Interpreting BEC in e^+e^- annihilation

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BEC Introduction

$$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \Rightarrow \frac{\rho_2(Q)}{\rho_0(Q)} \qquad Q^2 = (p_1 - p_2)^2$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$

$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent and other effects reducing BEC

Assuming $S(x)$ is a spherical Gaussian with radius $r \Longrightarrow$

$$R_2(Q) = 1 + \lambda\,\mathrm{e}^{-(Qr)^2}$$

Or, more generally, assuming $S(x)$ is a symmetric Lévy distribution
with index of stability α and scale parameter r

$$R_2(Q) = 1 + \lambda\,\mathrm{e}^{-|Qr|^\alpha} \quad , \qquad 0 < \alpha \leq 2$$

$e^+e^- \longrightarrow$ hadrons

- a clean environment for studying hadronization
- everything is jets – no spectators
- at $\sqrt{s} = M_Z$ almost all events are

  2-jet $e^+e^- \longrightarrow q\bar{q}$ or 3-jet $e^+e^- \longrightarrow q\bar{q}g$

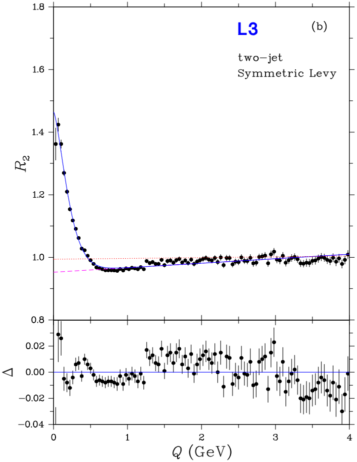
- event hadronization axis is the $q\bar{q}$ direction
  estimate by the thrust axis, *i*.e., axis $\vec{n}_T$ for which
  $T = \frac{\sum |\vec{p}_i \cdot \vec{n}_T|}{\sum |\vec{p}_i|}$ is maximal
- 3-jet events are planar.
  Estimate event plane by thrust, major axes.
  Major is analogous to thrust, but in plane perpendicular to $\vec{n}_T$.
- Require $\vec{n}_T$ within central tracking chamber
  $\Longrightarrow 4\pi$ acceptance

BEC – 'Classic' Parametrizations

$$R_2 = \frac{\rho(p_1,p_2)}{\rho_0(p_1,p_2)} = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$ $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^\alpha\right)$, $0 < \alpha \leq 2$
  α is index of stability
  Gaussian if $\alpha = 2$ $\alpha = 1.34 \pm 0.04$

Cannot accomodate the anticorrelation seen as a dip in R_2 below unity in the region $0.6 < Q < 1.5\,\mathrm{GeV}$



	Gauss	Edgeworth	sym. Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

L3, EPJC 71 (2011) 1648

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:

  $$\overline{x}^\mu(p^\mu) = a\tau p^\mu$$

  where for 2-jet events, $a = 1/m_\mathrm{t}$

  $\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the "longitudinal" proper time

  and $m_\mathrm{t} = \sqrt{E^2 - p_z^2}$ is the "transverse" mass
- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of production points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is

  $$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$
- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.

  $$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$
- Assume $\delta_\Delta(x^\mu - \overline{x}^\mu)$ is very narrow — a δ-function. Then

  $$R_2(p_1,p_2) = 1 + \lambda\,\mathrm{Re}\widetilde{H}\left(\frac{a_1Q^2}{2}\right)\widetilde{H}\left(\frac{a_2Q^2}{2}\right), \quad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction, $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\tfrac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\tfrac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right], \quad \alpha \neq 1$$
  where
  - α is the index of stability, $0 < \alpha \leq 2$;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$R_2(Q, a_1, a_2) = \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right\} \cdot (1 + \epsilon Q)$$

BEC in the τ-model

$$R_2(Q, a_1, a_2) = \gamma \left\{1 + \lambda \cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2}\right]\right\} \cdot (1 + \epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2}$
- Assume particle production begins immediately, $\tau_0 = 0$
- Then

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right] \cdot (1 + \epsilon Q)$$

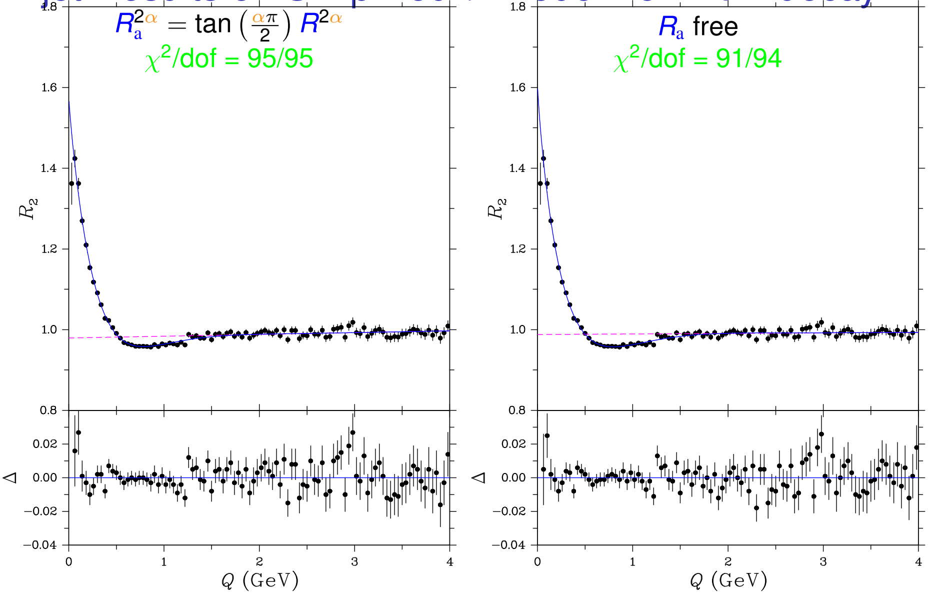
where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left(-|rQ|^{\alpha}\right)\right] (1 + \epsilon Q)$$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$

2-jet Results on Simplified τ-model from L3 Z decay



$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ agrees well with data

L3, EPJC 71 (2011) 1648

τ-model vs. sym. Lévy

- Simplified τ-model:

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right] \cdot (1 + \epsilon Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: Both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$ i.e. to the temporal distribution of production

- Symmetric Lévy parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left(-|rQ|^{\alpha}\right)\right] (1 + \epsilon Q)$$

- r describes the BEC peak
- the anticorrelation dip is NOT described
- BEC is related to the spatial distribution of the production points

But suppose we did not have the τ-model (or don't believe it):
What to do then?

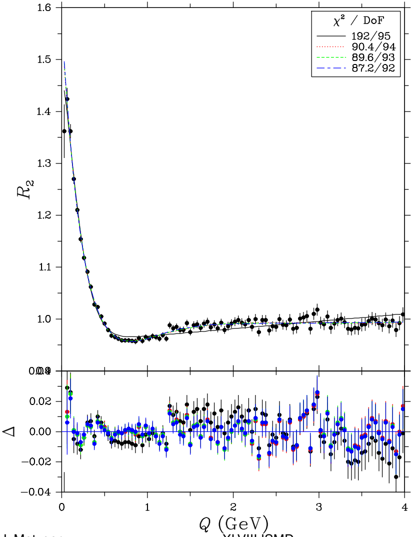
Lévy polynomials

Expand about the Symmetric Lévy distribution using Lévy Polynomials, l_i

Then the Symmetric Lévy parametrization becomes De Kock, Eggers, Csörgő, PoS WPCF 2011 (2011) 033

$$R_2(Q) = \gamma \left[1 + \lambda \exp\left(-|rQ|^{\alpha}\right)\left(1 + \sum c_i l_i\right)\right] \cdot (1 + \epsilon Q)$$

Csörgő, Pasechnik, Ster, arXiv.1807.02897

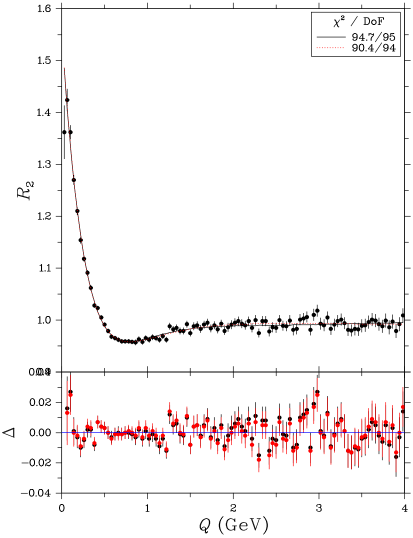


l_i are orthonormal

Fits to succeeding orders provide improved χ^2:

- Order 0: very bad χ^2
- Order 1: good χ^2
- Orders 2-3 give: only marginal further improvement

Lévy polynomials *vs.* τ-model



- χ^2 of order-1 Sym. Lévy polynomial fit is a bit better than τ-model
- but not much difference in fits difference is mainly for $Q > 1.5\,\mathrm{GeV}$

Lévy polynomials *vs.* τ-model

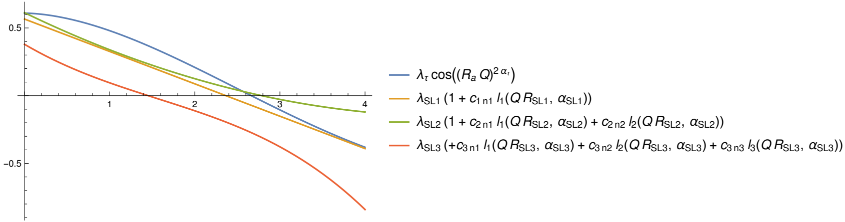
- Simplified τ-model:

$$R_2(Q) = \gamma\left[1 + \lambda\cos\left((R_a Q)^{2\alpha}\right)\exp\left(-(RQ)^{2\alpha}\right)\right]\cdot(1+\epsilon Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

- Symmetric Lévy polynomial parametrization:

$$R_2(Q) = \gamma\left[1 + \lambda \quad \left(1 + \sum c_i l_i\right) \quad \exp\left(-|rQ|^{\alpha}\right)\right]\cdot(1+\epsilon Q)$$



- τ-model describes dip by the cosine term
- Sym. Lévy by Lévy polynomial(s)

Lévy polynomials *vs.* τ-model

- Simplified τ-model:

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right] \cdot (1 + \epsilon Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

- Symmetric Lévy polynomial parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \quad (1 + \sum c_i l_i) \quad \exp\left(-|rQ|^{\alpha}\right)\right] \cdot (1 + \epsilon Q)$$

R_a	$2\alpha = 0.88 \pm 0.02$	$\lambda = 0.61 \pm 0.03$	$R = 0.78 \pm 0.04$ fm
SL order 1	$\alpha = 1.07 \pm 0.06$	$\lambda = 0.16 \pm 0.03$	$r = 0.54 \pm 0.03$ fm
SL order 2	$\alpha = 1.01 \pm 0.10$	$\lambda = 0.23 \pm 0.03$	$r = 0.43 \pm 0.04$ fm
SL order 3	$\alpha = 1.36 \pm 0.25$	$\lambda = 0.22 \pm 0.03$	$r = 0.54 \pm 0.05$ fm

Values of parameters differs between τ-model and Sym. Lévy
and between orders of Sym. Lévy

Does expansion improve the τ-model?

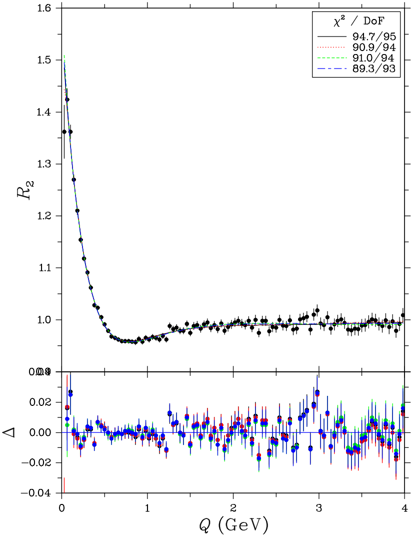
Lacking (so far) an orthogonal polynomial expansion for the asymmetric Lévy distribution $H(\tau)$ of the τ-model, we use a derivative expansion:

$$R_2(Q) = \gamma \left[1 + \lambda \left\{ \cos\left((R_a Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) + \sum c_n \frac{d^n}{dQ^n} \cos\left((R_a Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) \right\} \right] \cdot (1 + \epsilon Q)$$

	order 0	order 1	order 0, R_a free	order 1, R_a free
α	0.44 ± 0.01	0.43 ± 0.01	0.41 ± 0.02	0.40 ± 0.03
R (fm)	0.78 ± 0.04	0.84 ± 0.05	0.79 ± 0.04	0.83 ± 0.07
R_a (fm)	–	–	0.69 ± 0.04	0.60 ± 0.06
λ	0.61 ± 0.03	0.67 ± 0.05	0.63 ± 0.03	1 at limit
γ	0.979 ± 0.002	0.979 ± 0.002	0.988 ± 0.005	0.992 ± 0.006
ϵ	0.005 ± 0.001	0.005 ± 0.001	0.001 ± 0.002	0.000 ± 0.002
c_1	–	0.0008 ± 0.0005	–	0.072 ± 0.015
χ^2/DoF	$94.7/95$	$90.9/94$	$91.0/94$	$89.3/93$
CL	49%	57%	57%	59%

- Orders 0-1 $\sim 1\sigma$ difference
- Order 1 has somewhat better χ^2, as does order 0, R_a free

τ-model expansion



order	χ^2/DoF	CL
order 0	94.7/95	49%
order 1	90.9/94	57%
order 0, R_a free	91.0/94	57%
order 1, R_a free	89.3/93	59%

- Difference of two χ^2 is also a χ^2
- Small CL$(\chi_1^2 - \chi_2^2, \mathrm{DoF}_1 - \mathrm{DoF}_2)$ is reason to reject Hypothesis 1
- CL(94.7 – 90.9, 1 dof) = 5.1%
  Not small enough to reject order 0
- Other χ^2 differences are smaller; so CL larger
- expansion not needed
  R_a free does not give significant improvement

Conclusions

- Expansions provide a test of whether the assumed function is (approximately) correct
  and if only approximately, help to locate the differences
- for 2-jet events
  - for τ-model expansion is not needed;
    assumption that $H(\tau)$ is an asymmetric Lévy distribution is OK
  - for symmetric Lévy order-1 expansion is required;
    modification of the symmetric Lévy required is similar to that of the τ-model

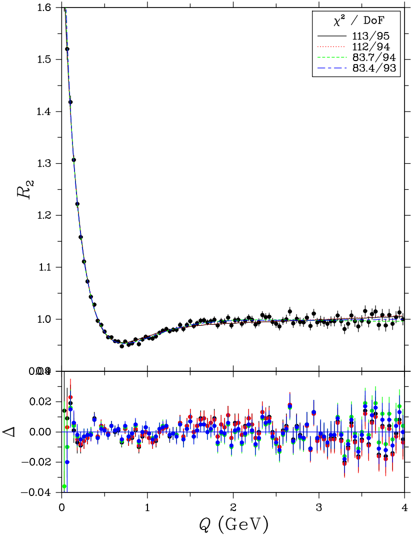
τ-model– 3-jet events

- at $\sqrt{s} = M_Z$ almost all events are

  2-jet $e^+e^- \longrightarrow q\bar{q}$ or 3-jet $e^+e^- \longrightarrow q\bar{q}g$

- for 2-jet events hadronizaton is basically 1+1 dimension, which lead in the τ-model to the dependence on
  τ, the longitudinal proper time
  m_t, the transverse mass
- for 3-jet events this is more complicated
  So, we might expect the τ-model to work less well

τ-model expansion – 3-jet events



order	χ^2/DoF	CL
order 0	113.2/95	10%
order 1	112.4/94	9%
order 0, R_a free	83.7/94	76%
order 1, R_a free	83.4/93	75%

- CL(113.2 – 112.4, 1 dof) = 37%
  CL(83.7 – 83.4, 1 dof) = 58%
  Order 1 gives no significant improvement
  expansion not needed
- However,
  CL(113.2 – 83.7, 1 dof)$= 6 \cdot 10^{-8}$
- R_a free does give significant improvement

Conclusions – 3-jet events

- τ-model expansion not needed
  $\Longrightarrow H(\tau) =$ asymmetric Lévy distribution is OK
- significant improvement is obtained letting R_a free
  i.e., by lessening the connection of simplified τ-model
  between the BEC peak and antisymmetric dip
  possibly due to the more complicated structure of the event

BACKUP

Lévy Polynomials

provide an expansion about

De Kock, Eggers, Csörgő, PoS WPCF 2011 (2011) 033

$$w(t \mid \alpha) = \exp(-t^{\alpha}) \quad , \qquad t \geq 0$$

Csörgő, Pasechnik, Ster, arXiv.1807.02897

Applied to BEC,

$$t = Qr$$

$$R_2(Q) \propto 1 + \lambda \exp(t^{\alpha}) \sum_{n=0}^{\infty} c_n l_n(t \mid \alpha)$$

$$l_j(t \mid \alpha) = \frac{1}{\sqrt{D_j D_{j+1}}} L_j(t \mid \alpha)$$

$$D_0(\alpha) = 1 \qquad L_0(t \mid \alpha) = 1$$

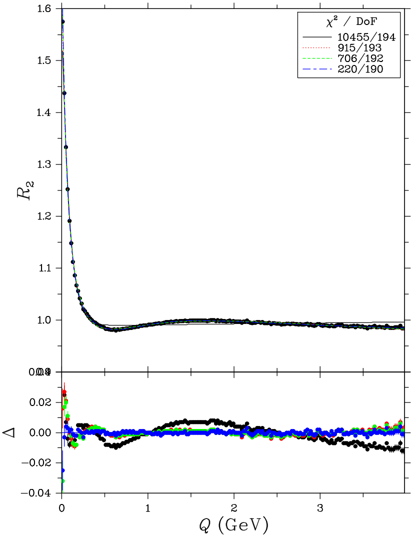
$$D_1(\alpha) = \mu_{0,\alpha} \qquad L_1(t \mid \alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & t \end{pmatrix}$$

$$D_2(\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} \end{pmatrix} \qquad L_2(t \mid \alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & t & t^2 \end{pmatrix}$$

etc., *where* $\mu_{n,\alpha} = \int_0^{\infty} \mathrm{d}t \, t^n \exp(-t^{\alpha}) = \frac{1}{\alpha} \Gamma\left(\frac{n+1}{\alpha}\right)$

l_i are orthonormal: $\int_0^{\infty} \mathrm{d}t \, \exp(-t^{\alpha}) l_n(t \mid \alpha) l_m(t \mid \alpha) = \delta_{n,m}$

Lévy polynomials in pp



CMS sees anticorrelation in pp at LHC

PRC97,064912(2018)

ATLAS also (unpublished) in PhD thesis
R. Astaloš http://hdl.handle.net/2066/143448

Using data from a figure in this thesis:

- Sym. Lévy: χ^2/DOF = 10455/194 – does not fit
- χ^2 of τ-model (R_a free) (Order 0) is much better 915/193
- χ^2 of τ-model (R_a free) (Order 1) is better 706/192
- Sym. Lévy polynomial (Order 4) is better 220/190