

Bose-Einstein Correlations in e^+e^- Annihilation: Introduction and recent L3 results on Parametrization and Reconstruction of the Source Function

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Introduction — BEC

q-particle density $\rho_q(p_1, \dots, p_q) = \frac{1}{\sigma_{\text{tot}}} \frac{d^q \sigma_q(p_1, \dots, p_q)}{dp_1 \dots dp_q}$, where σ_q is inclusive cross section

2-particle correlation: $\frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)}$

To study only BEC, not all correlations,
let $\rho_0(p_1, p_2)$ be the 2-particle density if no BEC
(= ρ_2 of the 'reference sample') and define

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} \cdot \frac{\rho_1(p_1)\rho_1(p_2)}{\rho_0(p_1, p_2)} = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)}$$

Since 2-π BEC only at small Q

$$Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$$

integrate over other variables: $R_2(Q) = \frac{\rho(Q)}{\rho_0(Q)}$

Assuming particles produced incoherently with spatial source density $S(x)$,

$$R_2(Q) = 1 + |\tilde{S}(Q)|^2$$

where $\tilde{S}(Q) = \int dx e^{iQx} S(x)$
— Fourier transform of $S(x)$

Assuming $S(x)$ is a Gaussian with radius $r \implies R_2(Q) = 1 + e^{-Q^2 r^2}$

$$R_2(Q) \propto 1 + \lambda e^{-Q^2 r^2}$$

Assumes

- incoherent average over source
 λ tries to account for
 - partial coherence
 - multiple (distinguishable) sources,
long-lived resonances
 - pion purity
- spherical (radius r) Gaussian density
of particle emitters
seems unlikely in e^+e^- annihilation
 - jets
- static source, *i.e.*, no t -dependence
certainly wrong

Nevertheless, this Gaussian formula is
the most often used parametrization
And it works fairly well
But what do the values of λ and r
actually mean?

When Gaussian parametrization does
not fit well, can expand about the
Gaussian (Edgeworth expansion).
Keeping only the lowest-order
non-Gaussian term,
 $\exp(-Q^2 r^2)$ becomes

$$\exp(-Q^2 r^2) \cdot \left[1 + \frac{\kappa}{3!} H_3(Qr) \right]$$

(H_3 is third-order Hermite polynomial)

Experimental Problems I

I. Pion purity

1. mis-identified pions – K, p
 - correct by MC. – But is it correct?
2. resonances
 - long-lived affect λ
BEC peak narrower than resolution
 - short-lived, e.g., ρ , - affect r
 - correct by MC. – But is it correct?
3. weak decays
 - $\sim 20\%$ of Z decays are $b\bar{b}$
like long-lived resonances,
decrease λ

- per Z: $17.0 \pi^\pm$, $2.3 K^\pm$, $1.0 p$
(15% non- π)

| Origin of π^+ in Z decay | (%) (JETSET 7.4) |
|---|---------------------|
| direct (string fragmentation) | 16 |
| decay (short-lived resonances) $\Gamma > 6.7 \text{ MeV}, \tau < 30 \text{ fm}$ (ρ , ω , K^* , Δ , ...) | 62 |
| decay (long-lived resonances) $\Gamma < 6.7 \text{ MeV}, \tau > 30 \text{ fm}$ | 22 |

Experimental Problems II

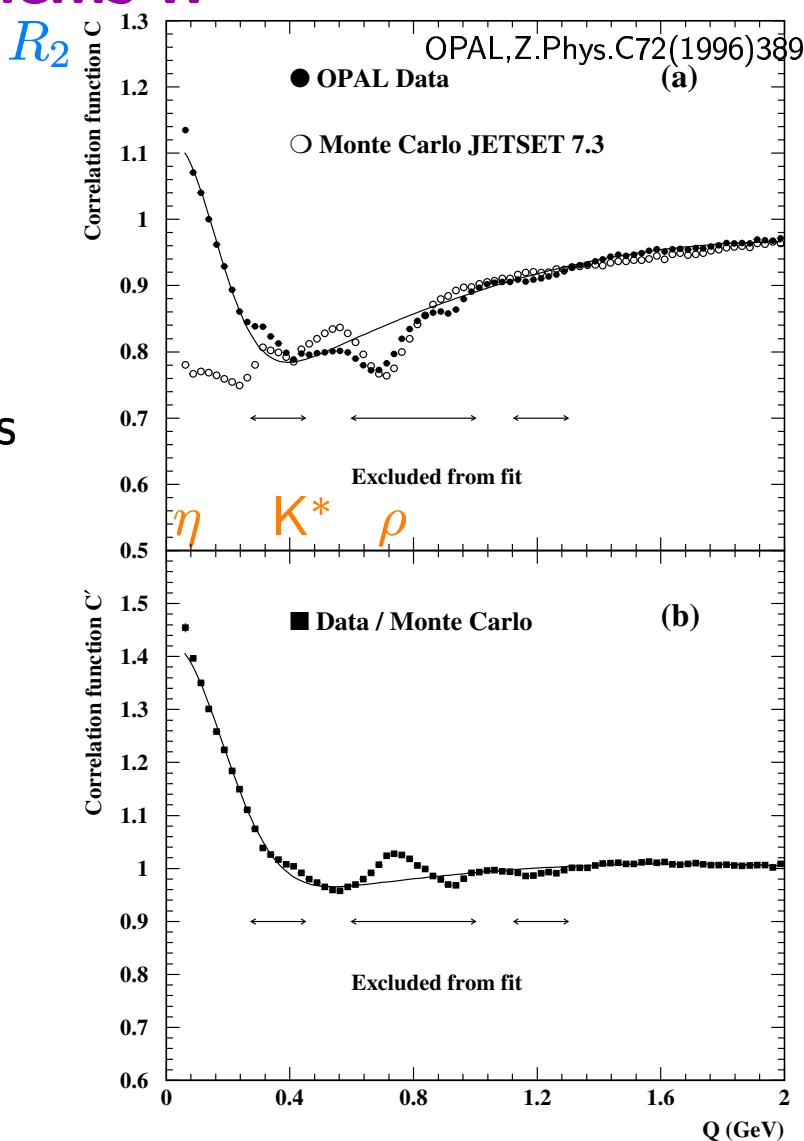
II. Reference Sample, ρ_0 — it does NOT exist

Common choices:

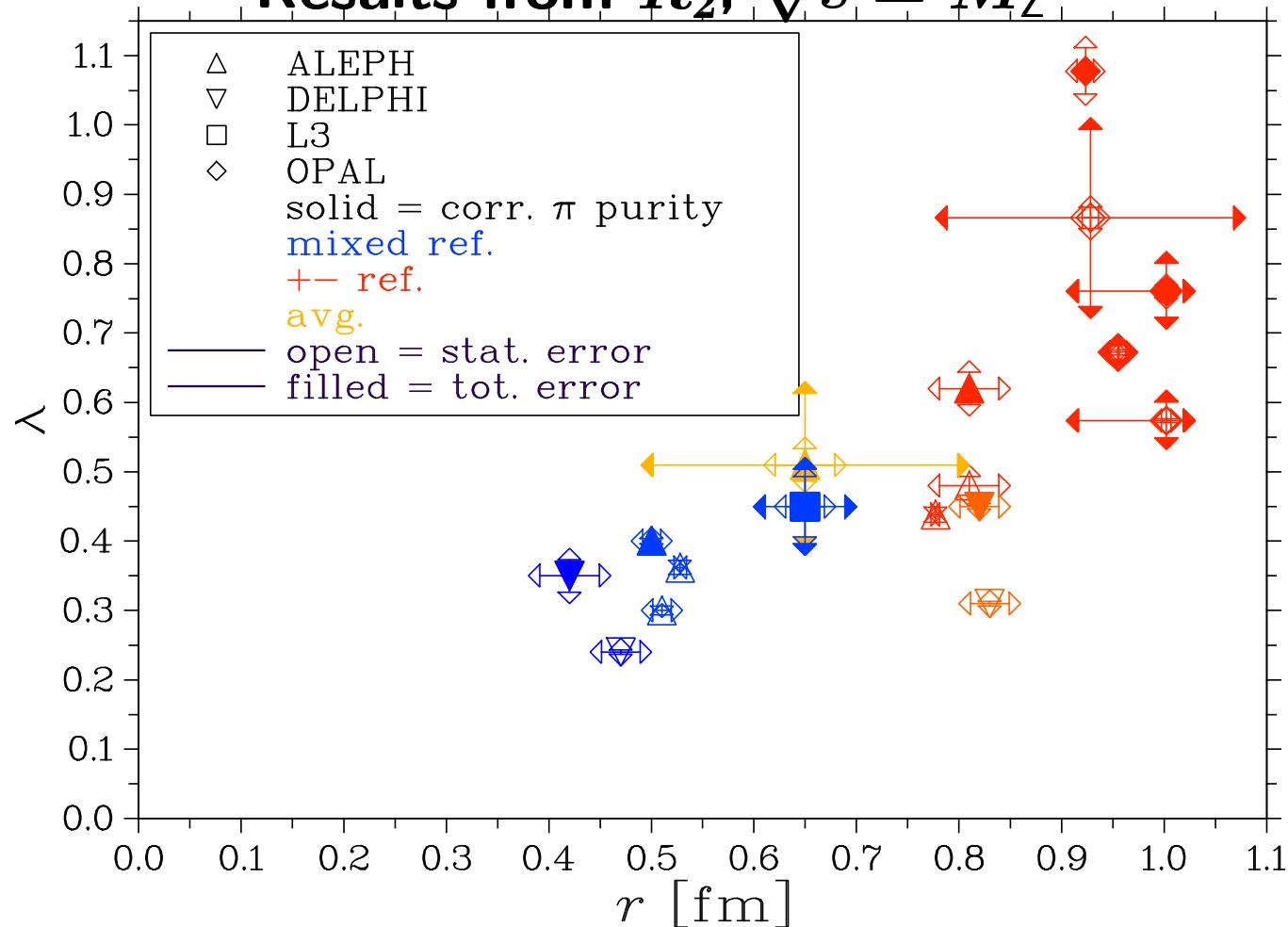
1. $+-$ pairs But different resonances than $++$
– correct by MC. – But is it correct?
2. Monte Carlo — But is it correct?
3. Mixed events – pair particles from different events
But destroys all correlations, not just BEC
– correct by MC. – But is it correct?
4. Mixed hemispheres (for 2-jet events)
– pair particle with particle reflected
from opposite hemisphere
But destroys all correlations
– correct by MC. – But is it correct?

To account for long-range correlations
inadequately removed by reference sample

$$R_2(Q) \propto (1 + \lambda e^{-Q^2 r^2})(1 + \delta Q)$$

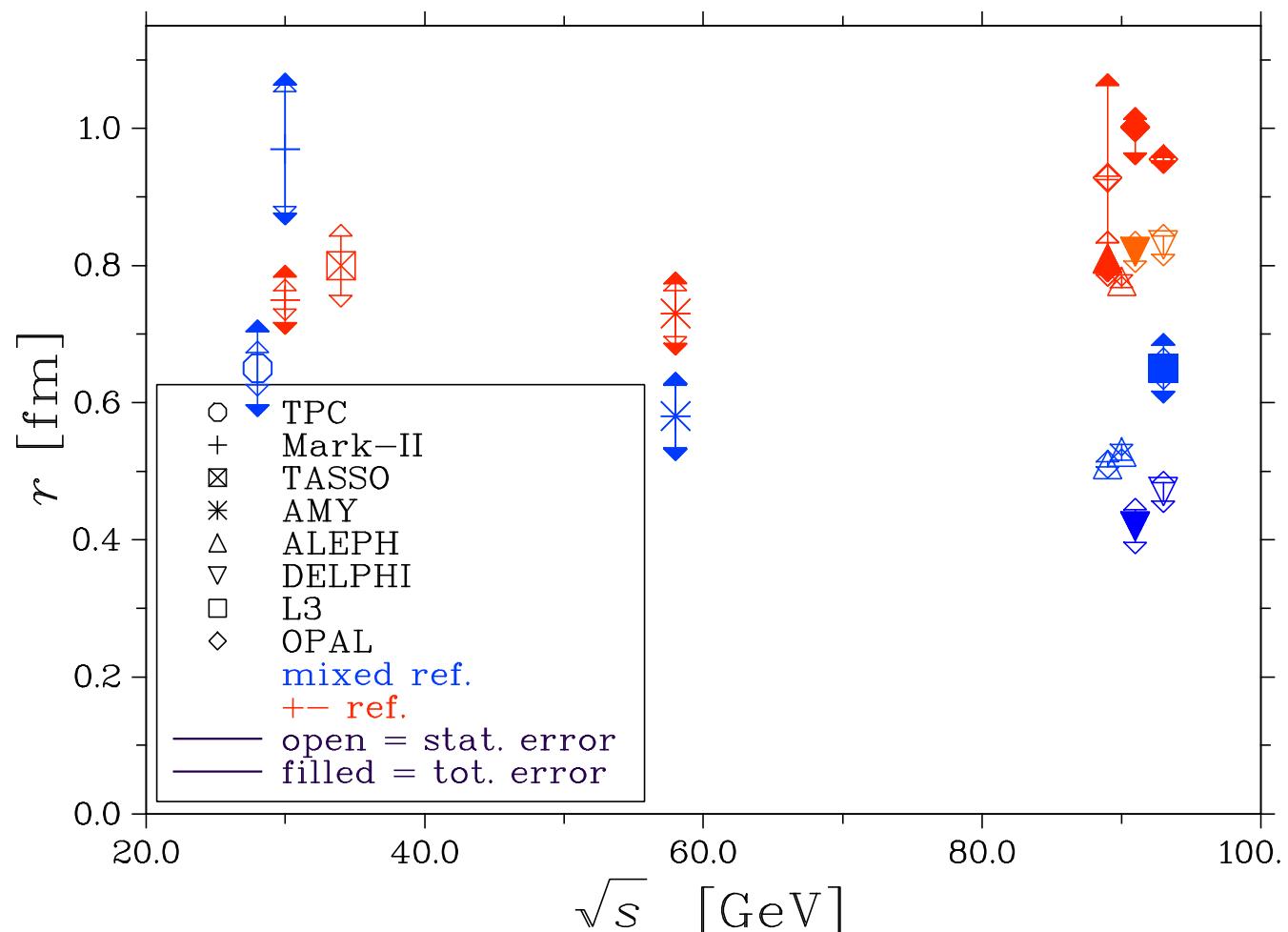


Results from R_2 , $\sqrt{s} = M_Z$



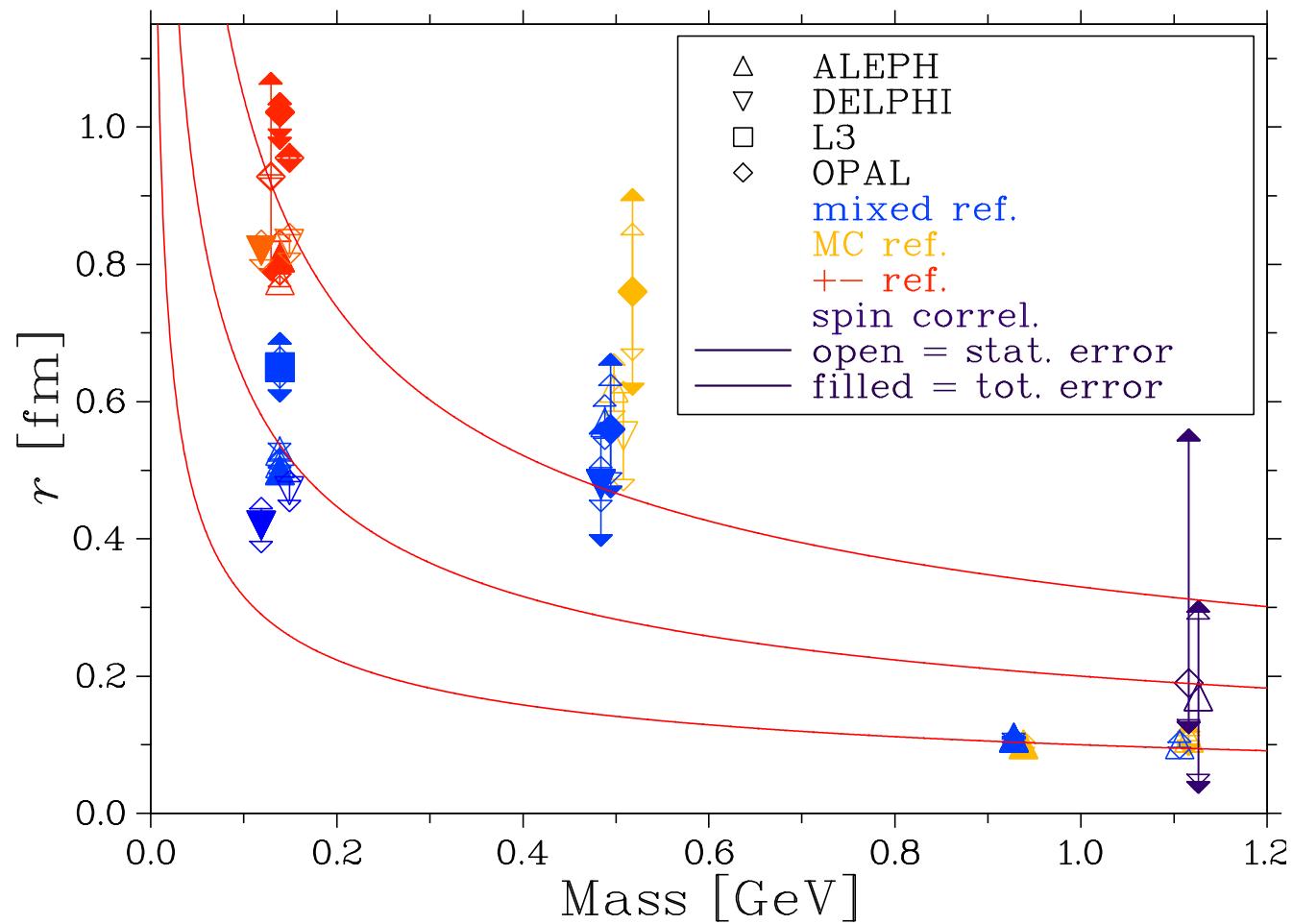
- correction for π purity increases λ
- mixed ref. gives smaller λ , r than $+-$ ref.

\sqrt{s} dependence of r



No evidence for \sqrt{s} dependence

Mass dependence of r — BEC and FDC



No evidence for $r \sim 1/\sqrt{m}$

$r(\text{mesons}) > r(\text{baryons})$

Elongation of the source

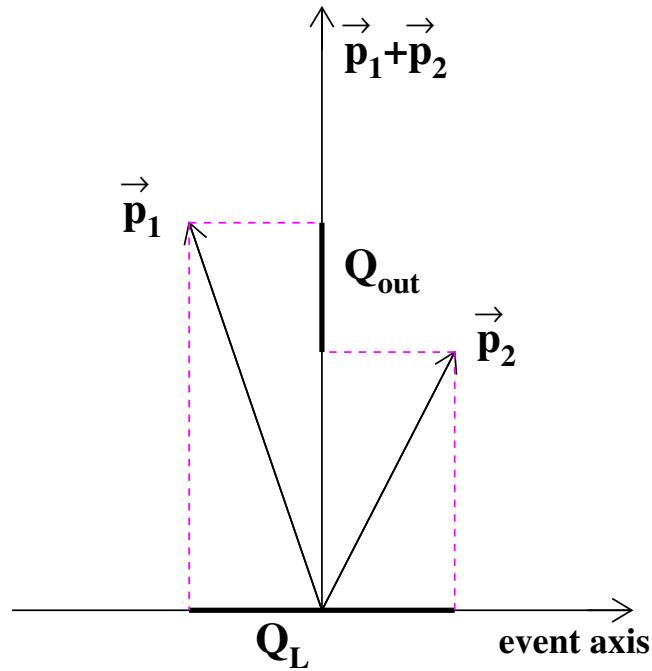
The usual parametrization assumes a symmetric Gaussian source

But, there is **no reason** to expect this symmetry in $e^+e^- \rightarrow q\bar{q}$.

Therefore, do a 3-dim. analysis in the Longitudinal Center of Mass System

LCMS:

Boost each π -pair along event axis (thrust or sphericity)



$$p_{L1} = -p_{L2}$$

$\vec{p}_1 + \vec{p}_2$ defines 'out' axis

$Q_{side} \perp (Q_L, Q_{out})$

the LCMS

Advantages of LCMS:

$$\begin{aligned} Q^2 &= Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 - (\Delta E)^2 \\ &= Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 (1 - \beta^2) \quad \text{where } \beta \equiv \frac{p_{\text{out}1} + p_{\text{out}2}}{E_1 + E_2} \end{aligned}$$

Thus, the energy difference,
and therefore the difference in emission time of the pions
couples only to the out-component, Q_{out} .

Thus,

Q_L and Q_{side} reflect only spatial dimensions of the source
 Q_{out} reflects a mixture of spatial and temporal dimensions.

Parametrization of R_2

Writing R_2 in terms of $\vec{Q} = (Q_L, Q_{\text{side}}, Q_{\text{out}})$: $R_2(\vec{Q}) = \frac{\rho(\vec{Q})}{\rho_0(\vec{Q})}$

We parametrize $R_2(\vec{Q})$ by a 3-dimensional Gaussian

$$R_2(Q_L, Q_{\text{out}}, Q_{\text{side}}) = \gamma \cdot (1 + \lambda G) \cdot B$$

where

- γ = normalization (≈ 1)
- λ = “incoherence”, or strength of BE effect
- G = azimuthally symmetric Gaussian:
$$G = \exp(-r_L^2 Q_L^2 - r_{\text{out}}^2 Q_{\text{out}}^2 - r_{\text{side}}^2 Q_{\text{side}}^2 + 2\rho_{L,\text{out}} r_L r_{\text{out}} Q_L Q_{\text{out}})$$
- $B = (1 + \delta Q_L + \varepsilon Q_{\text{out}} + \xi Q_{\text{side}})$ describes large Q (long-range correlations)

Elongation Results (L3)

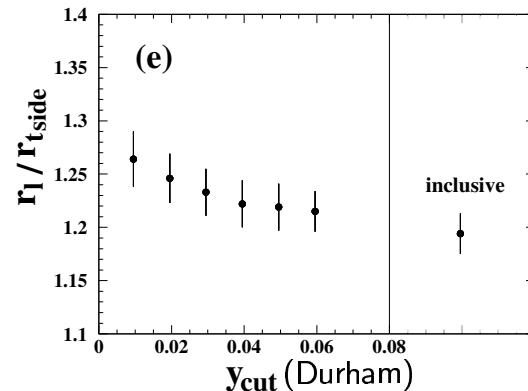
| parameter | Gaussian | Edgeworth |
|------------------------|--------------------------------------|--------------------------------------|
| λ | $0.41 \pm 0.01^{+0.02}_{-0.19}$ | $0.54 \pm 0.02^{+0.04}_{-0.26}$ |
| r_L (fm) | $0.74 \pm 0.02^{+0.04}_{-0.03}$ | $0.69 \pm 0.02^{+0.04}_{-0.03}$ |
| r_{out} (fm) | $0.53 \pm 0.02^{+0.05}_{-0.06}$ | $0.44 \pm 0.02^{+0.05}_{-0.06}$ |
| r_{side} (fm) | $0.59 \pm 0.01^{+0.03}_{-0.13}$ | $0.56 \pm 0.02^{+0.03}_{-0.12}$ |
| r_{out}/r_L | $0.71 \pm 0.02^{+0.05}_{-0.08}$ | $0.65 \pm 0.03^{+0.06}_{-0.09}$ |
| r_{side}/r_L | $0.80 \pm 0.02^{+0.03}_{-0.18}$ | $0.81 \pm 0.02^{+0.03}_{-0.19}$ |
| κ_L | — | $0.5 \pm 0.1^{+0.1}_{-0.2}$ |
| κ_{out} | — | $0.8 \pm 0.1 \pm 0.3$ |
| κ_{side} | — | $0.1 \pm 0.1 \pm 0.3$ |
| δ | $0.025 \pm 0.005^{+0.014}_{-0.015}$ | $0.036 \pm 0.007^{+0.012}_{-0.023}$ |
| ϵ | $0.005 \pm 0.005^{+0.034}_{-0.012}$ | $0.011 \pm 0.005^{+0.037}_{-0.012}$ |
| ξ | $-0.035 \pm 0.005^{+0.031}_{-0.024}$ | $-0.022 \pm 0.006^{+0.020}_{-0.025}$ |
| χ^2/DoF | 2314/2189 | 2220/2186 |
| C.L. (%) | 3.1 | 30 |

- $\rho_{L,\text{out}} = 0$ So fix to 0.
- Edgeworth fit significantly better than Gaussian
- $r_{\text{side}}/r_L < 1$ more than 5 std. dev. Elongation along thrust axis
- Models which assume a spherical source are too simple.

Elongation Results

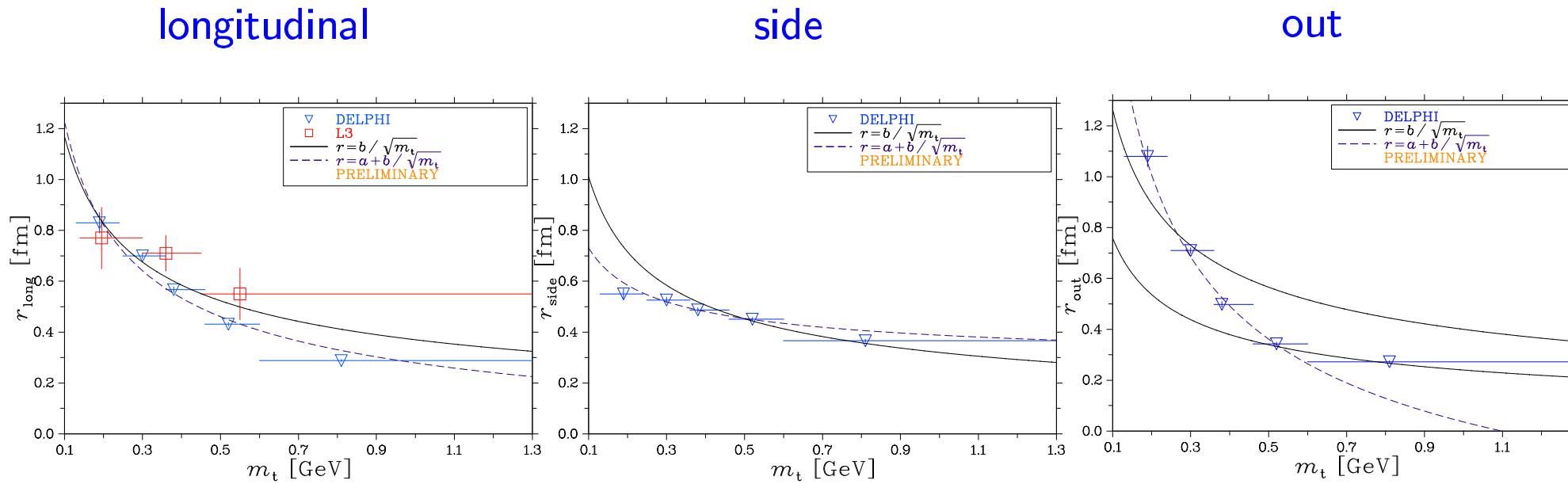
| | | | Gauss / Edgeworth | 2-D r_t/r_L | 3-D r_{side}/r_L |
|--------|-------|-------|----------------------|--------------------------|------------------------------------|
| DELPHI | mixed | 2-jet | Gauss | $0.62 \pm 0.02 \pm 0.05$ | — |
| ALEPH | mixed | 2-jet | Gauss | $0.61 \pm 0.01 \pm 0.??$ | — |
| | +,- | 2-jet | Gauss | $0.91 \pm 0.02 \pm 0.??$ | — |
| | mixed | 2-jet | Edgeworth | $0.68 \pm 0.01 \pm 0.??$ | — |
| | +,- | 2-jet | Edgeworth | $0.84 \pm 0.02 \pm 0.??$ | — |
| OPAL | +,- | 2-jet | Gauss | — | $0.82 \pm 0.02 \pm 0.01$ 0.05 |
| L3 | mixed | all | Gauss | — | $0.80 \pm 0.02 \pm 0.03$ 0.18 |
| | mixed | all | Edgeworth | — | $0.81 \pm 0.02 \pm 0.03$ 0.19 |

~20% elongation along thrust axis
(ZEUS finds similar results in ep)



OPAL:
Elongation larger
for narrower jets

Transverse Mass dependence of r



r decreases with m_t for all directions

but more like $r = a + b/\sqrt{m_t}$ - - -
than like $r = b/\sqrt{m_t}$ —

Smirnova&Lörstad, 7th Int. Workshop on Correlations and Fluctuations (1996)
Van Dalen, 8th Int. Workshop on Correlations and Fluctuations (1998)

Summary

- Comparison between experiments is difficult.
 - reference samples
 - MC corrections
- No evidence for \sqrt{s} dependence of r
- $r(\text{mesons}) > r(\text{baryons})$ — no evidence for $r \sim 1/\sqrt{m}$
- some evidence for approximate $1/\sqrt{m_t}$ dependence of r
- $\sim 20\%$ elongation along thrust axis — consistent with string model

New L3 Results

- $e^+e^- \rightarrow \text{hadrons}$ at $\sqrt{s} \approx M_Z$
- about 10^6 events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\text{cut}} = 0.006$
- use mixed events for reference sample

Beyond the Symmetric Gaussian?

Decompose Q in various ways:

$$R_2 = \gamma \cdot (1 + \lambda G) \cdot B$$

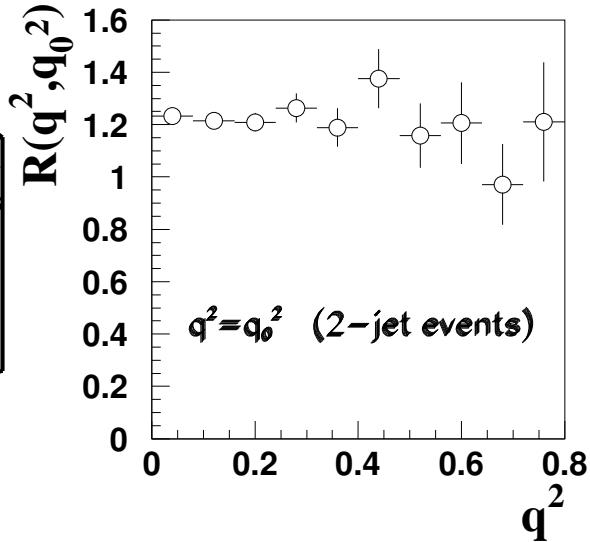
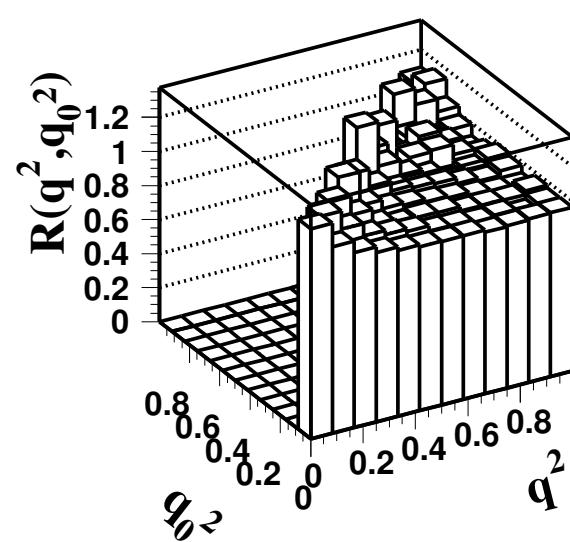
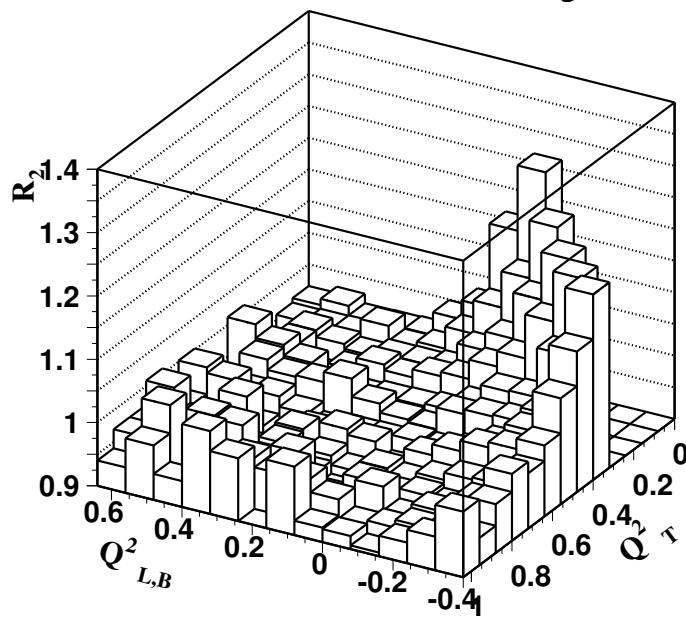
1. $G = \exp(-r_L^2 Q_L^2 - r_{\text{side}}^2 Q_{\text{side}}^2 - r_o^2 (Q_{\text{out}}^2 - (\Delta E)^2))$
 $G = \exp(-r_L^2 Q_L^2 - r_{\text{side}}^2 Q_{\text{side}}^2 - r_{\text{out}}^2 Q_{\text{out}}^2)$
2. $G = \exp(-r_\ell^2 (Q_L^2 - (\Delta E)^2) - r_T^2 (Q_{\text{side}}^2 + Q_{\text{out}}^2))$
3. $G = \exp(-r_1^2 (Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2) - r_0^2 (-(\Delta E)^2))$

| all events | | | | |
|---|-------------------------------------|--|-----------------------|--------------------------------|
| 1 | $r_L = 0.74 \pm 0.02 \text{ fm}$ | $r_{\text{side}} = 0.59 \pm 0.01 \text{ fm}$ | $\text{CL} = 3\%$ | $\sim 20\% \text{ elongation}$ |
| 2 | $r_\ell = 0.54 \pm 0.01 \text{ fm}$ | $r_T = 0.57 \pm 0.01 \text{ fm}$ | $\text{CL} < 10^{-5}$ | \approx |
| 3 | $r_1 = 0.57 \pm 0.07 \text{ fm}$ | $r_0 = 0.56 \pm 0.08 \text{ fm}$ | $\text{CL} = 0.2\%$ | $=$ |
| 2-jet events (Durham $y_{\text{cut}} = 0.006$) | | | | |
| 1 | OPAL | | | $\sim 20\% \text{ elongation}$ |
| 2 | $r_\ell = 0.51 \pm 0.02 \text{ fm}$ | $r_T = 0.55 \pm 0.02 \text{ fm}$ | $\text{CL} = 10^{-4}$ | \approx |
| 3 | $r_1 = 0.55 \pm 0.08 \text{ fm}$ | $r_0 = 0.53 \pm 0.09 \text{ fm}$ | $\text{CL} = 42\%$ | $=$ |

CLs none too good, but $R_2 \approx R_2(Q)$

confirms TASSO, Z.Phys.C71(1986)405

Beyond the Symmetric Gaussian?



$$Q_{L,B}^2 = Q_L^2 - (\Delta E)^2$$

$$Q_T^2 = Q_{\text{side}}^2 + Q_{\text{out}}^2$$

$$q_0^2 = (\Delta E)^2$$

$$q^2 = Q_L^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2$$

Conclusion: $R_2 \approx R_2(Q)$

Only consider $R_2(Q)$ for rest of talk.

Beyond the Gaussian

Assume static distribution of pion emitters in configuration space, $f(r)$ with characteristic function (Fourier transform), $\tilde{f}(Q)$

Then $R_2 = \gamma \cdot [1 + \lambda |\tilde{f}(Q)|^2] \cdot (1 + \delta Q)$

- $f(r)$ is Gaussian with mean $\mu = 0$ and variance R^2

$$\tilde{f}(Q) = \exp\left(\iota\mu Q - \frac{(\textcolor{teal}{R}Q)^2}{2}\right) \quad R_2 = \gamma \cdot [1 + \lambda \exp(-(RQ)^2)] \cdot (1 + \delta Q)$$

- approximately Gaussian – Edgeworth expansion

$$R_2 = \gamma \cdot [1 + \lambda \exp(-(RQ)^2) \cdot [1 + \frac{\kappa}{3!} H_3(RQ)]] \cdot (1 + \delta Q)$$

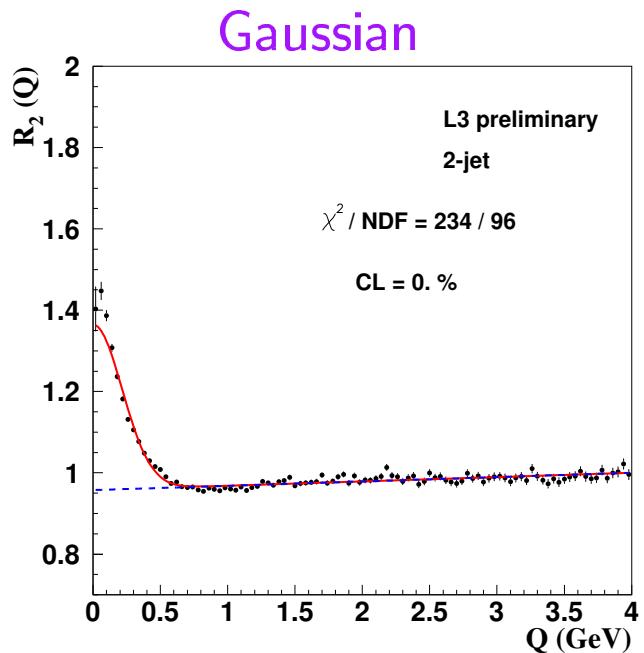
- $f(r)$ is a symmetric Lévy stable distribution with location parameter $x_0 = 0$, ‘width’ parameter R , and ‘index of stability’, $0 < \alpha \leq 2$

$$\tilde{f}(Q) = \exp\left(\iota x_0 Q - \frac{|\textcolor{teal}{R}Q|^\alpha}{2}\right) \quad R_2 = \gamma \cdot [1 + \lambda \exp(-(RQ)^\alpha)] \cdot (1 + \delta Q)$$

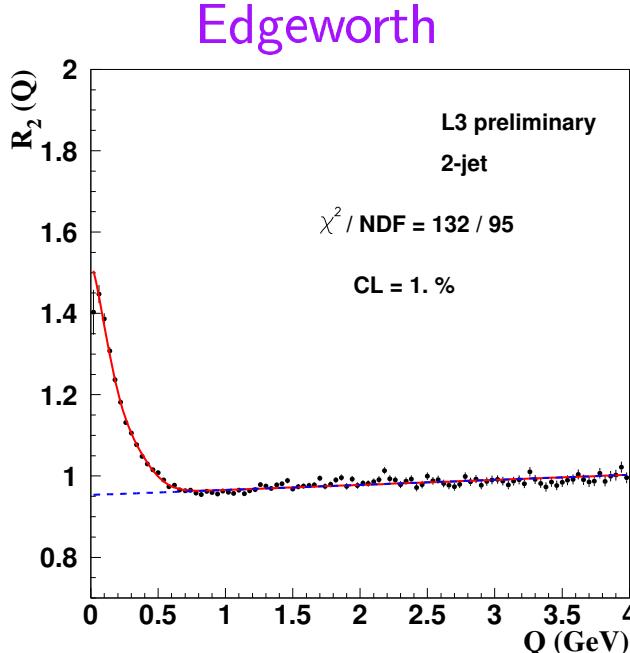
$\alpha = 2$ corresponds to Gaussian with $\mu = x_0$, variance R^2

$\alpha = 1$ corresponds to a Cauchy distribution for $f(r)$

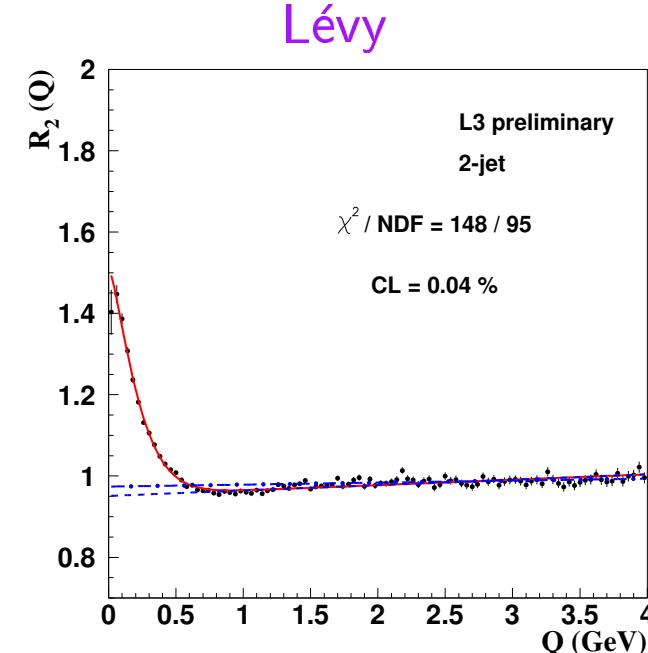
Beyond the Gaussian



Far from Gaussian.



$$\kappa = 0.71 \pm 0.06$$



$$\alpha = 1.34 \pm 0.04$$

Poor CLs. Edgeworth and Lévy better than Gaussian, but still poor
Problem is the dip of R_2 in the region $0.6 < Q < 1.5 \text{ GeV}$

Same conclusions for 3-jet events, all events.

Beyond the Gaussian

Summary:

- We have assumed a static source — **certainly wrong**
- BEC depends (at least approximately) only on Q
- r decreases with m_t , approximately as $1/\sqrt{m_t}$
may be due to correlation between momentum and production point

Białas *et al.*

Let's turn to a model incorporating these points.

BEC in the τ -model

The τ -model assumes avg. production point proportional to momentum:

Csörgő and Zimányi, Nucl.Phys.**A517**(1990)588.

$$\bar{x}^\mu(k^\mu) = dk^\mu, \text{ where for 2-jet events, } d = \tau/m_t \quad (1)$$

For 3-jet events, d is more complicated — so only consider 2-jet events from here on.

Here, $\tau = \sqrt{\bar{t}^2 - \bar{r}_z^2}$ is the “longitudinal” proper time
and $m_t = \sqrt{E^2 - p_z^2}$ is the “transverse” mass

With $\delta_\Delta(x^\mu - \bar{x}^\mu)$ the distribution of production points about their mean,
and $H(\tau)$ the distribution of τ ,

Emission function is

$$S(x, k) = \int_0^\infty d\tau H(\tau) \delta_\Delta(x - dk) \rho_1(k) \quad (2)$$

In the plane-wave approximation, the two-pion distribution is

Yano and Koonin, PL **B78**(1978)556.

$$\rho_2(k_1, k_2) = \int d^4x_1 d^4x_2 S(x_1, k_1) S(x_2, k_2) (1 + \cos([k_1 - k_2][x_1 - x_2])) \quad (3)$$

Assume $\delta_\Delta(x - dk)$ is very narrow — a δ -function. Then (1),(2),(3) lead to

$$R_2(k_1, k_2) = 1 + \lambda \operatorname{Re} \tilde{H}^2 \left(\frac{Q^2}{2m_t} \right) \text{ where } \tilde{H}(\omega) = \int d\tau H(\tau) \exp(i\omega\tau)$$

BEC in the τ -model

Assume a Lévy distribution for $H(\tau)$

Since no particle production before the interaction, $H(\tau)$ is one-sided.

Characteristic function of $H(\tau)$ is

$$\tilde{H}(\omega) = \exp \left[-\frac{1}{2} (\Delta\tau |\omega|)^\alpha \left(1 - i \operatorname{sign}(\omega) \tan \left(\frac{\alpha\pi}{2} \right) \right) + i\omega\tau_0 \right], \quad \alpha \neq 1$$

where

- α is the index of stability
- τ_0 is the proper time of the onset of particle production
- $\Delta\tau$ is a measure of the width of the dist.

Then,

$$R_2(Q, \bar{m}_t) = \gamma \left[1 + \lambda \cos \left(\frac{\tau_0 Q^2}{\bar{m}_t} + \tan \left(\frac{\alpha\pi}{2} \right) \left(\frac{\Delta\tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \exp \left(- \left(\frac{\Delta\tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \right] (1 + \delta Q)$$

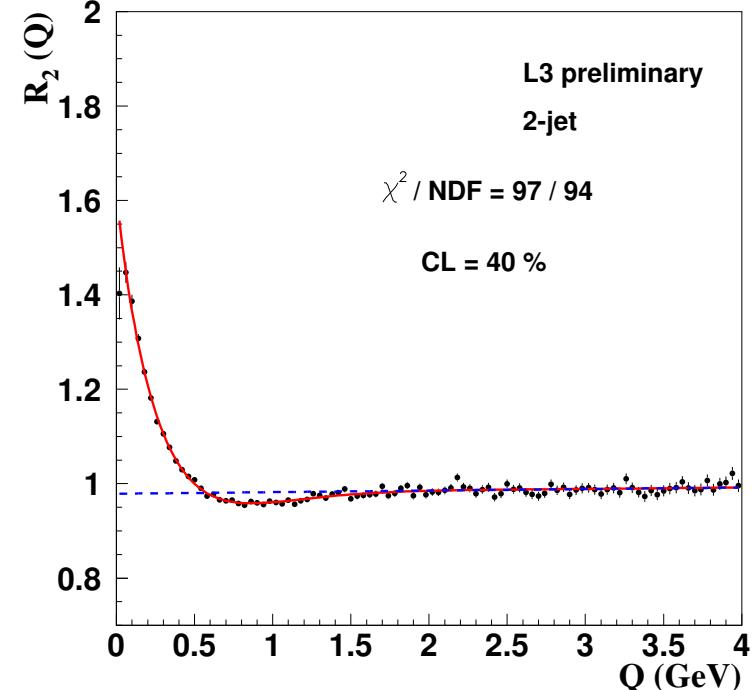
BEC in the τ -model

$$R_2(Q, \bar{m}_t) = \gamma \left[1 + \lambda \cos \left(\frac{\tau_0 Q^2}{\bar{m}_t} + \tan \left(\frac{\alpha \pi}{2} \right) \left(\frac{\Delta \tau Q^2}{2 \bar{m}_t} \right)^{\alpha} \right) \exp \left(- \left(\frac{\Delta \tau Q^2}{2 \bar{m}_t} \right)^{\alpha} \right) \right] (1 + \delta Q)$$

Before fitting in two dimensions (Q, \bar{m}_t) , assume an “average” \bar{m}_t dependence by introducing effective radius, $R = \sqrt{\Delta \tau / (2 \bar{m}_t)}$. Also assume $\tau_0 = 0$. Then:

$$R_2(Q) = \gamma \left[1 + \lambda \cos [(R_a Q)^{2\alpha}] \exp (-(R Q)^{2\alpha}) \right] (1 + \delta Q), \quad R_a^{2\alpha} = \tan \left(\frac{\alpha \pi}{2} \right) R^{2\alpha}$$

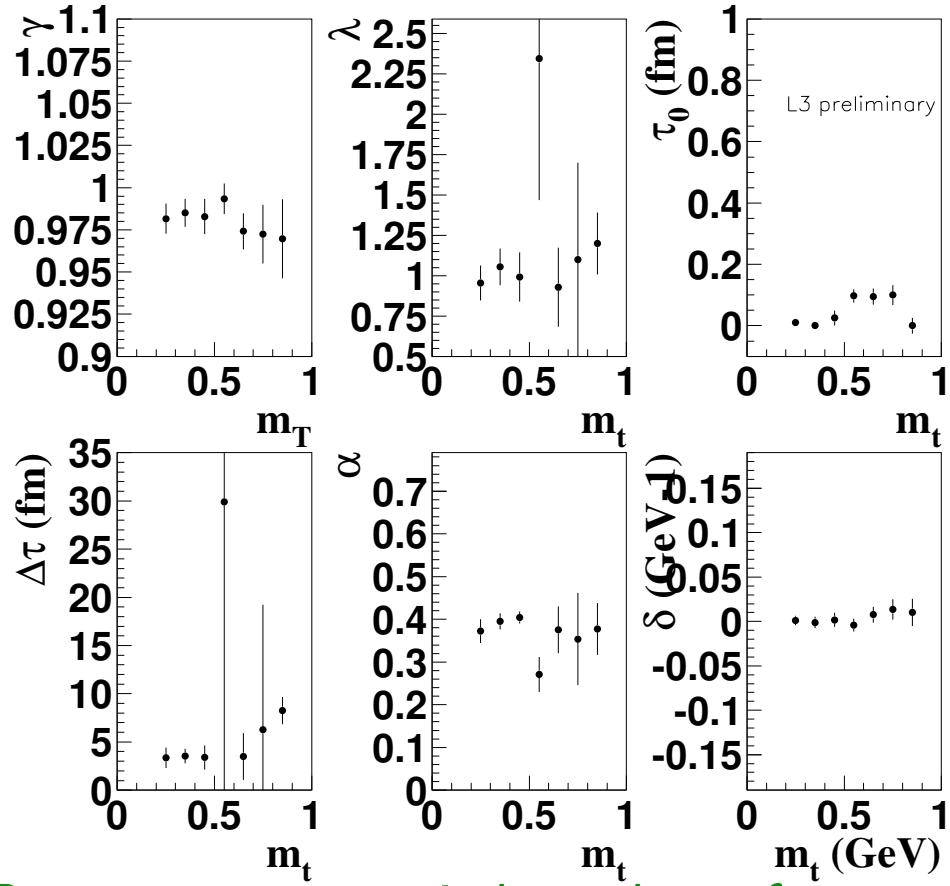
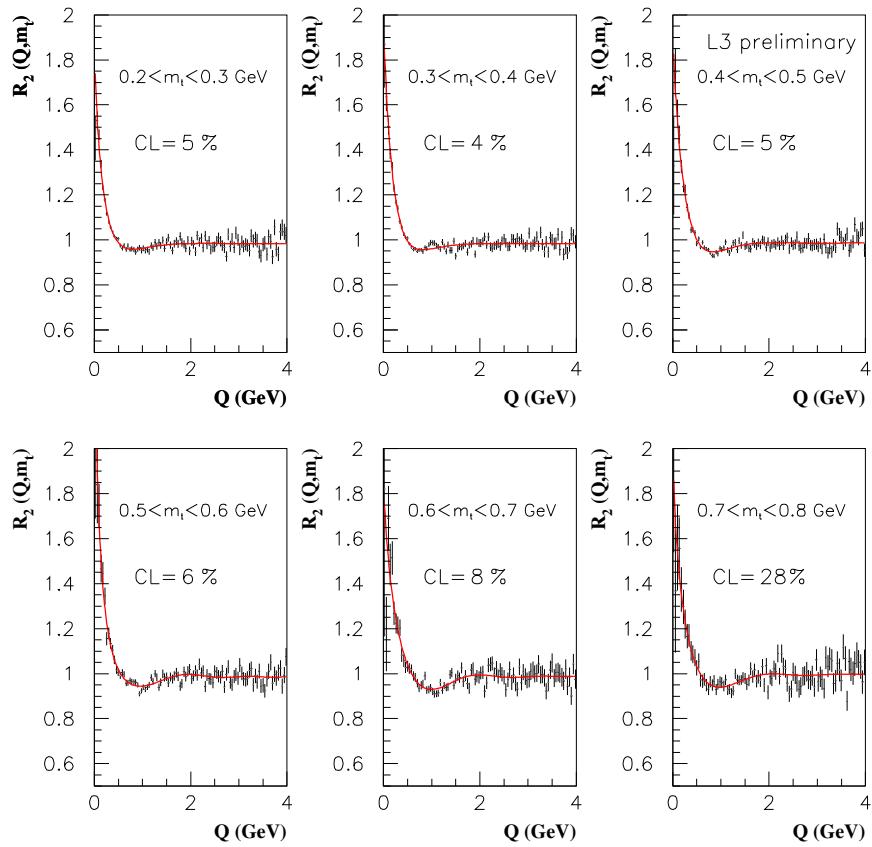
| parameter | R_a free | $R_a^{2\alpha} = \tan \left(\frac{\alpha \pi}{2} \right) R^{2\alpha}$ |
|---------------------|-------------------|--|
| α | 0.42 ± 0.02 | 0.42 ± 0.01 |
| λ | 0.67 ± 0.03 | 0.67 ± 0.03 |
| R (fm) | 0.79 ± 0.04 | 0.79 ± 0.03 |
| R_a (fm) | 0.59 ± 0.03 | — |
| δ | 0.003 ± 0.002 | 0.003 ± 0.001 |
| γ | 0.979 ± 0.005 | 0.979 ± 0.005 |
| χ^2/DoF | 97/94 | 97/95 |
| CL | 40% | 42% |



R_a free or not gives same results. – Good CL

BEC in the τ -model

Next we fit the full formula in m_t bins:



CLs are reasonable

Parameters consistent with “average m_t ” fit:

these fits: $\alpha \approx 0.38 \pm 0.04$ “average m_t ” fit: $\alpha = 0.42 \pm 0.02$

$$\Delta\tau \approx 3.6 \pm 0.6 \text{ fm}$$

$$\Delta\tau \approx 3.5 \text{ fm} - \text{from } R = 0.79 \text{ fm and } \bar{m}_t = 0.563 \text{ GeV}$$

Parameters approx. independent of m_t

$$\tau_0 \approx 0$$

From α to α_s

- LLA parton shower leads to a fractal in momentum space
fractal dimension is related to α_s
- Lévy dist. arises naturally from a fractal, or random walk, or anomalous diffusion
- strong momentum-space/configuration space correlation of τ -model
 \implies fractal in configuration space with same α
- generalized LPHD suggests particle dist. has same properties as gluon dist.
- Putting this all together leads to

Gustafson *et al.*

Metzler and Klafter, Phys.Rep.339(2000)

Csörgő *et al.*

$$\alpha_s = \frac{2\pi}{3}\alpha^2$$

- Using our value of $\alpha = 0.42 \pm 0.02$ yields $\alpha_s = 0.37 \pm 0.04$
- This value is reasonable for a scale of 1–2 GeV,
where production of hadrons takes place
cf., from τ decays $\alpha_s(m_\tau \approx 1.8 \text{ GeV}) = 0.35 \pm 0.03$

PDG

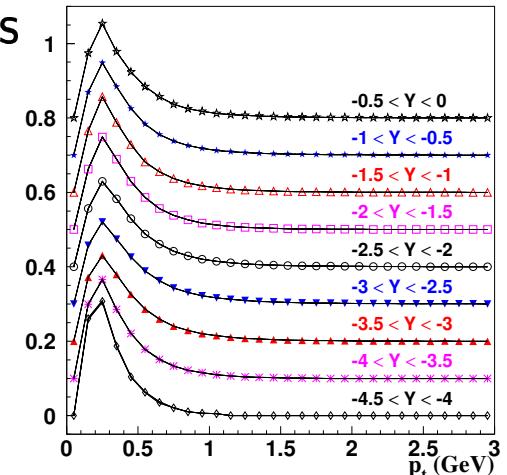
Emission function of 2-jet events

In the τ -model, the emission function in configuration space is

$$S(x) = \frac{d^4 n}{d\tau d^3 r} = \left(\frac{m_t}{\tau}\right)^3 H(\tau) \rho_1 \left(k = \frac{rm_t}{\tau}\right)$$

For simplicity, assume $S(r, z, t) = G(\eta) I(r) H(\tau)$
 $(\eta = \text{space-time rapidity})$

Strongly correlated $x, k \Rightarrow \eta = y$ and $r = p_t \tau / m_t$

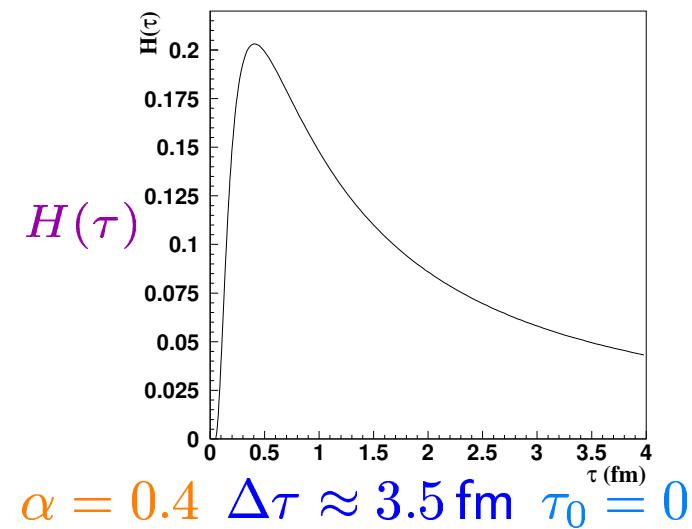


Factorization y, p_t OK

$$G(\eta) = N_y(\eta) \quad I(r) = \left(\frac{m_t}{\tau}\right)^3 N_{p_t}(rm_t/\tau)$$

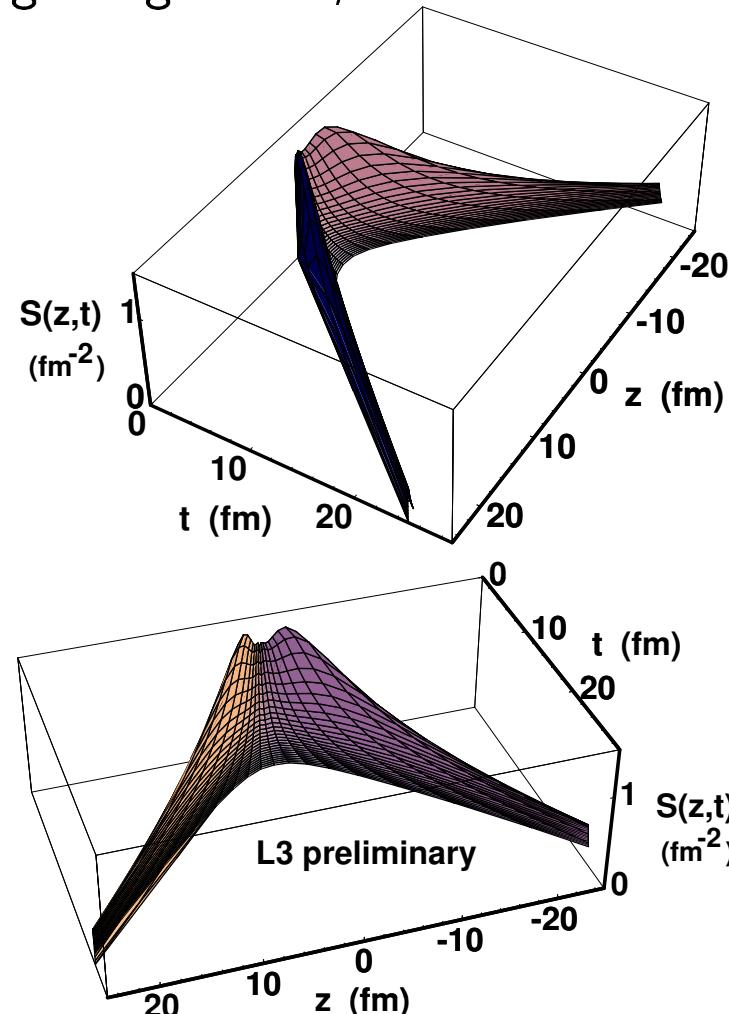
$(N_y, N_{p_t}$ are inclusive single-particle distributions)

So, using experimental N_y, N_{p_t} distributions
and $H(\tau)$ from BEC fits,
we can reconstruct the full emission function, S .



Emission function of 2-jet events

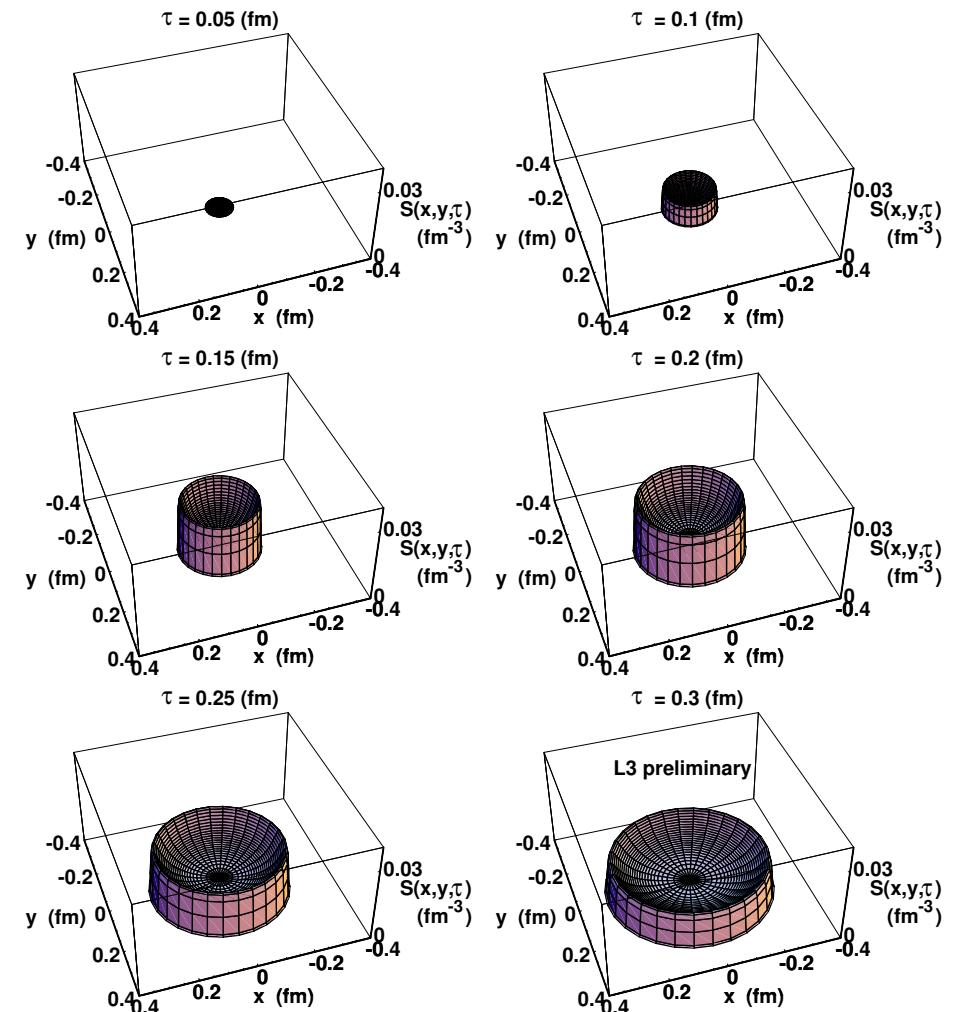
Integrating over r ,



“Boomerang shape”

Particle production is close to the light-cone

Integrating over z ,



Expanding ring

Summary

- Parametrizing R_2 as a function of Q only is a reasonably good approximation
- Symmetric Gaussian, Edgeworth, Lévy parametrizations of R_2 do not fit well
- The τ -model with a one-sided Lévy proper-time distribution leads to $R_2(Q, m_t)$, which successfully fits R_2 for 2-jet events
 - ★ both Q - and m_t -dependence described correctly
 - ★ Note: we found $\Delta\tau$ to be independent of m_t
 $\Delta\tau$ enters R_2 as $\Delta\tau Q^2/m_t$
In Gaussian parametrization, R enters R_2 as $R^2 Q^2$
Thus $\Delta\tau$ independent of m_t corresponds to $R \propto 1/\sqrt{m_t}$
- fractal dimension associated with Lévy α relates α to α_s
 $\alpha = 0.42 \pm 0.02$ corresponds to $\alpha_s = 0.37 \pm 0.04$, reasonable for a scale of 1–2 GeV
- Emission function shaped like a boomerang in z - t and an expanding ring in x - y
Particle production is close to the light-cone

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