Recent L3 results on Parametrization of BEC in e^+e^- Annihilation and Reconstruction of the Source Function

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II Workshop on Particle Correlation and Femtoscopy

São Paulo

10 September 2006

Introduction — BEC

q-particle density $\rho_q(p_1,...,p_q) = \frac{1}{\sigma_{\text{tot}}} \frac{\mathrm{d}^q \sigma_q(p_1,...,p_q)}{\mathrm{d}p_1 ... \mathrm{d}p_q}$, where σ_q is inclusive cross section

2-particle correlation: $\dfrac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)}$

To study only BEC, not all correlations,
let $\rho_0(p_1,p_2)$ be the 2-particle density if no BEC
($= \rho_2$ of the 'reference sample') and define

$$R_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \cdot \frac{\rho_1(p_1)\rho_1(p_2)}{\rho_0(p_1,p_2)} = \frac{\rho_2(p_1,p_2)}{\rho_0(p_1,p_2)}$$

Since 2-π BEC only at small Q

$$Q = \sqrt{-(p_1-p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$$

integrate over other variables: $R_2(Q) = \dfrac{\rho(Q)}{\rho_0(Q)}$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + |\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$
– Fourier transform of $S(x)$

Assuming $S(x)$ is a Gaussian with radius $r \Longrightarrow$
$R_2(Q) = 1 + e^{-Q^2 r^2}$

$$R_2(Q) = \gamma \cdot (1 + \lambda G(Q)) \cdot B \,, \quad G(Q) = e^{-Q^2 r^2}$$

- $\gamma =$ normalization (≈ 1)
- B tries to account for long-range correlations inadequately removed by reference sample, *e.g.*, $B = 1 + \delta Q$

Assumes

- incoherent average over source
  λ tries to account for
  - partial coherence
  - multiple (distinguishable) sources, long-lived resonances
  - pion purity
- spherical (radius r) Gaussian density of particle emitters
  seems unlikely in e^+e^- annihilation—jets
- static source, *i.e.*, no t-dependence
  certainly wrong

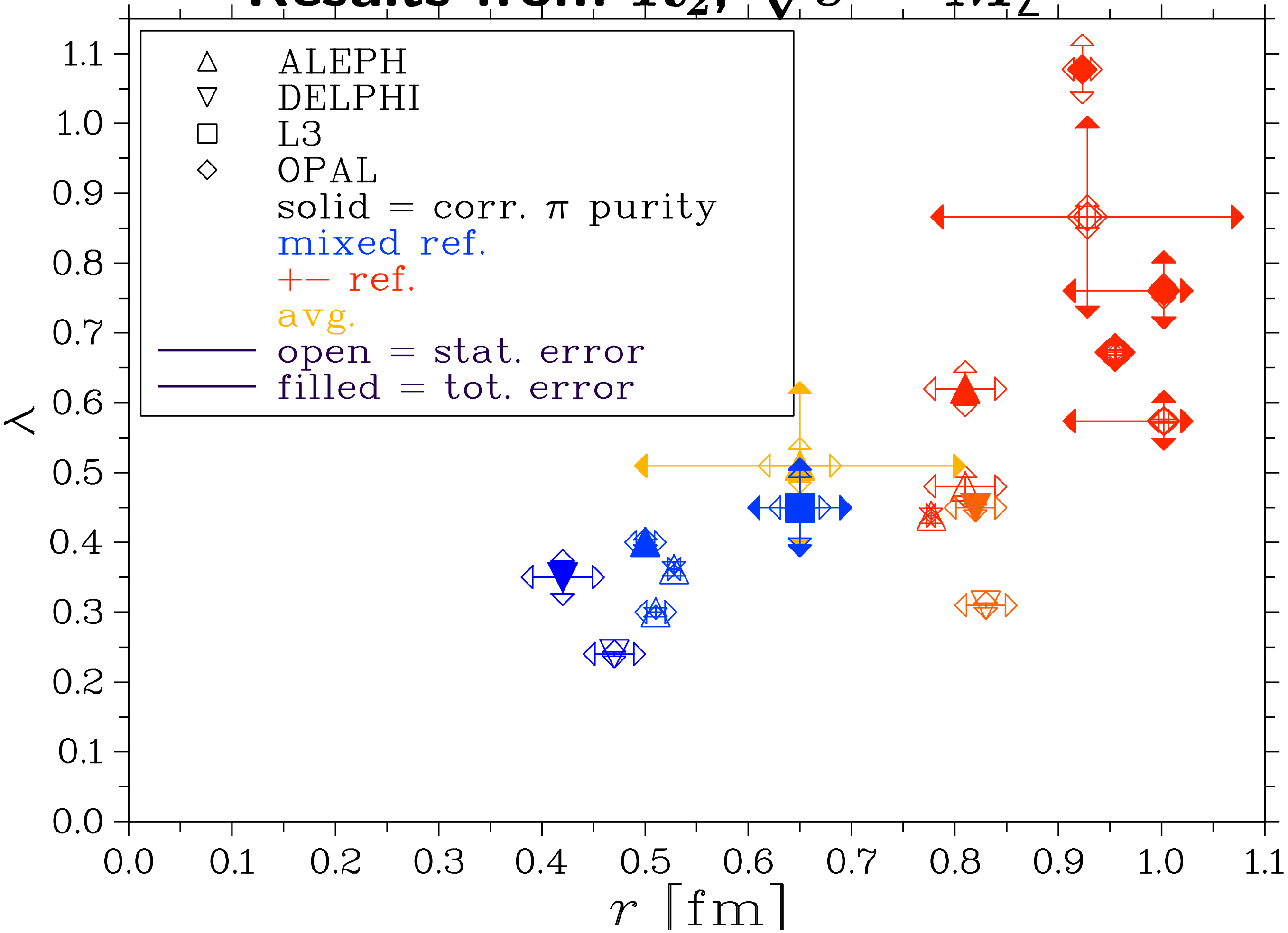
Nevertheless, this Gaussian formula is the most often used parametrization
And it works fairly well
But what do the values of λ and r actually mean?

When Gaussian parametrization does not fit well, can expand about the Gaussian (Edgeworth expansion).
Keeping only the lowest-order non-Gaussian term,
$\exp\left(-Q^2 r^2\right)$ becomes

$$\exp\left(-Q^2 r^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(Qr)\right]$$

(H_3 is third-order Hermite polynomial)

Results from R_2, $\sqrt{s} = M_Z$



– correction for π purity increases λ

– mixed ref. gives smaller λ, r than $+-$ ref.

Elongation of the source

The usual parametrization assumes a symmetric Gaussian source
But, there is no reason to expect this symmetry in $e^+e^- \rightarrow q\bar{q}$.
Therefore, do a 3-dim. analysis in the Longitudinal Center of Mass System

Advantages of LCMS:

$$
\begin{aligned}
Q^2 &= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 - (\Delta E)^2 \\
&= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 (1-\beta^2) \qquad \text{where } \beta \equiv \frac{p_{\mathrm{out}\,1} + p_{\mathrm{out}\,2}}{E_1 + E_2}
\end{aligned}
$$

Thus, the energy difference, and therefore the difference in emission time of the pions couples only to the out-component, Q_{out}.

So, Q_{L} and Q_{side} reflect only spatial dimensions of the source
Q_{out} reflects a mixture of spatial and temporal dimensions.

Parametrization: $R_2(Q_{\mathrm{L}}, Q_{\mathrm{out}}, Q_{\mathrm{side}}) = \gamma \cdot (1 + \lambda G) \cdot B$
where $G =$ azimuthally symmetric Gaussian:

$$G = \exp\left(-r_{\mathrm{L}}^2 Q_{\mathrm{L}}^2 - r_{\mathrm{out}}^2 Q_{\mathrm{out}}^2 - r_{\mathrm{side}}^2 Q_{\mathrm{side}}^2 + 2\rho_{\mathrm{L,out}} r_{\mathrm{L}} r_{\mathrm{out}} Q_{\mathrm{L}} Q_{\mathrm{out}}\right)$$

$$B = (1 + \delta Q_{\mathrm{L}} + \varepsilon Q_{\mathrm{out}} + \xi Q_{\mathrm{side}})$$

Elongation Results in the LCMS (L3)

parameter	Gaussian	Edgeworth
λ	$0.41 \pm 0.01^{+0.02}_{-0.19}$	$0.54 \pm 0.02^{+0.04}_{-0.26}$
r_{L} (fm)	$0.74 \pm 0.02^{+0.04}_{-0.03}$	$0.69 \pm 0.02^{+0.04}_{-0.03}$
r_{out} (fm)	$0.53 \pm 0.02^{+0.05}_{-0.06}$	$0.44 \pm 0.02^{+0.05}_{-0.06}$
r_{side} (fm)	$0.59 \pm 0.01^{+0.03}_{-0.13}$	$0.56 \pm 0.02^{+0.03}_{-0.12}$
$r_{\mathsf{out}}/r_{\mathsf{L}}$	$0.71 \pm 0.02^{+0.05}_{-0.08}$	$0.65 \pm 0.03^{+0.06}_{-0.09}$
$r_{\mathsf{side}}/r_{\mathsf{L}}$	$0.80 \pm 0.02^{+0.03}_{-0.18}$	$0.81 \pm 0.02^{+0.03}_{-0.19}$
κ_{L}	–	$0.5 \pm 0.1^{+0.1}_{-0.2}$
κ_{out}	–	$0.8 \pm 0.1 \pm 0.3$
κ_{side}	–	$0.1 \pm 0.1 \pm 0.3$
δ	$0.025 \pm 0.005^{+0.014}_{-0.015}$	$0.036 \pm 0.007^{+0.012}_{-0.023}$
ϵ	$0.005 \pm 0.005^{+0.034}_{-0.012}$	$0.011 \pm 0.005^{+0.037}_{-0.012}$
ξ	$-0.035 \pm 0.005^{+0.031}_{-0.024}$	$-0.022 \pm 0.006^{+0.020}_{-0.025}$
χ^2/DoF	2314/2189	2220/2186
C.L. (%)	3.1	30

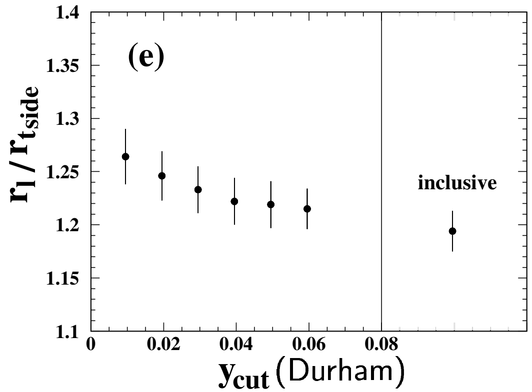
- $\rho_{\mathsf{L,out}} = 0$ So fix to 0.
- Edgeworth fit significantly better than Gaussian
- $r_{\mathsf{side}}/r_{\mathsf{L}} < 1$ more than 5 std. dev. Elongation along thrust axis
- Models which assume a spherical source are too simple.

Elongation Results

			Gauss / Edgeworth	2-D r_t/r_L	3-D r_{side}/r_L
DELPHI	mixed	2-jet	Gauss	$0.62 \pm 0.02 \pm 0.05$	—
ALEPH	mixed	2-jet	Gauss	$0.61 \pm 0.01 \pm 0.??$	—
	+,−	2-jet	Gauss	$0.91 \pm 0.02 \pm 0.??$	—
	mixed	2-jet	Edgeworth	$0.68 \pm 0.01 \pm 0.??$	—
	+,−	2-jet	Edgeworth	$0.84 \pm 0.02 \pm 0.??$	—
OPAL	+,−	2-jet	Gauss	—	$0.82 \pm 0.02 \pm^{0.01}_{0.05}$
L3	mixed	all	Gauss	—	$0.80 \pm 0.02 \pm^{0.03}_{0.18}$
	mixed	all	Edgeworth	—	$0.81 \pm 0.02 \pm^{0.03}_{0.19}$

∼20% elongation along thrust axis

(ZEUS finds similar results in ep)

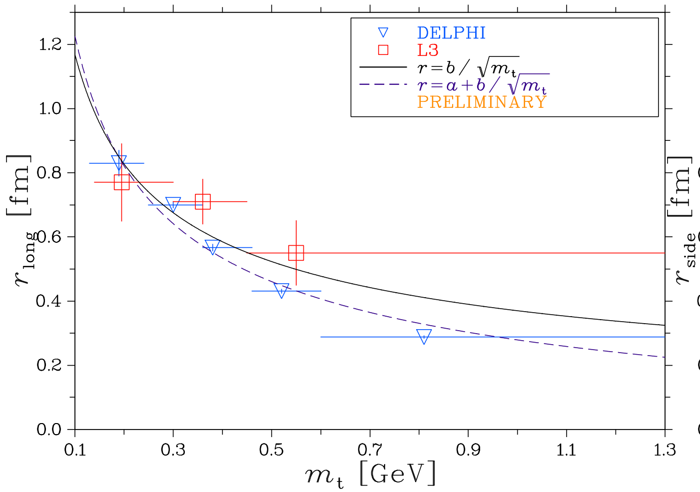


OPAL:

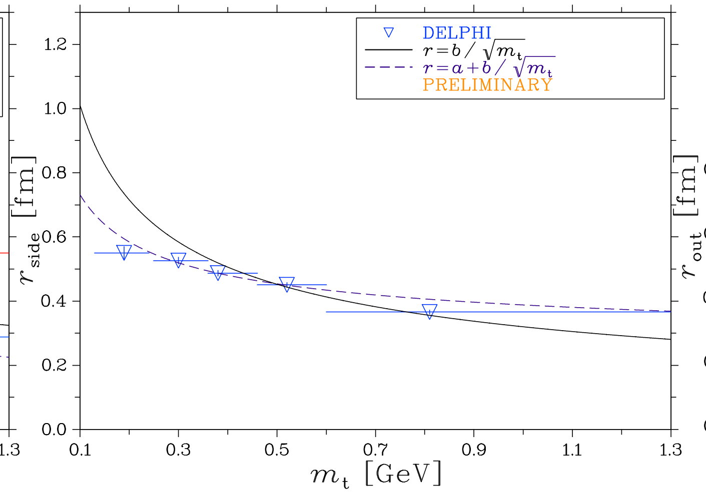
Elongation larger for narrower jets

Transverse Mass dependence of r

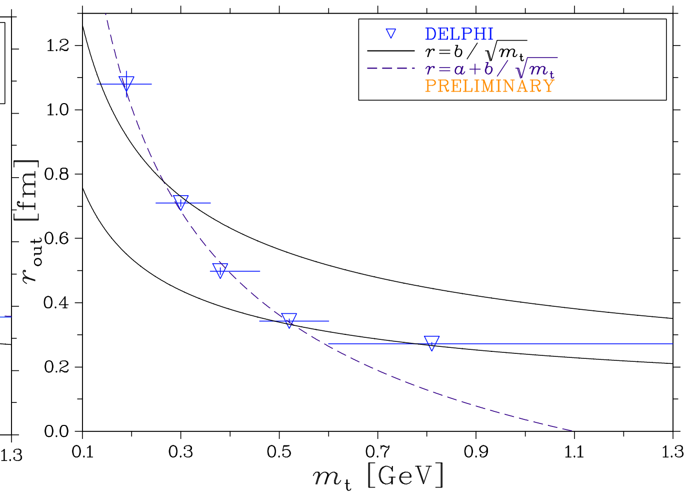
longitudinal



side



out



r decreases with m_t for all directions

Smirnova&Lörstad,7th Int.Workshop on Correlations and Fluctuations (1996)

Van Dalen,8th Int.Workshop on Correlations and Fluctuations (1998)

but more like $\quad r = a + b/\sqrt{m_t}$ - - - -

than like $\quad r = b/\sqrt{m_t}$ ———

New L3 Results

- $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about 10^6 events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\text{cut}} = 0.006$
- use mixed events for reference sample

Beyond the Symmetric Gaussian — Non-Symmetric?

Decompose Q in various ways in the LCMS: $R_2 = \gamma \cdot (1 + \lambda G) \cdot B$

1. $G = \exp\left(-r_\mathrm{L}^2 Q_\mathrm{L}^2 - r_\mathrm{side}^2 Q_\mathrm{side}^2 - r_\mathrm{o}^2 (Q_\mathrm{out}^2 - (\Delta E)^2)\right)$
   $G = \exp\left(-r_\mathrm{L}^2 Q_\mathrm{L}^2 - r_\mathrm{side}^2 Q_\mathrm{side}^2 - r_\mathrm{out}^2 Q_\mathrm{out}^2\right)$

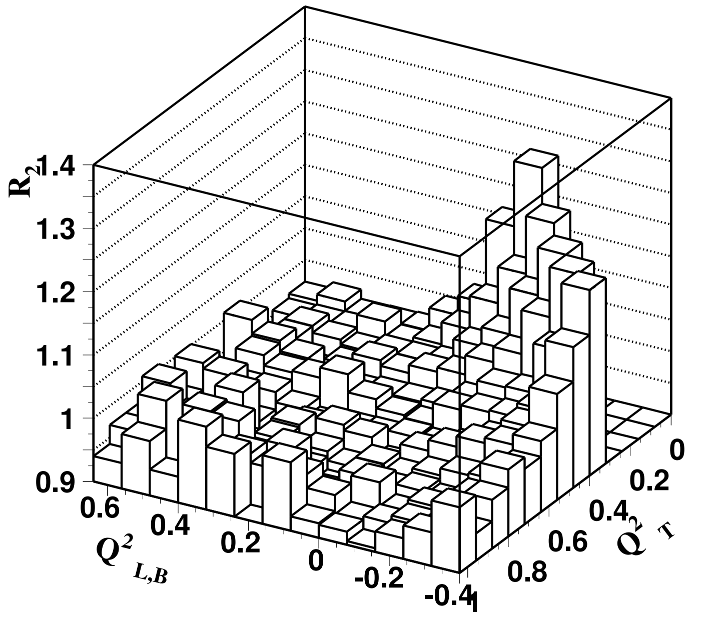
2. $G = \exp\left(-r_\ell^2 (Q_\mathrm{L}^2 - (\Delta E)^2) - r_\mathrm{T}^2 (Q_\mathrm{side}^2 + Q_\mathrm{out}^2)\right)$

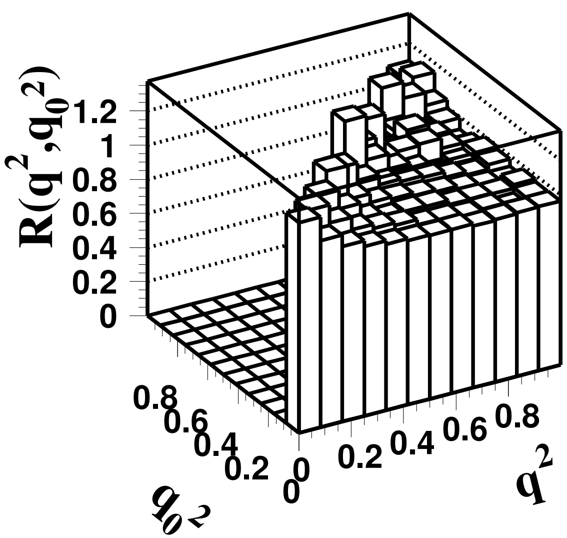
3. $G = \exp\left(-r_1^2 (Q_\mathrm{L}^2 + Q_\mathrm{side}^2 + Q_\mathrm{out}^2) - r_0^2 (-(\Delta E)^2)\right)$

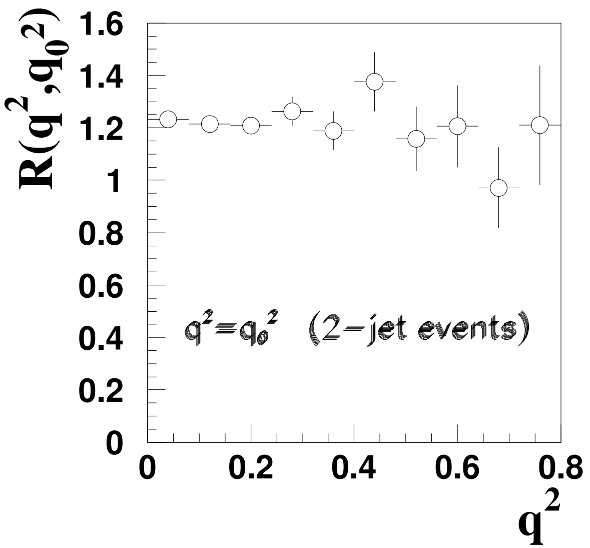
all events				
1	$r_\mathrm{L} = 0.74 \pm 0.02$ fm	$r_\mathrm{side} = 0.59 \pm 0.01$ fm	CL$= 3\%$	$\sim$20% elongation
2	$r_\ell = 0.54 \pm 0.01$ fm	$r_\mathrm{T} = 0.57 \pm 0.01$ fm	CL$< 10^{-5}$	$\approx$
3	$r_1 = 0.57 \pm 0.07$ fm	$r_0 = 0.56 \pm 0.08$ fm	CL$= 0.2\%$	$=$
2-jet events (Durham $y_\mathrm{cut} = 0.006$)				
1	OPAL			$\sim$20% elongation
2	$r_\ell = 0.51 \pm 0.02$ fm	$r_\mathrm{T} = 0.55 \pm 0.02$ fm	CL$= 10^{-4}$	$\approx$
3	$r_1 = 0.55 \pm 0.08$ fm	$r_0 = 0.53 \pm 0.09$ fm	CL$= 42\%$	$=$

CLs none too good, but $R_2 \approx R_2(Q)$ confirms TASSO, Z.Phys.**C71**(1986)405

Beyond the Symmetric Gaussian — Non-Symmetric?







$$Q^2_{\mathsf{L,B}} = Q^2_{\mathsf{L}} - (\Delta E)^2$$
$$Q^2_{\mathsf{T}} = Q^2_{\mathsf{side}} + Q^2_{\mathsf{out}}$$

$$q_0^2 = (\Delta E)^2$$
$$\mathbf{q}^2 = Q^2_{\mathsf{L}} + Q^2_{\mathsf{side}} + Q^2_{\mathsf{out}}$$

Conclusion: $R_2 \approx R_2(Q)$

Only consider $R_2(Q)$ for rest of talk.

Beyond the Gaussian

Assume static distribution of pion emitters in configuration space, $f(r)$
with characteristic function (Fourier transform), $\tilde{f}(Q)$

Then $\quad R_2 = \gamma \cdot \left[1 + \lambda |\tilde{f}(Q)|^2\right] \cdot (1 + \delta Q)$

- $f(r)$ is Gaussian with mean $\mu = 0$ and variance R^2

  $\tilde{f}(Q) = \exp\left(\imath\mu Q - \frac{(RQ)^2}{2}\right) \qquad R_2 = \gamma \cdot \left[1 + \lambda \exp\left(-(RQ)^2\right)\right] \cdot (1 + \delta Q)$

- approximately Gaussian – Edgeworth expansion

  $$R_2 = \gamma \cdot \left[1 + \lambda \exp\left(-(RQ)^2\right) \cdot \left[1 + \tfrac{\kappa}{3!} H_3(RQ)\right]\right] \cdot (1 + \delta Q)$$

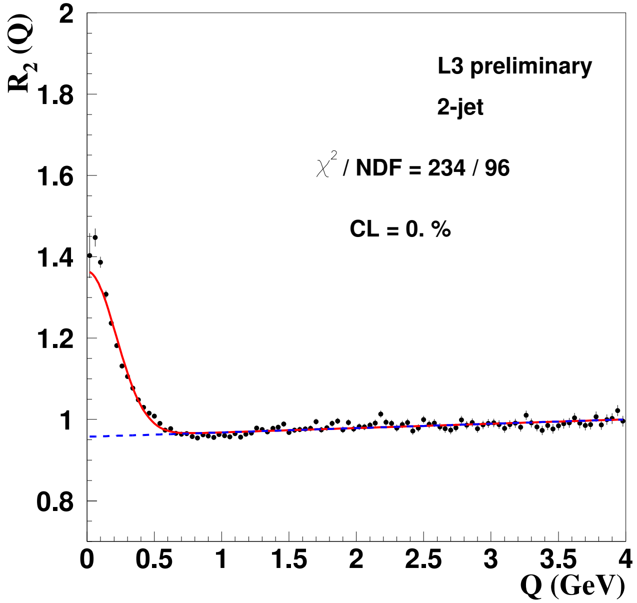
- $f(r)$ is a symmetric Lévy stable distribution with location parameter $x_0 = 0$,
  'width' parameter R, and 'index of stability', $0 < \alpha \leq 2$

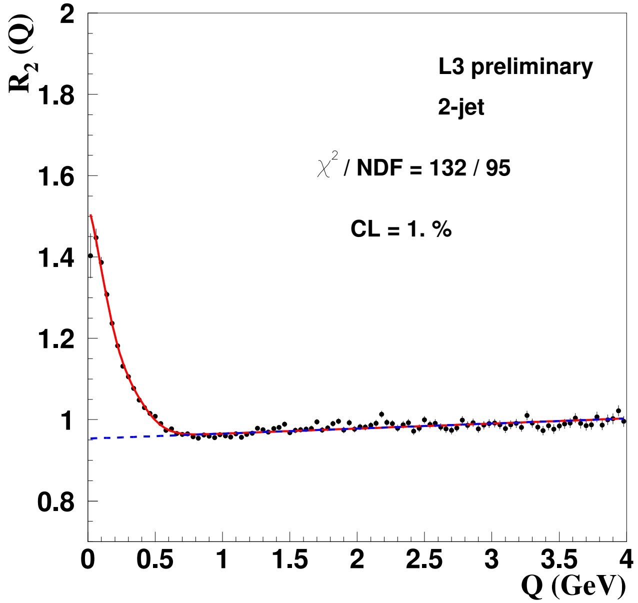
  $\tilde{f}(Q) = \exp\left(\imath x_0 Q - \frac{|RQ|^\alpha}{2}\right) \qquad R_2 = \gamma \cdot \left[1 + \lambda \exp\left(-(RQ)^\alpha\right)\right] \cdot (1 + \delta Q)$

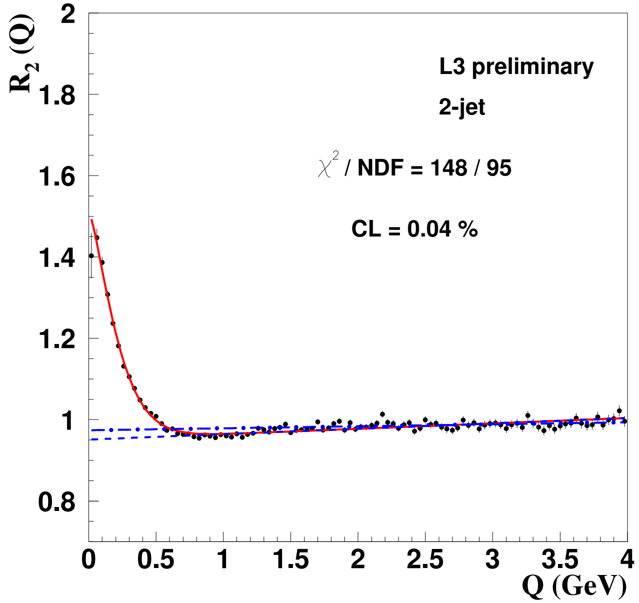
  $\alpha = 2$ corresponds to Gaussian with $\mu = x_0$, variance R^2
  $\alpha = 1$ corresponds to a Cauchy distribution for $f(r)$

Beyond the Gaussian

Gaussian	Edgeworth	Lévy







Far from Gaussian.	$\kappa = 0.71 \pm 0.06$	$\alpha = 1.34 \pm 0.04$

Poor CLs. Edgeworth and Lévy better than Gaussian, but still poor
Problem is the dip of R_2 in the region $0.6 < Q < 1.5$GeV

Same conclusions for 3-jet events, all events.

Beyond the Gaussian

Summary:

- We have assumed a static source — certainly wrong
- BEC depends (at least approximately) only on Q
- r decreases with m_{t}, approximately as $1/\sqrt{m_{\mathrm{t}}}$
  may be due to correlation between momentum and production point   Białas *et al.*

Let's turn to a model incorporating these points.

BEC in the τ-model

The τ-model assumes avg. production point proportional to momentum:

Csörgő and Zimányi, Nucl.Phys.**A517**(1990)588.

$$\overline{x}^\mu(k^\mu) = dk^\mu \text{ , where for 2-jet events, } d = \tau / m_\mathrm{t} \qquad (1)$$

For 3-jet events, d is more complicated — so only consider 2-jet events from here on.

Here, $\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the "longitudinal" proper time

and $m_\mathrm{t} = \sqrt{E^2 - p_z^2}$ is the "transverse" mass

With $\delta_\Delta(x^\mu - \overline{x}^\mu)$ the distribution of production points about their mean,
and $H(\tau)$ the distribution of τ,
Emission function is

$$S(x,k) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - dk)\rho_1(k) \qquad (2)$$

In the plane-wave approximation, the two-pion distribution is Yano and Koonin, PL **B78**(1978)556.

$$\rho_2(k_1,k_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,k_1)S(x_2,k_2)\left(1+\cos\left([k_1-k_2][x_1-x_2]\right)\right) \qquad (3)$$

Assume $\delta_\Delta(x - dk)$ is very narrow — a δ-function. Then (1),(2),(3) lead to

$$R_2(k_1,k_2) = 1 + \lambda \,\mathrm{Re}\widetilde{H}^2\left(\frac{Q^2}{2\overline{m}_\mathrm{t}}\right) \quad \text{where } \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

Assume a Lévy distribution for $H(\tau)$
Since no particle production before the interaction, $H(\tau)$ is one-sided.
Characteristic function of $H(\tau)$ is

$$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathsf{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right], \quad \alpha \neq 1$$

where

- α is the index of stability
- τ_0 is the proper time of the onset of particle production
- $\Delta\tau$ is a measure of the width of the dist.

Then,

$$R_2(Q, \overline{m}_{\mathrm{t}}) = \gamma\left[1 + \lambda\cos\left(\frac{\tau_0 Q^2}{\overline{m}_{\mathrm{t}}} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2\overline{m}_{\mathrm{t}}}\right)^{\alpha}\right)\exp\left(-\left(\frac{\Delta\tau Q^2}{2\overline{m}_{\mathrm{t}}}\right)^{\alpha}\right)\right](1 + \delta Q)$$

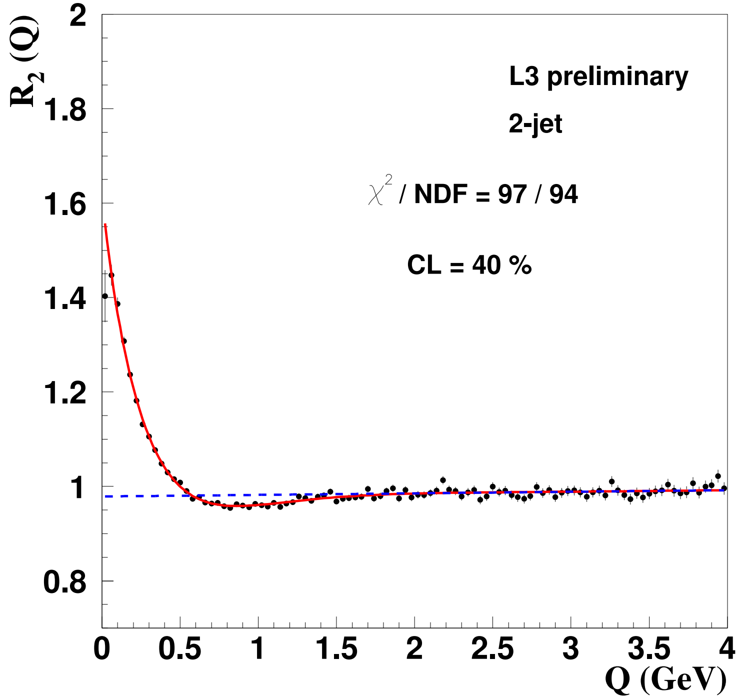
BEC in the τ-model – 2-jet events

$$R_2(Q,\overline{m}_t) = \gamma\left[1+\lambda\cos\left(\frac{\tau_0 Q^2}{\overline{m}_t}+\tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right)\exp\left(-\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right)\right](1+\delta Q)$$

Before fitting in two dimensions $(Q,\overline{m}_t)$, assume an "average" $\overline{m}_t$ dependence by introducing effective radius, $R=\sqrt{\Delta\tau/(2\overline{m}_t)}$. Also assume $\tau_0=0$. Then:

$$R_2(Q) = \gamma\left[1+\lambda\cos\left[(R_a Q)^{2\alpha}\right]\exp\left(-(RQ)^{2\alpha}\right)\right](1+\delta Q)\ , \qquad R_a^{2\alpha}=\tan\left(\frac{\alpha\pi}{2}\right)R^{2\alpha}$$

parameter	R_a free	$R_a^{2\alpha}=\tan\left(\frac{\alpha\pi}{2}\right)R^{2\alpha}$
α	0.42 ± 0.02	0.42 ± 0.01
λ	0.67 ± 0.03	0.67 ± 0.03
R (fm)	0.79 ± 0.04	0.79 ± 0.03
R_a (fm)	0.59 ± 0.03	—
δ	0.003 ± 0.002	0.003 ± 0.001
γ	0.979 ± 0.005	0.979 ± 0.005
χ^2/DoF	97/94	97/95
CL	40%	42%



R_a free or not gives same results. – Good CL

BEC in the τ-model – 3-jet events

$$R_2(Q,\overline{m}_\mathrm{t}) = \gamma\left[1+\lambda\cos\left(\frac{\tau_0 Q^2}{\overline{m}_\mathrm{t}} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2\overline{m}_\mathrm{t}}\right)^\alpha\right)\exp\left(-\left(\frac{\Delta\tau Q^2}{2\overline{m}_\mathrm{t}}\right)^\alpha\right)\right](1+\delta Q)$$

Although derived for 2-jet events, *i.e.*, using $d=\tau/m_\mathrm{t}$, lets try it on 3-jet data

Assuming an effective radius, $R=\sqrt{\Delta\tau/(2\overline{m}_\mathrm{t})}$. and $\tau_0=0$. Then:

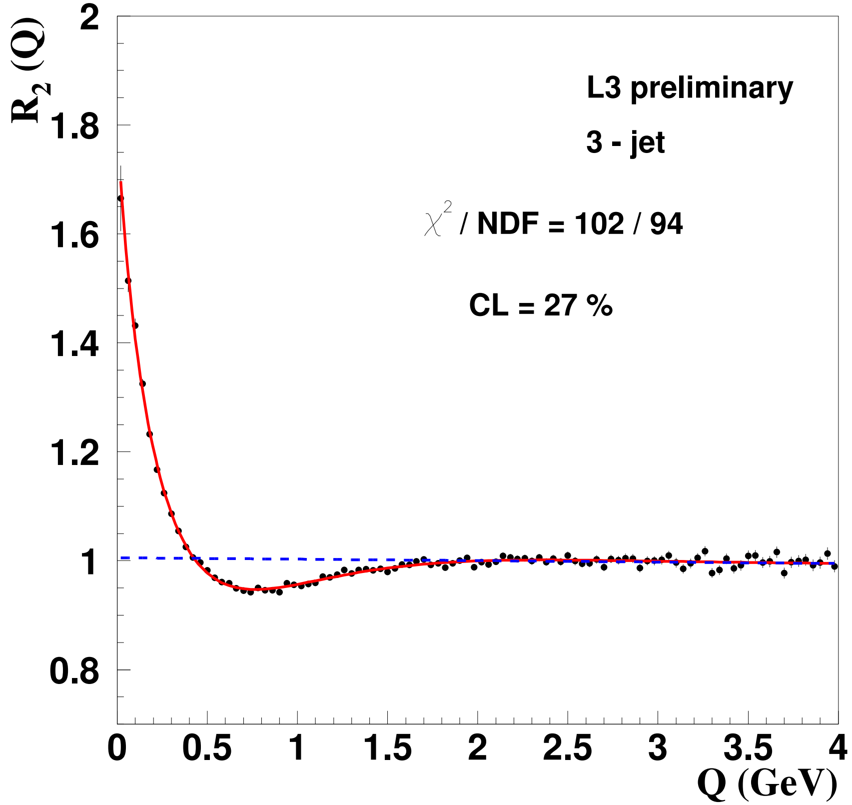
$$R_2(Q) = \gamma\left[1+\lambda\cos\left[(R_\mathrm{a}Q)^{2\alpha}\right]\exp\left(-(RQ)^{2\alpha}\right)\right](1+\delta Q)\ , \qquad R_\mathrm{a}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right)R^{2\alpha}$$

parameter	R_a free	$R_\mathrm{a}^{2\alpha}=\tan\left(\frac{\alpha\pi}{2}\right)R^{2\alpha}$
α	0.35 ± 0.01	0.44 ± 0.01
λ	0.84 ± 0.04	0.77 ± 0.04
R (fm)	0.89 ± 0.03	0.84 ± 0.04
R_a (fm)	0.88 ± 0.04	—
δ	-0.003 ± 0.002	0.010 ± 0.001
γ	1.001 ± 0.005	0.972 ± 0.001
χ^2/DoF	102/94	174/95
CL	27%	10^{-6}

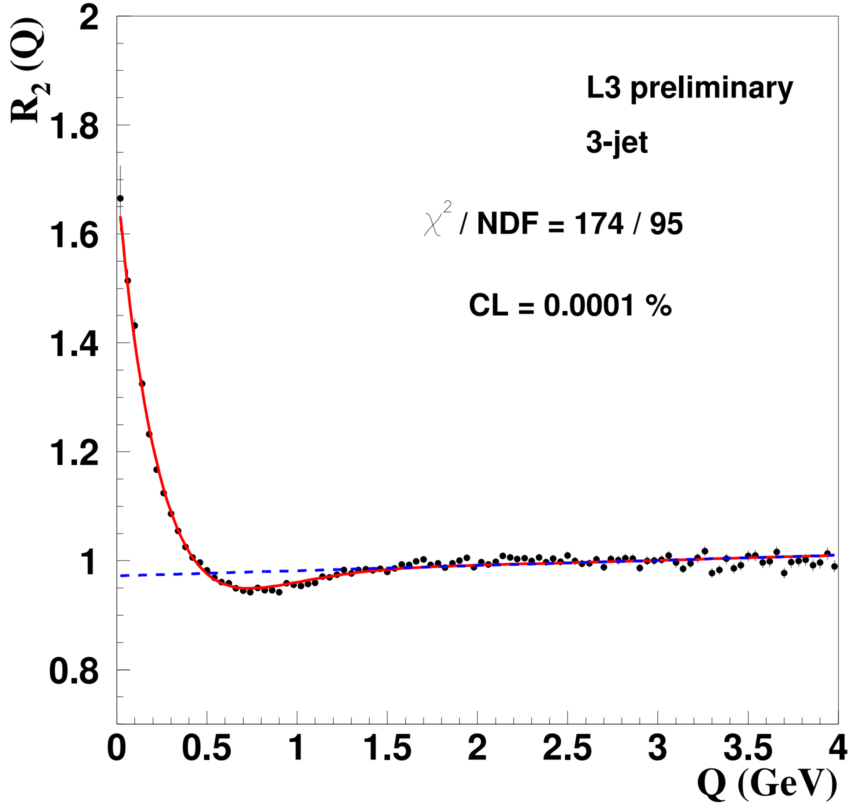
R_a free fits well, but poor fit with R_a set to model relationship

BEC in the τ-model – 3-jet events

R_a free



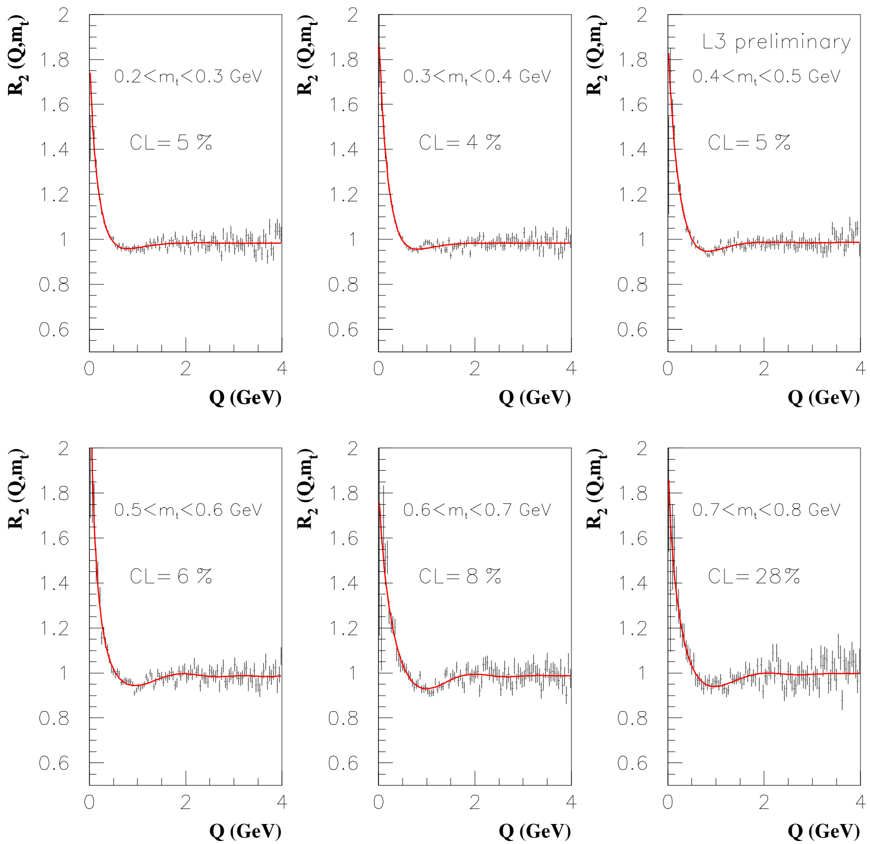
R_a set to model

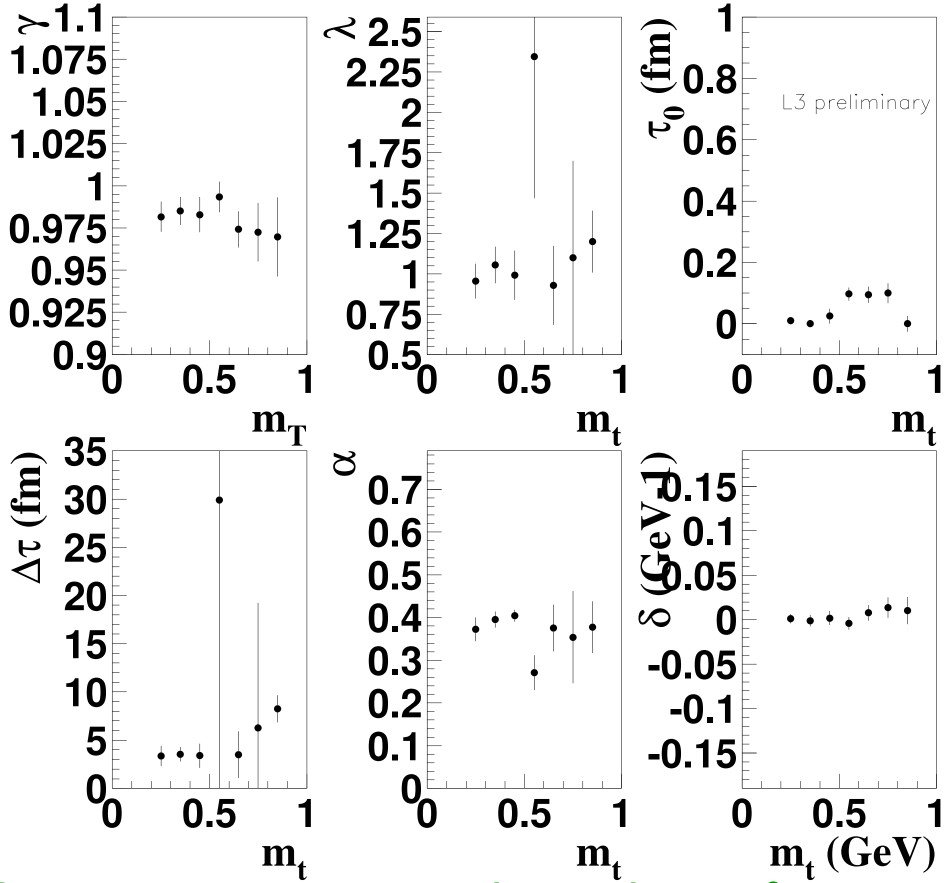


R_a free fits well, but poor fit with R_a set to model relationship

BEC in the τ-model – 2-jet events

Next we fit the full formula in m_t bins:





CLs are reasonable

Parameters approx. independent of m_t

Parameters consistent with "average m_t" fit: $\tau_0 \approx 0$

these fits: $\alpha \approx 0.38 \pm 0.04$ "average m_t" fit: $\alpha = 0.42 \pm 0.02$

$\Delta\tau \approx 3.6 \pm 0.6\,\text{fm}$ $\Delta\tau \approx 3.5\,\text{fm}$ – from $R = 0.79\,\text{fm}$ and $\overline{m}_t = 0.563\,\text{GeV}$

From α to α_s

- LLA parton shower leads to a fractal in momentum space
  fractal dimension is related to α_s Gustafson *et al.*
- Lévy dist. arises naturally from a fractal, or random walk, or anomalous diffusion

  Metzler and Klafter, Phys.Rep.**339**(2000)1.
- strong momentum-space/configuration space correlation of τ-model
  $\Longrightarrow$ fractal in configuration space with same α
- generalized LPHD suggests particle dist. has same properties as gluon dist.
- Putting this all together leads to Csörgő *et al.*

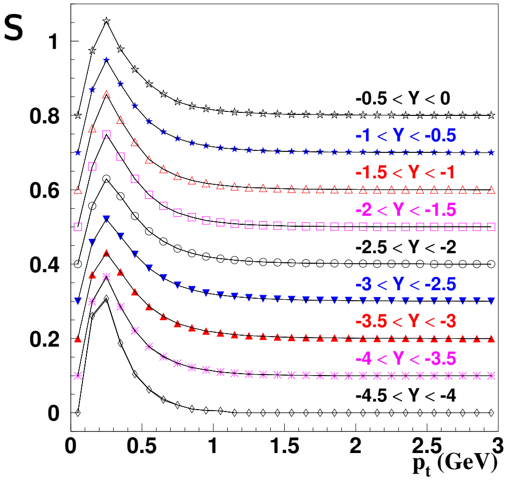
$$\alpha_s = \frac{2\pi}{3}\alpha^2$$

- Using our value of $\alpha = 0.42 \pm 0.02$ yields $\alpha_s = 0.37 \pm 0.04$
- This value is reasonable for a scale of 1–2 GeV,
  where production of hadrons takes place
  cf., from τ decays $\alpha_s(m_\tau \approx 1.8\,\text{GeV}) = 0.35 \pm 0.03$ PDG

Emission function of 2-jet events

In the τ-model, the emission function in configuration space is

$$S(x) = \frac{\mathrm{d}^4 n}{\mathrm{d}\tau \mathrm{d}^3 r} = \left(\frac{m_\mathrm{t}}{\tau}\right)^3 H(\tau)\rho_1\left(k = \frac{r m_\mathrm{t}}{\tau}\right)$$



Factorization y,p_t OK

For simplicity, assume $\quad S(r, z, t) = G(\eta) I(r) H(\tau)$

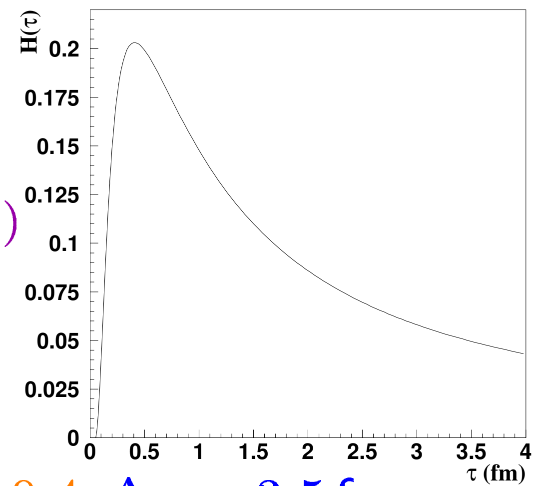
(η = space-time rapidity)

Strongly correlated $x, k \Longrightarrow \eta = y$ and $r = p_\mathrm{t}\tau/m_\mathrm{t}$

$$G(\eta) = N_y(\eta) \qquad I(r) = \left(\frac{m_\mathrm{t}}{\tau}\right)^3 N_{p_\mathrm{t}}(r m_\mathrm{t}/\tau)$$

(N_y, N_{p_t} are inclusive single-particle distributions)

$H(\tau)$

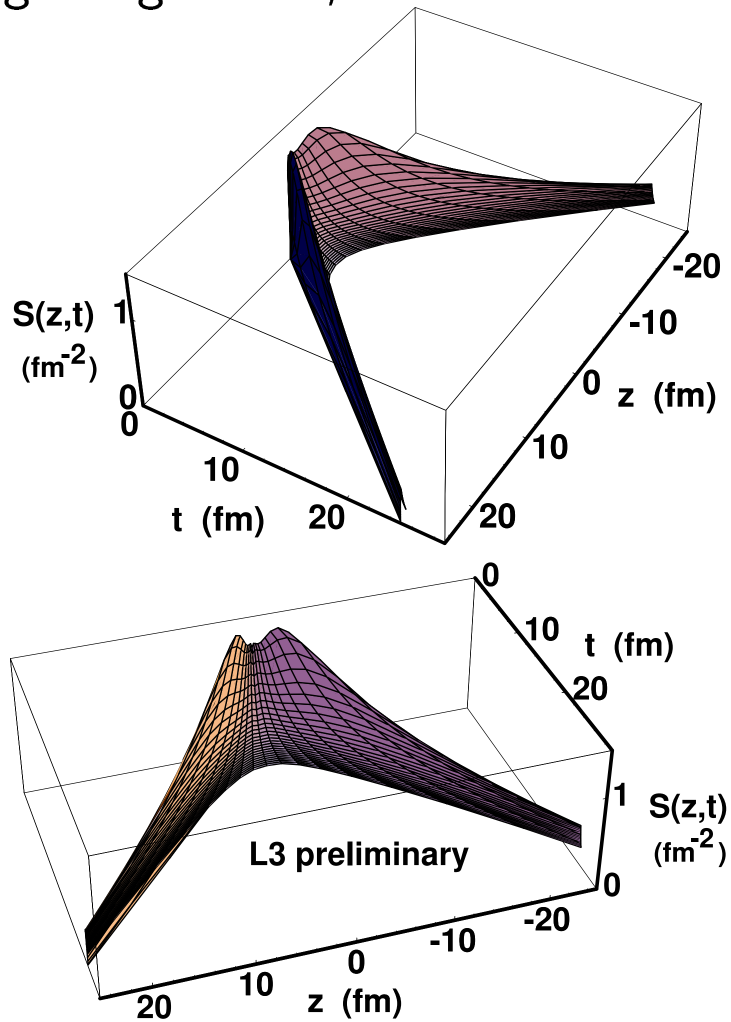


So, using experimental N_y, N_{p_t} distributions
and $H(\tau)$ from BEC fits,
we can reconstruct the full emission function, S.

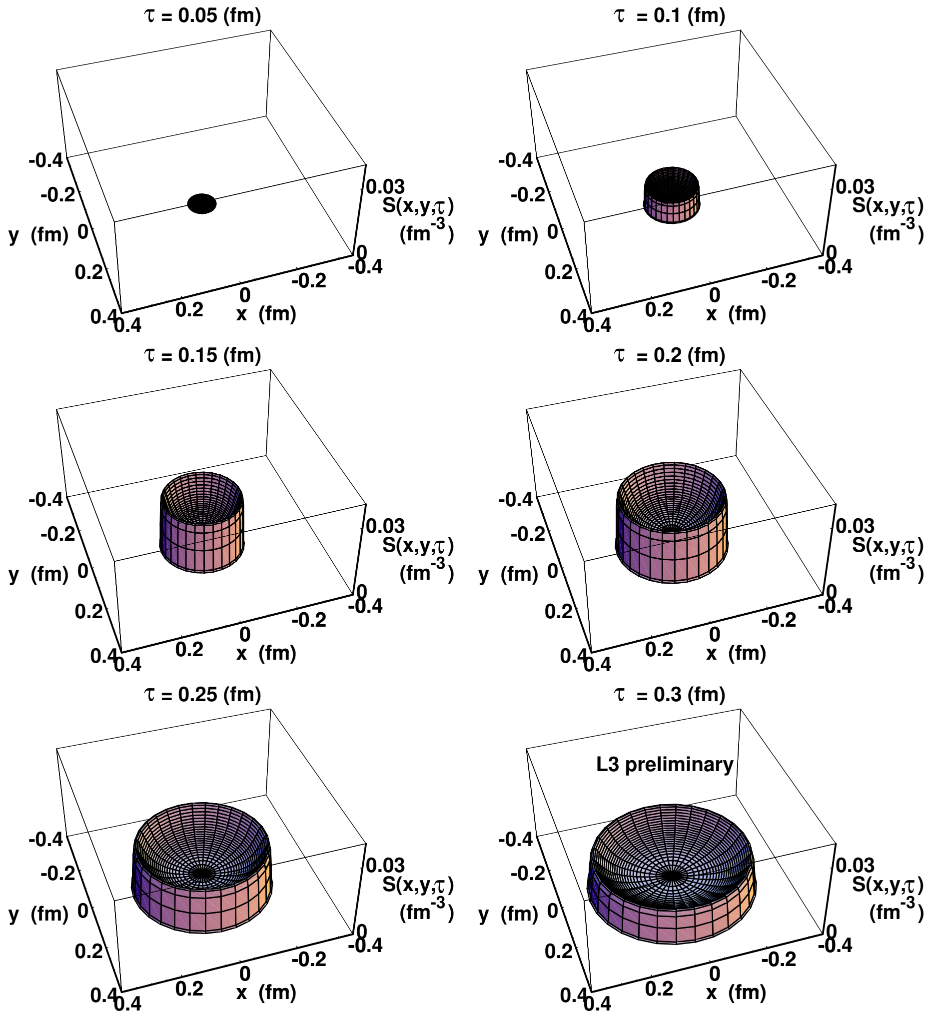
$\alpha = 0.4 \quad \Delta\tau \approx 3.5\,\mathrm{fm} \quad \tau_0 = 0$

Emission function of 2-jet events

Integrating over r,



Integrating over z,



"Boomerang shape"

Particle production is close to the light-cone

Expanding ring

Summary

- Parametrizing R_2 as a function of Q only is a reasonably good approximation
- Symmetric Gaussian, Edgeworth, Lévy parametrizations of R_2 do not fit well
- The τ-model with a one-sided Lévy proper-time distribution leads to $R_2(Q, m_\mathrm{t})$, which successfully fits R_2 for 2-jet events
  - ⋆ both Q- and m_t-dependence described correctly
  - ⋆ Note: we found $\Delta\tau$ to be independent of m_t
    $\Delta\tau$ enters R_2 as $\Delta\tau Q^2/m_\mathrm{t}$
    In Gaussian parametrization, R enters R_2 as R^2Q^2
    Thus $\Delta\tau$ independent of m_t corresponds to $R \propto 1/\sqrt{m_\mathrm{t}}$
- fractal dimension associated with Lévy α relates α to α_s
  $\alpha = 0.42 \pm 0.02$ corresponds to $\alpha_\mathrm{s} = 0.37 \pm 0.04$, reasonable for a scale of 1–2 GeV
- Emission function shaped like a boomerang in z-t and an expanding ring in x-y
  Particle production is close to the light-cone

Acknowledgements: This is the Ph.D. thesis work of Tamás Novák
Tamás Csörgő provided most of the theory.