Ex. 3 Consider $U(x) = e^{\frac{i}{2}\vec{\tau}\cdot\vec{\varphi}(x)} \in SU(2)$, with $\varphi^1(x), \varphi^2(x), \varphi^3(x) \in \mathbb{R}$ and τ^1, τ^2, τ^3 the usual Pauli spin matrices. Introduce a doublet field $\Phi(x)$ that transforms under $SU(2)$ according to

$\Phi(x) \to \Phi'(x) = U(x)\,\Phi(x)$ (as expected for a fundamental doublet).

Prove that $\Phi^c(x) \equiv i\tau^2\,\Phi^*(x)$ ($*$ = complex conjugation) has the same $SU(2)$ transformation property as $\Phi(x)$.

Hint: first figure out what happens if you bring τ^2 to the other side of $(\tau^j)^*$ for $j=1,2,3$.