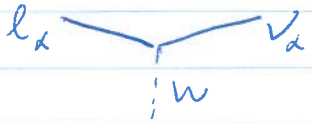
Neutrino's

Weak interactions of neutrino's: $W \left(\begin{pmatrix} \nu \\ e \end{pmatrix}_L \right) W$

define the flavour eigenstates of neutrino's.

Interactions with W always involve a specific flavour of charged lepton, and the same flavour neutrino.



$\alpha = e, \mu, \tau$ $\qquad |\nu_\alpha\rangle$

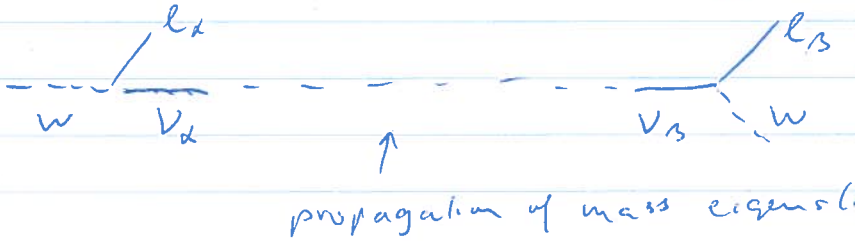
On the other hand, the mass eigenstates $|\nu_i\rangle$ $i = 1, 2, 3$ have a definite mass m_i.

Writing the connection between the two types of states

$$|\nu_\alpha| = \sum_i U^*_{\alpha i} |\nu_i\rangle \quad \text{or} \quad |\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle$$

U: Pontecorvo - Maki - Nakagawa - Sakata matrix.

Suppose we have a source of neutrino's of flavour α
Elsewhere we have a detector capable of measuring neutrino's of flavour β (by detecting corresponding lepton ℓ_β)



$$Amp(\nu_\alpha \to \nu_\beta) = \sum_i U^*_{\alpha i} \; Prop(\nu_i) \; U_{\beta i}$$

Prop(ν_i) : plane wave solution of Schrödinger eq:

$$|\nu_i(t)\rangle = |\nu_i(0)\rangle e^{-iE_i t}$$

To start easy, let's assume 2 flavours only : ν_α, ν_β , ν_1, ν_2

$$U^* = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad \left(\text{and } U = \begin{pmatrix} \cos\theta & -\sin \\ \sin\theta & \cos \end{pmatrix} \right.$$

then the probability to detect flavour β when starting out with flavour α is :

$$P(\nu_\alpha \to \nu_\beta) = |Amp(\nu_\alpha \to \nu_\beta)|^2 = \left| \sum_i U^*_{\alpha i} e^{-iE_i t} U_{\beta i} \right|^2$$

$$= \frac{1}{2} \sin^2 2\theta \left[1 - \cos(E_1 - E_2)t \right]$$

$$E_1 - E_2 = \sqrt{p^2 + m_1^2} - \sqrt{p^2 + m_2^2} \approx \frac{m_1^2 - m_2^2}{2p} \approx \frac{m_1^2 - m_2^2}{2E} = \frac{\Delta m^2_{12}}{2E}$$

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2}{4E} \cdot t \right)$$

$$= \sin^2 2\theta \, \sin^2 \left[1.27 \, \Delta m^2 (eV^2) \cdot \frac{L\,(km)}{E\,(GeV)} \right]$$

The above equations assume propagation in vacuum.

Some observations :

- $\beta \neq \alpha$ only if $\Delta m^2 \neq 0$
  Observation of neutrino oscillations $\Rightarrow$ neutrino's have mass
- We can only say something about Δm^2, not about m
- $\beta \neq \alpha$ only if U non-diagonal ($\theta \neq 0$)
  $\Rightarrow$ mixing

- $\sin^2(t)$, or $\sin^2\left(\frac{L}{E}\right)$ implies: oscillation back-and-forth
- ν oscillations do not change the total flux, only the distribution over different flavours
- Experimental approach:
  - disappearance measurements ($P(\nu_\alpha \to \nu_\alpha) \neq 1$)
  - appearance measurements ($P(\nu_\alpha \to \nu_\beta) \neq 0$)

A specific choice of L/E is sensitive to a specific Δm^2
⇒ play with beam, detector distance.

Experimental evidence:

▷ Solar neutrino deficit

- Davis, Homestake mine $\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$ $E_\nu > 0.8$

  prediction: $9.3^{+1.2}_{-1.4}$ SNU

  measurement: 2.33 ± 0.25 SNU

  $1\,\text{SNU} = \frac{1\ \nu\ \text{interaction}}{10^{36}\ {}^{37}Cl\ \text{atoms} \cdot s}$

- Super Kamiokande low E, but directional $E_\nu > 5$ MeV

  $\frac{N_{obs}}{N_{th}} : 0.47 \pm 0.02$

- Gallium experiments $\nu_e + {}^{71}Ga \to {}^{71}Ge + e^-$

  sensitive to low energy ν_e (0.25 MeV)

  prediction: 129^{+8}_{-6} SNU

  measurement: 66.1 ± 3.1 SNU

SNO: 1000 tons heavy water

CC interactions of ν_e : $\nu_e + D \rightarrow p + p + e$

electron scattering of ν_e : $\nu_e + e \rightarrow e + \nu_e$ (CC)

NC interactions of all ν : $\nu_x + D \rightarrow p + n + \nu_x$

ES of all ν : $\nu_x + e \rightarrow \nu_x + e$ (NC)

Result : sum of flux agrees well with predictions.
ν_e flux only 1/3 of total

▷ Atmospheric neutrino's :

- Deficit seen in ν_μ component compared to ν_e component

Naïve expectation :

$p + A \rightarrow \pi(k) + X$

$\pi \rightarrow \mu + \nu_\mu$ $\quad$ $\mu \rightarrow e + \nu_e + \nu_\mu$

Kamiokande : $R = \dfrac{(\nu_\mu/\nu_e)_{data}}{(\nu_\mu/\nu_e)_{MC}} \simeq 0.7 \pm 0.1$

Super K : clear dependence on zenith angle = L

- Accelerator expts : K2K $\langle E_\nu \rangle = 1.3$ GeV L = 250 km

MINOS multi-GeV wide-band beam L = 7 iron-scintillator

OPERA 17 GeV, L = 730 km
looks for ν_τ appearance
emulsion

Consistent interpretation : (atmospheric ν only + acceleration)

$\sin^2 2\theta \sim 1$ $\quad$ (> 0.95)

$\Delta m^2 \simeq 2.32 \cdot 10^{-3}$ eV^2

Interpretation of solar ν results trickier: matter effects

Mikheyev – Smirnov – Wolfenstein effect

ν_e ($\bar{\nu}_e$) have interactions in matter that ν_μ, ν_τ don't have

Forward scattering amplitude of ν_e in matter interferes with unscattered amplitudes
⇒ effective change in mass & mixing angle

flavour basis

$$i\frac{d}{dt}\Psi_f(t) = H\Psi_f(t)$$

mass basis

$$i\frac{d}{dt}\Psi_m(t) = H\Psi_m(t)$$

$$H_m = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \qquad \Psi_f(t) = e^{-iH_f t} \qquad \Psi_m(t) = e^{-iH_m t}$$

$$\Psi_f(t) = U^{-1} e^{-iH_m t} U \qquad U: \text{PMNS matrix}$$

In matter with electron density N_e this changes:

$$H_f = U^{-1}\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} U + \sqrt{2}\, G_F N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Solution (assuming 2 flavours)

$$\Delta_m m^2 \simeq \Delta m^2 \sqrt{(a - \cos 2\theta)^2 + \sin^2 2\theta}$$

$$\sin^2 2\theta_m \simeq \sin^2 2\theta \,/\, \left((a - \cos 2\theta)^2 + \sin^2 2\theta\right)$$

with $a = \dfrac{2\sqrt{2}\, E\, G_F N_e}{\Delta m^2}$ (inverted hierarchy: $-\Delta m^2$)

resonant effect if $a = \cos 2\theta$

Plays a major role in the sun: produced ν_e come out as pure mixture ν_e, ν_μ, ν_τ at surface.
In this solution: $\Delta m^2_{\text{solar}} = 7 \cdot 10^{-5}\ \text{eV}^2$ $\theta_{\text{solar}} \sim 32°$

Matter effects also play a role in long-baseline oscillations in the earth! (say $L > 300$ km or so)

Extending to 3 flavours:

$$|\nu_\alpha\rangle = U_{PMNS}\,|\nu_i\rangle$$

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} e^{i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv UV$$

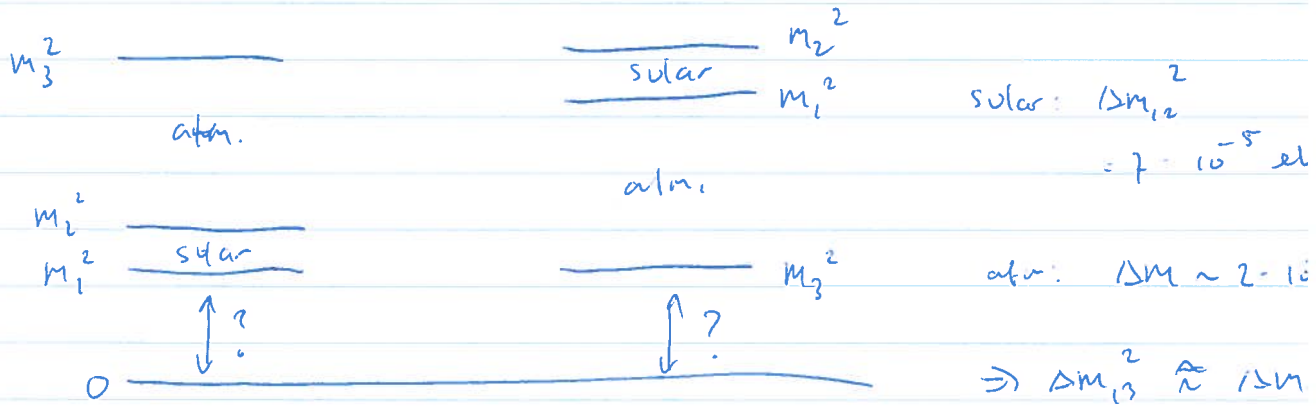
$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

α: Majorana phases (can be transformed away for Dirac neutrino's)

δ: CP-violation phase

3 angles $\theta_{12}, \theta_{13}, \theta_{23}$, 3 mass differences



normal hierarchy

inverted hierarchy

CPT invariance: $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = P(\nu_\beta \to \nu_\alpha)$

In general, however: $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$

CP violation

CP-violation occurs if U is truely complex: $\delta \neq 0$ or π

θ_{13} finally measured around 2010

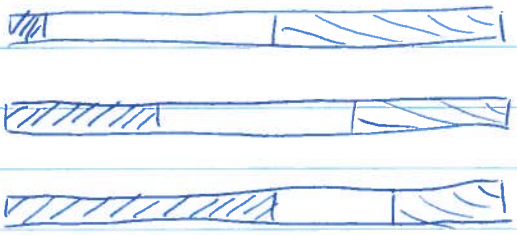
reactor ν, short baseline · Daya Bay, Reno, Chooz

$\theta_{13} \sim 8°$

Fitting it all together

$\theta_{12} \sim 32°$

$\theta_{23} \sim \pi/4$



$c_{12} = 0.85$

$s_{12} = 0.54$

$s_{13} = 0.14$

$$U \simeq \begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\frac{s_{12}}{\sqrt{2}} = 0.38$

$\frac{c_{12}}{\sqrt{2}} = 0.60$

Tri-bimaximal mixing: $\theta_{23} = \frac{\pi}{4}$ $\theta_{13} = 0$ $\theta_{12} = 35°$ $(s_{12} = 1/\sqrt{3})$

$$\begin{pmatrix} \sqrt{2/3} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\frac{1}{\sqrt{3}} = 0.57$

$\frac{1}{\sqrt{6}} = 0.41$

$\sqrt{2/3} = 0.82$

Remaining unknown: mass hierarchy and δ

δ: lack of CP-violation $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$

asymmetry: $A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)}$

$\propto \frac{\Delta m^2_{21}}{4E_\nu} \frac{L \sin 2\theta_{12}}{\sin \theta_{13}} \sin \delta_{CP}$

T2K, NOvA ($L = 295$ km, ~~1300~~ 800 km)

Hyper K: upgraded beam and bigger detector
⇒ hard to measure δ

DUNE: beam FNAL → Homestake 1300 km
long distance: matter effects need to be taken into account
⇒ DUNE can measure hierarchy as well as δ

But earlier: use atmospheric ν and matter effects:
ICH3Net (ORCA) or IceCube (PINGU)

Challenges to the 3-flavour paradigm?

- LSND experiment @ Los Alamos
  claimed observing $\bar{\nu}_\mu \to \bar{\nu}_e$ $E_\nu = 50$ MeV $l = 30$ m
  $\Delta m \sim 1$ eV ?
  MiniBoone experiment @ FNAL could not disprove it.
  Now MicroBoone @ FNAL will try again.

- Reactor ν flux : observed rate $\bar{\nu}_e$ few percent lower than calculated.
  (ν_e disappearance, also $\Delta m \sim 1$ eV)
  Now Daya Bay: Pu neutrino rate ok, U^{235} low
  Modelling issue ?

- Radioactive sources ^{51}Cr, ^{37}Ar for calibration of Ga solar ν
  $\nu_e + {}^{71}Ga \to {}^{71}Ge + e$
  $R \sim 0.86 \pm 0.05$? ν_e disappearance ?

- Big Bang nucleosynthesis and CMB anisotropies count the effective number of degrees of freedom N_{eff}.
  Light sterile ν mixing with normal ν would contribute.
  For a while $N_{eff} = 4$ seemed preferred.
  Latest data (Planck) more consistent with $N_{eff} = 3$
  Anyway, $m \sim 1$ eV not easy to accomodate

A fourth, sterile neutrino, would be a solution.
However, latest data disfavours it.

Neutrino mass :

Oscillations only give us Δm^2, not m_i

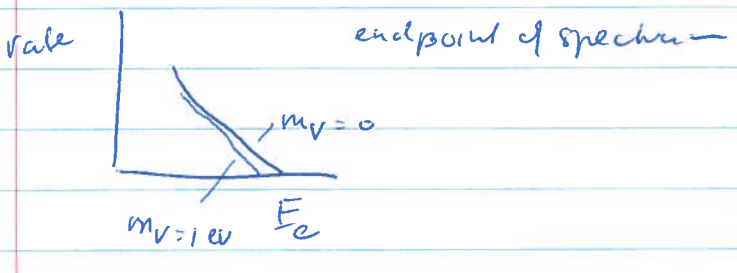
Before oscillations: measurement with ν from a dominant flavour

$m_{\nu_e} < 2$ eV end point tritium β decay

$m_{\nu_\mu} < 170$ keV

$m_{\nu_\tau} < 18.2$ MeV

In β decay, one really measures $|m_\beta| = \sum_i |U_{ei}| m_i$



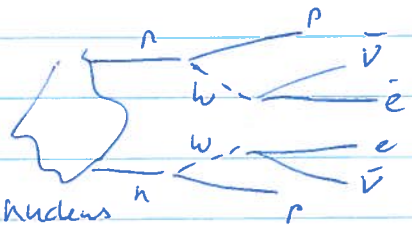
From cosmology :

Neutrino contribution to Ω (structure formation) $\sum_i m_{\nu_i} < 11$ eV

CMB + BAO + ... $\sum_i m_i <$ 0.12 eV (aggressive)
0.7 eV (less aggressive)

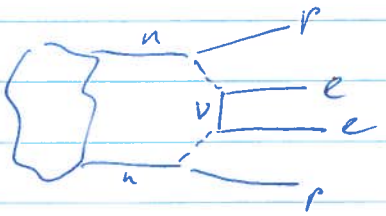
Note : if largest $\Delta m^2 = 2.3 \cdot 10^{-3}$ eV^2
then there must be a ν_i with $m_{\nu_i} \geq 0.05$ eV

Double β decay:



$2n \rightarrow 2p + 2e + 2\bar{\nu}_e$

Neutrinoless double β decay



$2n \rightarrow 2p + 2e$

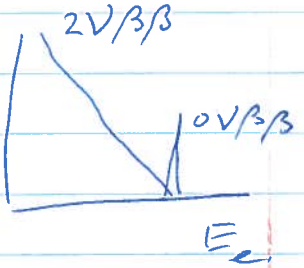
$0\nu\beta\beta$ decay : only if ν is Majorana $\nu = \bar{\nu}$

Decay rate $\sim |M|^2 \langle m_{\beta\beta} \rangle^2$ M nuclear matrix elements

$\langle m_{\beta\beta} \rangle = \sum_i U_{ei}^2 m_i$

Experimental measurement:

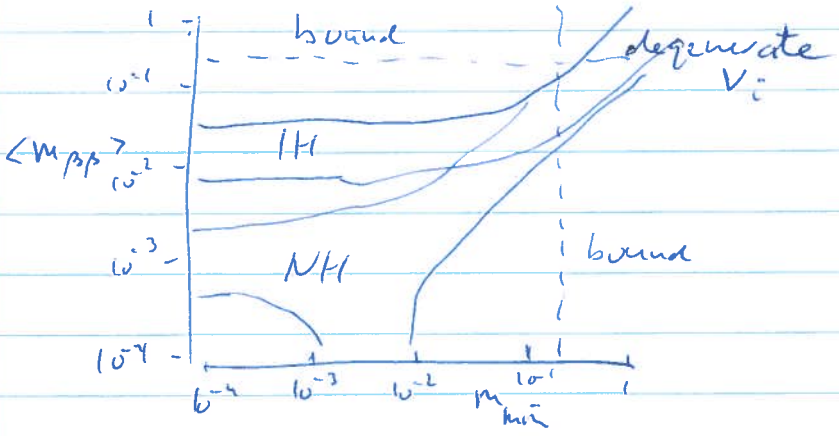
Needs an appropriate isotope



$2\nu\beta\beta$ has been observed, $t_{1/2} \sim 10^{18} - 10^{21}$ year

$0\nu\beta\beta$ only lower limits $t_{1/2} > 10^{24} - 10^{25}$ years

$m_{\beta\beta} < 0.2 - 0.4$ eV (uncertainty!) @ 90% CL



Experiments demand very low backgrounds!