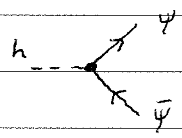
Ex. 6 Consider a theory described by the Lagrangian

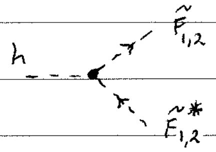
$$\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu - m_F)\psi + \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) - \tfrac{1}{2}m_h^2 h^2 + \sum_{i=1}^{2}\left[(\partial_\mu \tilde F_i^\dagger)(\partial^\mu \tilde F_i) - m_{\tilde F_i}^2 \tilde F_i^\dagger \tilde F_i\right] + \mathcal{L}_{int}\,.$$

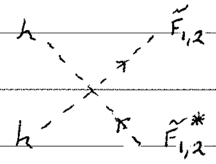
Ingredients: - ψ is a Dirac field, $\bar\psi = \psi^\dagger\gamma^0$ its adjoint and $m_F \in \mathbb{R}$ its mass;
- h is a real scalar field, with mass $m_h \in \mathbb{R}$;
- $\tilde F_{1,2}$ are two complex scalar fields, with masses $m_{\tilde F_{1,2}} \in \mathbb{R}$.

$$\mathcal{L}_{int} = -g_F\, h\bar\psi\psi - h_F(h^2 + 2vh)(\tilde F_1^\dagger \tilde F_1 + \tilde F_2^\dagger \tilde F_2) \qquad (v = 246\ \mathrm{GeV}),$$

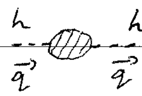
with $g_F = m_F/v \in \mathbb{R}$ a Yukawa coupling and $h_F \in \mathbb{R}$ a scalar coupling. The corresponding Feynman rules for these interactions read



$= -i g_F$,


$= -2ivh_F$,


$= -2ih_F$.

(i) Write down the expression for the one-loop self-energy 

of the h-field (keeping only 1-particle irreducible contributions).
<u>Don't calculate the integrals</u>, but work out the expression up to the point that performing the integrals would be the only thing left to do.

(ii) Indicate the degree of divergence of the various terms in your expression, using power-counting arguments.

(iii) - Show that the <u>leading divergences cancel</u> if $h_F = g_F^2$.
- Show that only one term remains if also $m_{\tilde F_1} = m_{\tilde F_2} = m_F$.