Bose-Einstein Correlations, the τ-model, and jets in e^+e^- annihilation

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Outline

1. Old stuff — Eur. Phys. J. C (2011) 71:1648

   25 pages — quickly summarize

2. New stuff

BEC Introduction

$$R_2 = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} = \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$
$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius $r \Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-Q^2 r^2}$$

intro

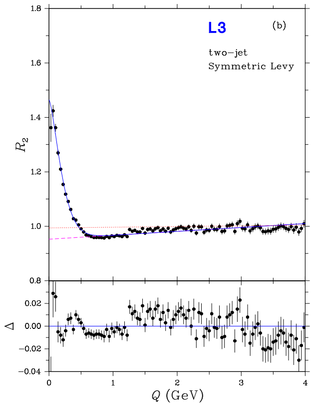
The L3 Data

- $e^+e^- \longrightarrow \mathrm{hadrons}$ at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign pairs of well measured charged tracks from about $0.8 \cdot 10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\mathrm{cut}} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, "3-jet events"
- use mixed events for reference sample, ρ_0

Results – ‘Classic’ Parametrizations

$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$
  $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^{\alpha}\right)$
  $0 < \alpha \leq 2$
  $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:
$$\overline{x}^\mu(p^\mu) = a\tau p^\mu$$
where for 2-jet events, $a = 1/m_t$
$$\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2} \text{ is the “longitudinal” proper time}$$
and $m_t = \sqrt{E^2 - p_z^2}$ is the “transverse” mass
- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is
$$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$
- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.
$$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1+\cos\left([p_1-p_2][x_1-x_2]\right)\right)$$
- Assume $\delta_\Delta(x - a\tau p)$ is very narrow — a δ-function. Then
$$R_2(p_1,p_2) = 1 + \lambda \,\mathrm{Re}\widetilde{H}\left(\frac{a_1Q^2}{2}\right)\widetilde{H}\left(\frac{a_2Q^2}{2}\right), \qquad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction, $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right]\;, \qquad \alpha \neq 1$$
  where
  - α is the index of stability;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$R_2(Q, a_1, a_2) = \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right]\right\}\cdot(1+\epsilon Q)$$

BEC in the τ-model

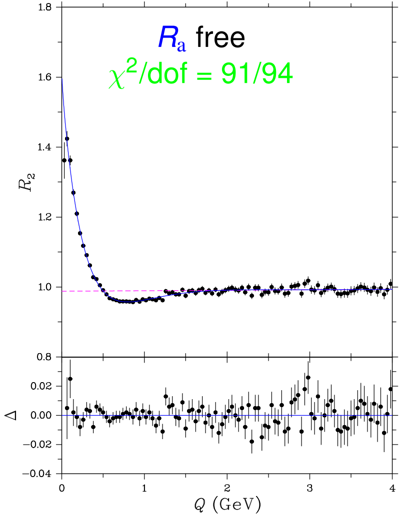
$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2} \right] \cdot \exp\left[- \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

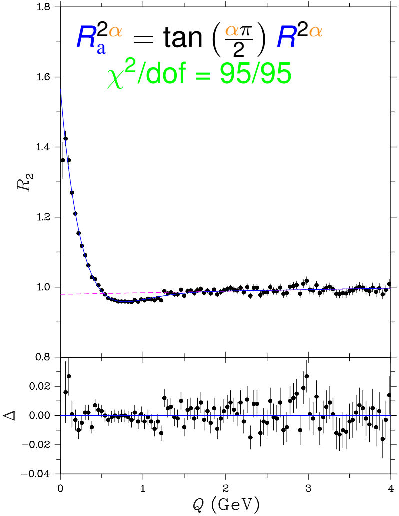
Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then
  $$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha} \right) \exp\left(- (RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$
  where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

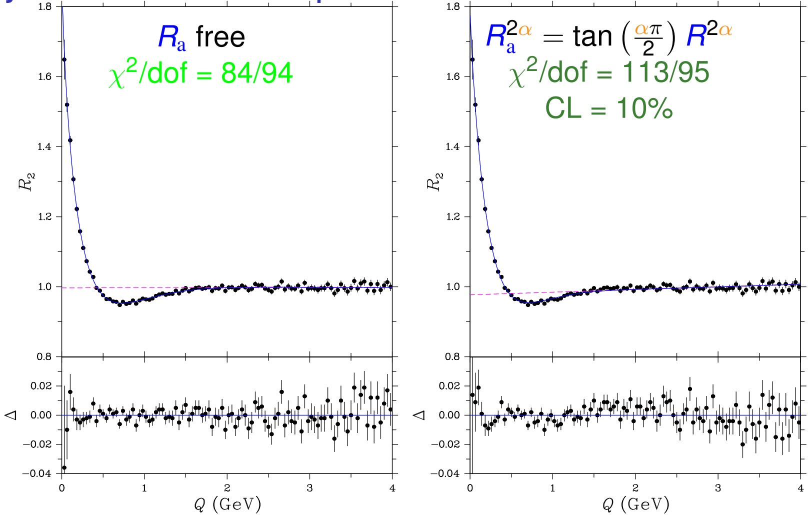
  Compare to sym. Lévy parametrization:
  $$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left[-|rQ|^{\alpha} \right] \right] (1 + \epsilon Q)$$

2-jet Results on Simplified τ-model from L3 Z decay





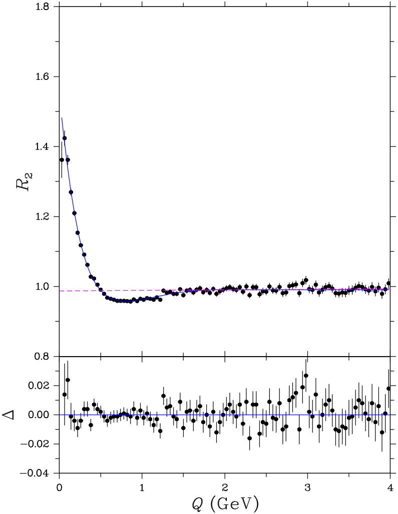
3-jet Results on Simplified τ-model from L3 Z decay



R-summary

Full τ-model for 2-jet events — $a = 1/m_t$

$$R_2(Q, m_{t1}, m_{t2}) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (m_{t1}+m_{t2})}{2(m_{t1} m_{t2})} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}} \right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}} \right] \right\} \cdot (1 + \epsilon Q)$$

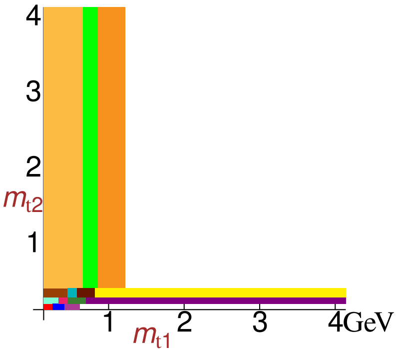


- Fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin, $m_{t1} > m_{t2}$
- $\tau_0 = 0.00 \pm 0.02$ so fix to 0
- χ^2/dof = 90/95

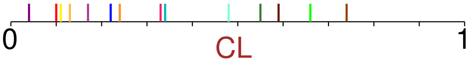
fit

Full τ-model for 2-jet events

- τ-model predicts dependence on m_t, $R_2(Q, m_{t1}, m_{t2})$
- Parameters α, $\Delta\tau$, τ_0 are independent of m_t
- λ (strength of BEC) can depend on m_t



- divide m_{t1}-m_{t2} plane in regions (equal statistics)
- in each region fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin with α, $\Delta\tau$, fixed to values found for entire plane and $\tau_0 = 0$ fits



Elongation?

- Previous results using fits of Gaussian or Edgeworth found (in LCMS) $R_{\mathrm{side}}/R_{\mathrm{L}} \approx 0.64$
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ-model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function?
  or is the τ-model in need of modification?
- So, we modify *ad hoc* the τ-model description to allow elongation (more on this later)
- and find $R_{\mathrm{side}}/R_{\mathrm{L}} = 0.61 \pm 0.02$ – elongation is real
- Perhaps, a should be different for transverse/longitudinal

$$\overline{x}^{\mu}(p^{\mu}) = a\,\tau p^{\mu}, \qquad a = 1/m_{\mathrm{t}} \text{ for 2-jet}$$

restQdep

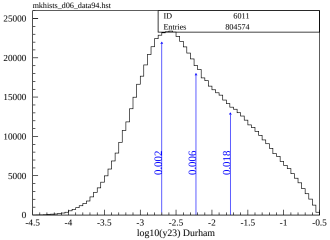
Outline

New stuff — very preliminary

Are BEC sensitive to jet structure?

Jets

- Jets — Durham algorithm
- y_{23} is value of y_{cut} where number of jets changes from 2 to 3
- force event to have 3 jets
- define regions of y_{23}:

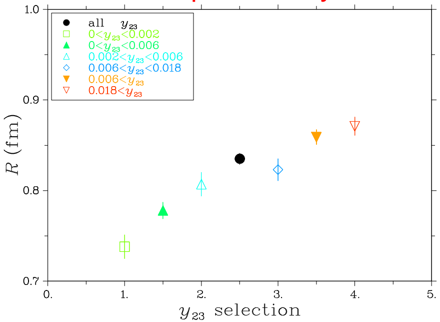


$y_{23} < 0.002$	narrow two-jet	or		
$0.002 < y_{23} < 0.006$	less narrow two-jet		$y_{23} < 0.006$	two-jet
$0.006 < y_{23} < 0.018$	narrow three-jet		$0.006 < y_{23}$	three-jet
$0.018 < y_{23}$	wide three-jet			

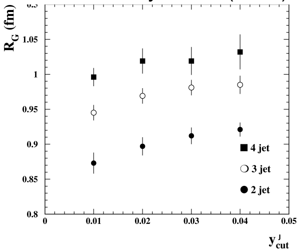
To stabilize fits against large correlation of α, R, fix $\alpha = 0.443$

Jets

L3 preliminary



OPAL: Z.Phys. C72 (1996) 389



- *R* increases as number of jets increases
- or as jets become more separated

Jets - Elongation

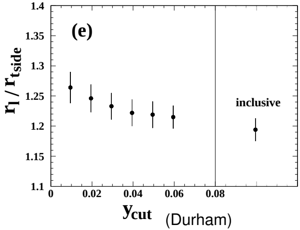
Results in LCMS frame: Longitudinal = Thrust axis ▸ LCMS

	$R_{\mathrm{L}}/R_{\mathrm{side}}$
L3	$1.25 \pm 0.03^{+0.36}_{-0.05}$
OPAL	$1.19 \pm 0.03^{+0.08}_{-0.01}$

(ZEUS finds similar results in ep)
$\sim$25% elongation along thrust axis

OPAL:

Elongation larger for narrower jets



(Durham)

LCMS and the Simplified τ-model

Consider 2 frames:

1. LCMS: $Q^2 = Q_{\rm L}^2 + Q_{\rm side}^2 + Q_{\rm out}^2 - (\Delta E)^2$
   $= Q_{\rm L}^2 + Q_{\rm side}^2 + Q_{\rm out}^2\left(1-\beta^2\right)\ , \qquad \beta = \frac{p_{1\rm out}+p_{2\rm out}}{E_1+E_2}$
2. LCMS-rest: $Q^2 = Q_{\rm L}^2 + Q_{\rm side}^2 + q_{\rm out}^2\ , \qquad q_{\rm out}^2 = Q_{\rm out}^2\left(1-\beta^2\right)$

$q_{\rm out}$ is $Q_{\rm out}$ boosted (β) along out direction to rest frame of pair

In simplified τ-model, replace R^2Q^2 by

1. $A^2 = R_{\rm L}^2Q_{\rm L}^2 + R_{\rm side}^2Q_{\rm side}^2 + \rho_{\rm out}^2Q_{\rm out}^2$
2. $B^2 = R_{\rm L}^2Q_{\rm L}^2 + R_{\rm side}^2Q_{\rm side}^2 + r_{\rm out}^2q_{\rm out}^2$

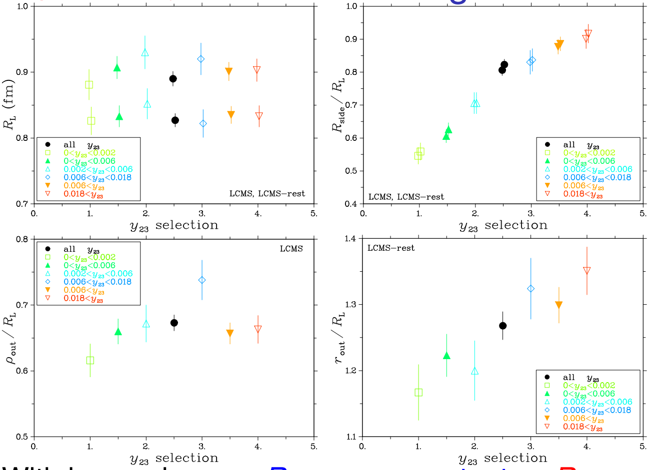
Then in τ-model, for case 1:

$$R_2(Q_{\rm L}, Q_{\rm side}, Q_{\rm out}) = \gamma\left[1 + \lambda\cos\left(\tan\left(\frac{\alpha\pi}{2}\right)A^{2\alpha}\right)\exp\left(-A^{2\alpha}\right)\right] \cdot\left(1 + \epsilon_{\rm L}Q_{\rm L} + \epsilon_{\rm side}Q_{\rm side} + \epsilon_{\rm out}Q_{\rm out}\right)$$

and comparable expression for case 2, $R_2(Q_{\rm L}, Q_{\rm side}, q_{\rm out})$

L3 preliminary

Jets - Elongation



Note:
$R_{\mathrm{side}} < R_{\mathrm{L}}$
$r_{\mathrm{out}} > R_{\mathrm{L}}$
Not azimuthally symmetric not even for narrow 2-jet trigger bias??

With increasing y_{23}, R_{L}, $\rho_{\mathrm{out}} \approx$ constant, R_{side}, r_{out} increase

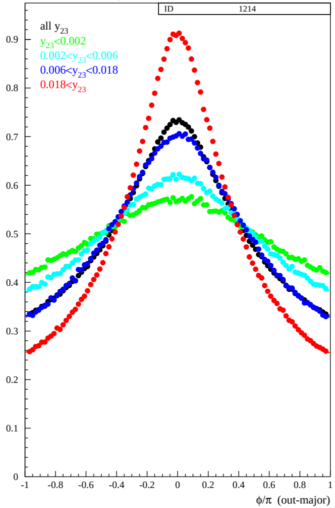
narrow 2-jet limit: $R_{\mathrm{side}} \approx R_{\mathrm{L}}/2$ $\quad r_{\mathrm{out}} \approx 1.1 R_{\mathrm{L}}$

wide 3-jet limit: $R_{\mathrm{side}} \approx R_{\mathrm{L}}$ $\quad r_{\mathrm{out}} \approx 1.4 R_{\mathrm{L}}$

ϕ major-out

- event plane $\equiv$ (thrust,major)
- out direction tends to be in the event plane out $\approx$ in
- side direction tends to be out of the plane side $\approx$ out
- this tendency increases with y_{23}
- suggests that lack of azimuthal symmetry is due to difference in fragmentation in and out of the event plane

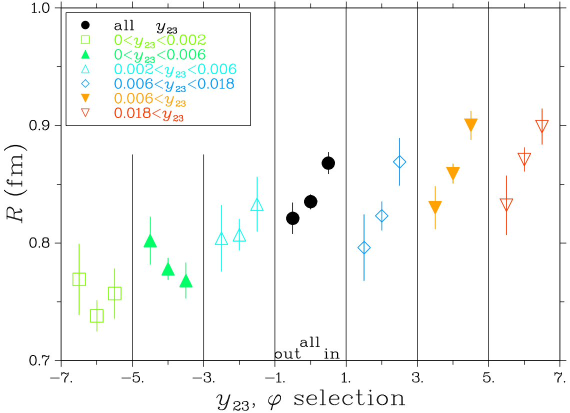
L3 preliminary



in/out of event plane

use only tracks with ϕ(trk-major) $< 45°$ in plane

ϕ(trk-major) $> 45°$ out of plane



L3 preliminary

R larger in the event plane

New Stuff — Summary

	narrow 2-jet limit	wide 3-jet limit
$R \approx$	0.7 fm	0.9 fm
$R_{\mathrm{L}} \approx$	0.9 fm – constant	
$\rho_{\mathrm{out}} \approx$	0.66 fm – constant	
	$R_{\mathrm{side}} \approx R_{\mathrm{L}}/2$	$R_{\mathrm{side}} \approx R_{\mathrm{L}}$
	$r_{\mathrm{out}} \approx 1.1 R_{\mathrm{L}}$	$r_{\mathrm{out}} \approx 1.4 R_{\mathrm{L}}$

out direction $\approx$ in event plane

side direction $\approx$ out of event plane

R in $\approx$ R out of plane	R in $>$ R out of plane

Acknowledgments

- Tamás Novák, Tamás Csörgő, Wolfram Kittel
  were instrumental for the 'Old Stuff'

- I take full responsibility for the 'New Stuff'