§2.3 The neutrino sector (neutrino masses and mixing)

The second natural extension of the Standard Model involves the neutrino sector, since there is not a unique way to implement neutrino masses.

Standard-Model-style neutrino masses: as already indicated on p. 22+23, an explicit Yukawa interaction $-(f_{\nu_e})_{AB}\,\bar{L}'_{A_L}\,\Phi^c\,\nu'_{B_R}$ + h.c. can be added to the Standard Model in order to obtain neutrino mass terms $-\frac{v}{\sqrt{2}}(f_{\nu_e})_{AB}\,\bar{\nu}'_{A_L}\,\nu'_{B_R}$ + h.c.. Here pure singlet right-handed neutrino modes have been added, which are not subject to electroweak or strong interactions. In analogy with the quark sector, gauge and mass eigenstates need not coincide in the lepton sector as well. This gives rise to lepton mixing in the charged-current interactions (Pontecorvo-Maki-Nakagawa-Sakata):

$$\bar{\nu}'_{A_L}\gamma_\mu e'_{A_L} = \bar{\nu}_{A_L}\gamma_\mu (S_\nu^\dagger S_e)_{AB}\, e_{B_L} \equiv \bar{\nu}_{A_L}\gamma_\mu (U^\dagger_{PMNS})_{AB}\, e_{B_L}$$

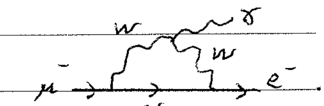
and
$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

with $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$.

This unitary PMNS-matrix describes lepton mixing. (being assigned to the neutrinos rather than the charged leptons). It involves 3 mixing angles and 1 CP-violating phase, just as argued for the CKM-matrix.

*) If all neutrino masses would be equal (e.g. all 0), then mixing would be irrelevant as each linear combination of mass eigenstates would be a mass eigenstate $\Rightarrow$ gauge eigenstate = mass eigenstate.

*) Neutrino mixing induces rare decays such as $\mu^- \to e^-\gamma$:



*) A non-unit lepton mixing matrix U_{PMNS} implies that the $\nu_{e,\mu,\tau}$ neutrinos, which are produced in weak-interaction processes, are linear superpositions of the neutrino mass eigenstates $\nu_{1,2,3}$. Quantum mechanically these three mass-eigenstate components evolve differently with time. As a result, the $\nu_{e,\mu,\tau}$ neutrinos can oscillate into each other while traveling a certain distance!

(neutrinos are stable)

This can be observed by measuring the neutrinos as gauge eigenstates (through weak int.).

(in the detector)

Neutrino oscillations were observed in 1998 @ Superkamiokande

$\Rightarrow (\nu_{e,\mu,\tau}) \neq (\nu_{1,2,3}) \Rightarrow$ massive neutrinos exist!

(see Paul's lecture)

Majorana neutrinos as an alternative: a generic mass term for fermions takes the form $-m(\bar{\psi}_{1_L}\psi_{2_R} + \bar{\psi}_{1_R}\psi_{2_L} + h.c.)$ for any combination of two spinor fields $\psi_{1,2}$. Since electroweak symmetry breaking leaves the $U(1)_{e.m.}$ symmetry intact, in order to guarantee that the photon remains massless, the fields $\psi_{1,2}$ are required to describe particles with the same electrostatic charge. We exploited this fact in the discussion of lepton mixing on p. 33. However, neutrinos are neutral and therefore do not possess quantum numbers that would allow us to differentiate between particles and antiparticles experimentally. We only know that a neutral fermion field should be combined with a charged lepton field to form a $SU(2)_L$ doublet under the weak interactions. The experimental evidence for this construction was based solely on helicity (chirality) considerations --- at no point did we have to demand that the doublet partners of the charged leptons should refer to particles rather than antiparticles. This opens up a new avenue of possibilities: ψ_{2_R} could either be a pure singlet right-handed neutrino mode (as on p. 33) or a charge-conjugated version of the left-handed neutrino mode that features in a $SU(2)_L$ doublet. Before we go into the details of this additional option, we first collect some properties of charge conjugation of spinor fields.

Intermezzo on charge conjugation for fermions:

$C = i\gamma^2\gamma^0$ in Dirac repr.

$$\psi \xrightarrow[\text{conj.}]{\text{charge}} \psi^c \equiv C\bar{\psi}^T, \quad \text{with } C^\dagger = C^{-1} \text{ such that } C^{-1}\gamma^\mu C = -(\gamma^\mu)^T.$$

$$\Rightarrow C^{-1}\gamma^5 C = (\gamma^5)^T$$

$$\text{Hence,}\quad \bar{\psi}^c \equiv (C\bar{\psi}^T)^\dagger\gamma^0 = (\psi^\dagger\gamma^0)^{T\dagger}C^{-1}\gamma^0 = \psi^T(\gamma^0)^T C^{-1}\gamma^0 = -\psi^T C^{-1}\underbrace{\gamma^0 C C^{-1}\gamma^0}_{I} = -\psi^T C^{-1}.$$

$$L/R \xrightarrow{C} R/L: \quad (\psi_{R/L})^c = C(\bar{\psi}_{R/L})^T = C P_{L/R}^T\bar{\psi}^T = C[C^{-1}P_{L/R}C]\bar{\psi}^T = P_{L/R}\psi^c,$$

$$(\bar{\psi}_{R/L})^c = -(\psi_{R/L})^T C^{-1} = -\psi^T P_{R/L}^T C^{-1} = -\psi^T[C^{-1}P_{R/L}C]C^{-1} = \bar{\psi}^c P_{R/L}.$$

By means of charge conjugation a left-handed fermion field is turned into a right-handed fermion field. A useful identity for the following discussion will be:

$$\bar{\psi}_R\psi_L = \bar{\psi}P_L P_L\psi \xrightarrow{\text{fermions}} -\psi^T P_L^T P_L^T\bar{\psi}^T = \underbrace{-\psi^T C^{-1}}_{\bar{\psi}^c}P_L\underbrace{CC^{-1}}_{I}P_L\underbrace{C\bar{\psi}^T}_{\psi^c} = \bar{\psi}^c P_L P_L\psi^c = (\bar{\psi}_L)^c(\psi_R)^c$$

and similarly $\bar{\psi}_L\psi_R = (\bar{\psi}_R)^c(\psi_L)^c$. Finally, since $C^T = -C$ we have $(\psi^c)^c = \psi$.

<u>Most general Lagrangian for the free neutrino:</u>

$$\mathcal{L}_\nu^{free} = \bar\nu\, i\gamma^\mu\partial_\mu \nu - m_D(\bar\nu_L \nu_R + h.c.) - \frac{m_L}{2}\big[\overbrace{(\bar\nu_L)^c \nu_L}^{\text{part of doublet}} + h.c.\big] - \frac{m_R}{2}\big[\overbrace{(\bar\nu_R)^c \nu_R}^{\text{pure singlet}} + h.c.\big] .$$

Next we introduce two so-called <u>Majorana fields</u> $N_1 \equiv \dfrac{\overset{L}{\nu_L} + \overset{R}{(\nu_L)^c}}{\sqrt2}$ and $N_2 \equiv \dfrac{\overset{R}{\nu_R} + \overset{L}{(\nu_R)^c}}{\sqrt2}$ comprised entirely of ν_L or ν_R. These fields satisfy the Majorana condition $N_i = N_i^c$ $(i=1,2)$. <u>Lepton number is violated for $m_{L/R} \neq 0$</u>, since ν and ν^c have <u>opposite fermion number</u>.

half the d.o.f. of a Dirac field

Relations: $\bar N_1 N_1 = \frac12[\bar\nu_L\nu_L + (\bar\nu_L)^c(\nu_L)^c + \bar\nu_L(\nu_L)^c + (\bar\nu_L)^c\nu_L] = \frac12[(\bar\nu_L)^c\nu_L + h.c.]$,

$\bar N_2 N_2 = \frac12[(\bar\nu_R)^c\nu_R + h.c.]$,

$\bar N_1 N_2 + \bar N_2 N_1 = \frac12[\bar\nu_L\nu_R + (\bar\nu_L)^c(\nu_R)^c + h.c.] = \bar\nu_L\nu_R + h.c.$

$$\Rightarrow \mathcal{L}_\nu^{free} = \bar N_1 i\gamma^\mu\partial_\mu N_1 + \bar N_2 i\gamma^\mu\partial_\mu N_2 - \overbrace{m_D}^{\text{Dirac mass}}(\bar N_1 N_2 + \bar N_2 N_1) - \overbrace{m_L \bar N_1 N_1 - m_R \bar N_2 N_2}^{\text{Majorana masses}}$$

$$= \bar N_1 i\gamma^\mu\partial_\mu N_1 + \bar N_2 i\gamma^\mu\partial_\mu N_2 - (\bar N_1, \bar N_2)\underbrace{\begin{pmatrix} m_L & m_D \\ m_D & m_R\end{pmatrix}}_{\equiv\,[M^\nu]\ \text{neutrino mass matrix}}\begin{pmatrix} N_1 \\ N_2\end{pmatrix} .$$

<u>Special cases:</u> 1) $m_{L,R} = 0,\ m_D \neq 0 \Rightarrow$ Dirac-mass case (like in Standard Model), with the Dirac field written as two Majorana fields with identical mass: $\nu = \nu_L + \nu_R = \sqrt2(P_L N_1 + P_R N_2) = \frac{N_1+N_2}{\sqrt2} - \gamma^5\frac{N_1 - N_2}{\sqrt2}$. mass m_D

cf. complex number written as two real numbers

2) $m_R = m_D = 0,\ m_L \neq 0 \Rightarrow$ <u>the neutrino mass is obtained by solely resorting to the ν_L field</u> and lepton number is violated by the mass term $-m_L \bar N_1 N_1$. The <u>massive neutrino</u> is a <u>Majorana neutrino</u> in this case!

we can do without ν_R!

3) $m_L = 0,\ m_{R,D} \neq 0 \Rightarrow$ in this case ν_R is needed for all mass terms. The mass term $-m_R \bar N_2 N_2$ violates lepton number, just as in case 2. The mass eigenvalues equal $\frac12\{m_R \pm \sqrt{m_R^2 + 4m_D^2}\}$, which opens up a theoretically very interesting possibility: <u>if $m_R \gg m_D$, then M^ν has one very small and one very large eigenvalue</u>

Seesaw mechanism
$$\begin{cases} \boxed{-m_\nu \approx -m_D^2/m_R} \rightarrow \text{eigenstate } \nu' = N_1 - N_2\, m_D/m_R \\ m_N \approx m_R \rightarrow \text{eigenstate } N = N_2 + N_1\, m_D/m_R \end{cases}$$

two Majorana fields with different mass instead of one Dirac field!

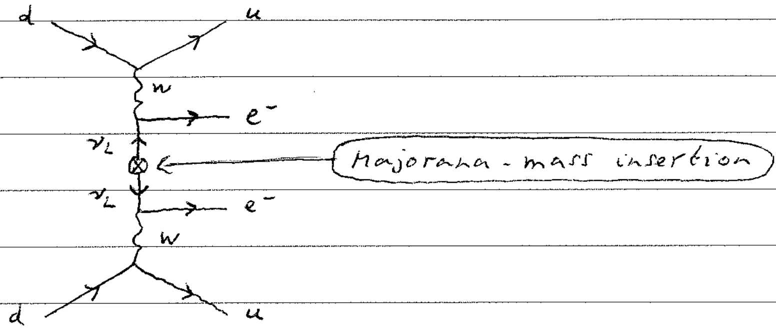
$\Rightarrow$ <u>ν' dominated by "observable" ν_L mode and naturally light,</u>
<u>N dominated by unobservable ν_R mode and very heavy.</u>

Remark: a negative value for the mass in a fermionic mass term can be straightforwardly turned into a positive value by writing

$$\psi = \gamma^5\psi', \quad \bar\psi = (\gamma^5\psi')^\dagger\gamma^0 = \psi'^\dagger\gamma^5\gamma^0 = -\bar\psi'\gamma^5 \quad\Rightarrow\quad \bar\psi\psi = -\bar\psi'\psi'.$$

Neutrinoless double-β decay: a Majorana mass term $-\frac{m_L}{2}[(\bar\nu_L)^c\nu_L + h.c.]$ violates lepton number by two units, since ν_L and $(\bar\nu_L)^c$ have the same lepton number $L=+1$. This induces the possibility of neutrinoless double-β decay, such as $\underset{(\bar u s)}{K^-} \to \underset{(u\bar d)}{\pi^+} e^- e^-$ or nucleus $(A,Z) \to$ nucleus$'(A,Z+2)\, e^-e^-$:

e.g.



This would be the smoking gun for Majorana fermions

Sources of the ν-mass terms: in order to discuss the symmetry breaking aspects that underlie the above-given neutrino mass terms, we first list the Standard Model quantum numbers of the relevant neutrino modes

ν_L : $Q=0$, $y=-1$, $I_3=1/2 \longrightarrow \bar\nu_L, (\nu_L)^c$: $Q=0$, $y=+1$, $I_3=-1/2$,

ν_R and $(\nu_R)^c$: $Q=y=I=0$ (pure singlet sterile neutrino).

Case 1: $m_D\,\bar\nu_L\nu_R$ follows from $\bar\nu_L\langle\Phi^c\rangle_0\nu_R$ provided that $\langle\Phi^c\rangle_0$ corresponds to $y=-1$, $I_3=1/2$, as is indeed the case for $\Phi^c = i\tau^2\Phi^*$ in the Standard Model.

Case 2: $m_L(\bar\nu_L)^c\nu_L$ follows from $(\bar\nu_L)^c\nu_L\langle\chi\rangle_0$ provided that $\langle\chi\rangle_0$ corresponds to $y=2$, $I_3=-1$. In this case χ should (at least) be part of an isospin triplet $(I=1)$.

Case 3: the mass term $-\frac{m_R}{2}[(\bar\nu_R)^c\nu_R + h.c.]$ can be added without breaking gauge invariance (being a pure singlet). So, either no additional scalar field is needed or a singlet scalar field should be added to provide m_R.

Going beyond the Standard Model and naturally small ν-masses:

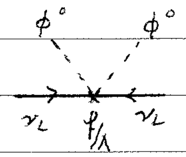
$\boxed{m_L}$: the Majorana mass term $\propto m_L$ can be generated directly by adding a scalar isospin triplet $\chi = \begin{pmatrix} H^{++} \\ H^+ \\ H^0 \end{pmatrix}$ to the Standard Model, giving rise to six additional physical degrees of freedom ($H^{++}, H^{--}, H^{\pm}$, ρ, massless Majoron). This, however, will have an

$\hookrightarrow \mathrm{Re}\,H^0, \mathrm{Re}\,\phi^0$ $\quad\hookrightarrow \mathrm{Im}\,H^0, \mathrm{Im}\,\phi^0$

$\langle \phi^0 \rangle_0 = v/\sqrt{2}$ $\quad$ $\langle H^0 \rangle_0 = v_\chi/\sqrt{2}$

impact on the ρ-parameter at leading order: $\rho = \dfrac{\frac{1}{2}v^2 + v_\chi^2}{\frac{1}{2}v^2 + 2v_\chi^2} = \dfrac{v^2 + 2v_\chi^2}{v^2 + 4v_\chi^2}$ (see p.28)

$\Rightarrow$ the very precisely measured value of ρ severely constrains the ratio v_χ^2/v^2!
In order to avoid the ρ-parameter constraints, we could restrict ourselves to the Standard Model doublet fields for providing the mass term $\propto m_L$.
To this end two Higgs doublets ($I = 1/2$) have to be combined into a scalar triplet ($I=1$), resulting in a non-renormalizable (effective) Lagrangian term containing $\underbrace{-(\bar{\nu}_L)^c \nu_L \phi^0 \phi^0}_{\text{dimension}=5} \dfrac{f}{\Lambda}$:



$\leftarrow$ should follow from an underlying new physics model @ scale Λ that allows lepton number violation (see below).

natural coupling strength

Plugging in the vev: $m_L = f v^2/\Lambda \lesssim \mathcal{O}(0.1\,\text{eV})$ for $v = 246$ GeV and $f = \mathcal{O}(1)$ if $\Lambda \gtrsim \mathcal{O}(10^{15}\,\text{GeV})$ $\Rightarrow$ would explain the smallness of the neutrino mass, something which the Standard Model cannot do!

$\boxed{m_R}$: in principle no new physics is strictly needed, but it might be used to set the scale of $m_R \gg m_D$ that would trigger the seesaw mechanism
$\Rightarrow m_\nu = m_D^2/m_R \lesssim \mathcal{O}(0.1\,\text{eV})$ for $m_D \leq m_\tau = 1.78$ GeV if $m_R = \mathcal{O}(10^{11}\,\text{GeV})$,
naturally linking the neutrino mass to the charged lepton mass!

SM singlet

Concept: larger gauge group $\xrightarrow[\text{scale }\mu_1]{\langle \Phi_1 \rangle_0}$ $SU(3)_c \times SU(2)_L \times U(1)_Y$ $\xrightarrow[\text{scale }\mu_2]{\langle \Phi_2 \rangle_0}$ $SU(3)_c \times U(1)_{e.m.}$

such that $m_R \sim \mu_1 \gg \mu_2 \sim m_D$.

Example: Left-Right symmetric model, which postulates perfect parity invariance (non-chirality) at high energies and breaks it spontaneously at a certain lower energy scale

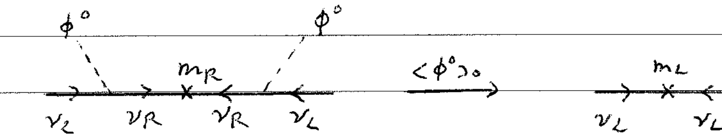
occurs in certain GUT models

$SU(2)_R \times SU(2)_L \times U(1) \xrightarrow{\langle \Phi_R \rangle_0} SU(2)_L \times U(1)_Y \xrightarrow{\langle \Phi_L \rangle_0} U(1)_{e.m.}$ $\quad$ as in SM

gives masses to $SU(2)_R$ gauge bosons $W_R^\pm, Z_R$

The coupling of $W_R^\pm$ to fermion doublets is suppressed by $\sim m_W^2/m_{W_R}^2$ and right-handed fermion doublets behave approximately as $SU(2)_L$ singlets.

m_L from m_D and m_R:



with $m_L = \mathcal{O}(m_D^2/m_R)$.

Additional aspects of Majorana neutrinos:

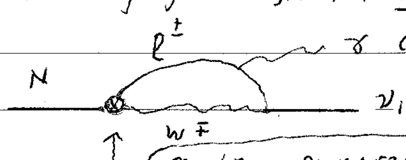
- Majorana fields are not capable of absorbing phases, so if the active (weak interaction) neutrinos happen to be Majorana fermions then two unabsorbable phases have to be added to the PMNS matrix:

$$U_{PMNS} \to U_{PMNS} \begin{pmatrix} e^{i\varphi_1/2} & & 0 \\ & e^{i\varphi_2/2} & \\ 0 & & 1 \end{pmatrix} \leftarrow \text{more sources of CP violation}$$

- Sterile neutrinos can be added at will to the Standard Model, since they only interact gravitationally. Such a minimal (neutrino) extension of the Standard Model is referred to as the νMSM. If the associated m_R coefficients are large enough we may end up with two distinct sets of three mass eigenstates: three very light Majorana states that are dominated by the three ν_L modes (active neutrinos) and three substantially heavier Majorana states that are dominated by the sterile ν_R modes. Since the gauge eigenstates are admixtures of all six mass eigenstates (although some mixing parameters m_D/m_R may be rather small), the 3x3 matrix U_{PMNS} will not be unitary anymore. Moreover, the heavier states might help us solve some of the problems that the Standard Model has left behind.

*) $\mathcal{O}$(10 keV) Majorana neutrinos might be viable candidates for dark matter!
These particles are only subject to gravitational interactions and m_D/m_R-suppressed weak interactions (ν_L component in N), and they are sufficiently heavy to escape cosmological constraints (e.g. too fast diffusion).
↑ light ν's not suited

Smoking gun signal: monochromatic X-ray photons from its decay



↑ m_D/m_R suppressed ν_L component

← e.g. 3.56 keV line seen by ESA's X-ray Multi-Mirror Mission (XMM-Newton) in 2014

*) $\gtrsim \mathcal{O}(10^8$ GeV) Majorana neutrinos might help explain the matter-antimatter asymmetry in the universe: Majorana neutrinos in principle decay democratically into leptons and antileptons (as they are their own antiparticles). However, owing to CP violation in the lepton sector the balance can tilt slightly towards the matter side!