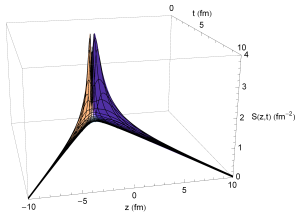




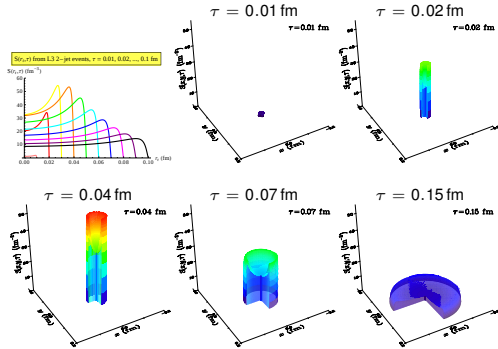


Integrating over z ,

Integrating over r ,



“Boomerang” shape

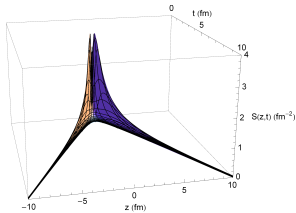


Expanding ring

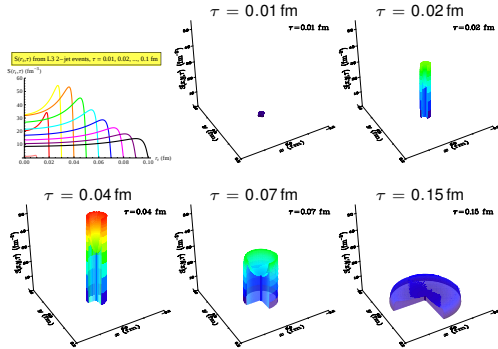
Particle production is close to the light-cone

Integrating over z ,

Integrating over r ,



“Boomerang” shape

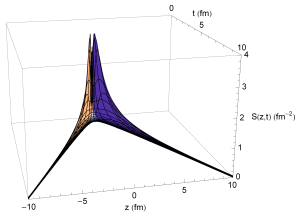


Expanding ring

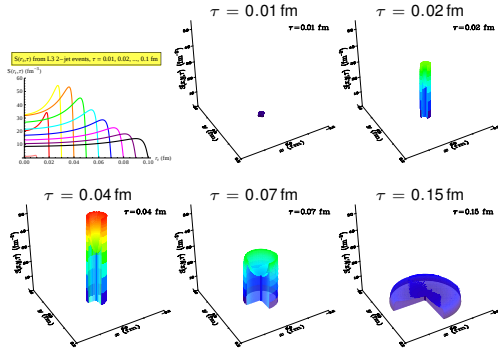
Particle production is close to the light-cone

Integrating over z ,

Integrating over r ,



“Boomerang” shape



Expanding ring

Particle production is close to the light-cone

