

*) Yang-Mills interactions between three or four gauge bosons:
 the kinetic gauge-boson term $\frac{1}{2} \text{Tr}(W_{\mu\nu} W^{\mu\nu}) = -\frac{i}{2} w_{\mu\nu}^a w^{a\mu\nu}$
 contains three distinct terms involving two, three and four
 gauge fields $-\frac{1}{4} (\partial_\mu w_\nu^a - \partial_\nu w_\mu^a)(\partial^\mu w^\nu - \partial^\nu w^\mu)$,

(just like in QED)

$$\begin{array}{c} \text{triple gauge} \\ \text{couplings (TGC)} \end{array} \rightarrow +g(\partial_\mu w_\nu^a) w^{b,\mu} w^{c,\nu} f^{abc}, \quad \text{absent in QED: } Q_8 = 0$$

$$\begin{array}{c} \text{quartic gauge} \\ \text{couplings (QGC)} \end{array} \rightarrow -\frac{g^2}{4} f^{abe} f^{cde} w_\mu^a w_\nu^b w_\lambda^c w^\mu_\lambda$$

Feynman rules for interaction vertices: — = fermion, ~~~ = gauge boson

$$\begin{array}{c} l \\ j \end{array} \xrightarrow{a,m} \sim = -ig \gamma^\mu (T^a) \frac{e_j}{e_l} \quad (l, j = \text{multiplet label}; a = \text{gauge-boson label}),$$

used: $\partial_\lambda \rightarrow -i \cdot \vec{v}$ (incoming momentum)

$$\begin{array}{c} a,m \\ b,\nu \\ c,\rho \\ d,\sigma \end{array} \xrightarrow{q} = -ig f^{abc} [g^{m\nu} \delta^a_{\rho} + g^{n\rho} \delta^a_{\nu} + g^{m\rho} \delta^a_{\nu}],$$

$$\begin{array}{c} a,m \\ b,\nu \\ c,\rho \\ d,\sigma \end{array} \xrightarrow{q} = -ig^2 [f^{abe} f^{cde} (g^{m\nu} g^{ro} - g^{m\rho} g^{n\nu}) + f^{ace} f^{bde} (g^{m\nu} g^{ro} - g^{m\rho} g^{n\nu}) \\ + f^{ade} f^{bce} (g^{m\nu} g^{ro} - g^{m\rho} g^{n\nu})],$$

$$\begin{array}{c} j \\ i \end{array} \xrightarrow{p} \ell = \frac{i(p+m)}{p^2 - m^2 + i\varepsilon} \frac{d\ell_i}{dp}, \quad a_m \xrightarrow{p} b_\nu = \frac{-ig_{\mu\nu}}{p^2 + i\varepsilon} \frac{d\ell_i}{dp}.$$

Pauli spin matrices

$$\Rightarrow w_\mu = \frac{1}{2} w_\mu^a \sigma^a = \frac{1}{2} \vec{w}_\mu \cdot \vec{\sigma} = \frac{1}{2} \begin{pmatrix} w_\mu^3 & \frac{1}{2} w_\mu^+ \\ w_\mu^- + i w_\mu^2 & -w_\mu^3 \end{pmatrix} = \frac{1}{2} w_\mu^+ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = T w_\mu^+ + \frac{T^+ w_\mu^- + T^- w_\mu^+}{\sqrt{2}},$$

$$u(x) = e^{-\frac{i}{2} q(x) \sigma^a} = e^{-\frac{i}{2} \vec{q}(x) \cdot \vec{\sigma}} = \cos(\varphi(x)/2) I_2 + i \frac{\vec{q}(x) \cdot \vec{\sigma}}{\varphi(x)} \sin(\varphi(x)/2),$$

$$\text{with } q(x) = |\vec{q}(x)| = \sqrt{q^a(x) q^a(x)}.$$

This gauge group will be needed for the gauge-theory description of the weak interactions and the associated w^\pm and Z gauge bosons [although only part of the Z gauge boson belongs to $SU(2)$].

* SU(3): matter triplets and gauge octets, $T^a = \lambda^a/2$ ($a=1, \dots, 8$)
in terms of the Gell-Mann λ -matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

This gauge group is used in the gauge-theory description of the strong interactions, called Quantum Chromodynamics (QCD).
The associated gauge bosons are called gluons.

↑ indicated by ~~for~~

Chapter 2: Building towards the Standard Model

In the next step we discuss the experimental input that has led to the gauge-group ingredients of the Standard Model, which is a gauge-theory description of the strong, weak and hypercharge interactions based on the gauge group $\underbrace{\text{SU}(3)_c}_{\text{strong int.}} \times \underbrace{\text{SU}(2)_L}_{\text{electroweak int.}} \times \underbrace{\text{U}(1)_Y}_{\text{hypercharge}}$. The electromagnetic

interactions are automatically contained in this description, hidden in the electroweak $\text{SU}(2)_L \times \text{U}(1)_Y$ sector.

$\text{SU}(3)_c$, c=colour: matter triplets (quarks) and gauge octets (gluons)

Quantum Chromodynamics (QCD) \rightarrow Gauge coupling: g_s = strong interaction strength.

$$\begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$$

$$T^a_{\mu\nu} \quad (a=1, \dots, 8)$$

Why? *) spin- $\frac{1}{2}$ quarks confined inside hadrons have an intrinsic property (quantum number) called colour \Rightarrow colour multiplets!

This follows from the existence of the Δ^{++} baryon (=unbound state with $L=0$ and $S=3/2$) Fermions \rightarrow at least three colours!

↑ spatial + spin symmetric quarks

*) Consider the reaction $e^+e^- \rightarrow p\bar{p}$ ($p\bar{p} = \pi^+, \pi^-, (\eta)$) for unpolarized e^+e^- beams, which is dominated by virtual photon exchange at $\mathcal{O}(\text{GeV})$ energies:



$$\sigma_{e^+ e^- \rightarrow p\bar{p}}^{\text{unpol.}} \underset{\text{QFT}}{\text{Ex. 22}} \frac{\pi q^2 Q_F^2}{3E^2} \sqrt{1 - m_F^2/E^2} \left[1 + \frac{m_F^2}{2E^2} \right] \Theta(E - m_F)$$

for each type of fermion f .

production threshold

threshold for different flavours of quarks

$$\text{Hence, } R = \frac{\sigma_{e^+ e^- \rightarrow \text{hadrons}}}{\sigma_{e^+ e^- \rightarrow \mu^+ \mu^-}^{\text{unpol.}}} \underset{Q_F = -1}{\underset{E \gg m_\mu}{\rightarrow}} \sum_q Q_q^2 \sqrt{1 - m_q^2/E^2} \left[1 + \frac{m_q^2}{2E^2} \right] \Theta(E - m_q)$$

$$\text{Given that each quark flavour } q = \underset{\downarrow}{d}, \underset{\uparrow}{u}, \underset{\downarrow}{s}, \underset{\uparrow}{c}, \underset{\downarrow}{b}, \underset{\uparrow}{t} \quad Q_d = +2/3 \text{ charge}$$

$$Q_s = -1/3 \text{ charge}$$

comes in N_c colours, R will display resonances followed by steps given by $R \rightarrow N_c \sum_{\text{active flavours}} Q_q^2$ involving all (active) flavours for which $m_q \ll E$.

$E > 1.5 \text{ GeV}$: d, u, s, c active flavours $\Rightarrow R$ approaches $\frac{10}{9} N_c$ in the limit that $E \gg m_c$.

$E > 4.7 \text{ GeV}$: d, u, s, c, b active flavours $\Rightarrow R$ approaches $\frac{11}{9} N_c$ in the limit that $E \gg m_b$.

see figure

Data suggest that $N_c = 3$. Also the decay $\pi^0 \rightarrow 2\gamma$ supports $(\frac{1}{2} (u\bar{u} - d\bar{d}) \text{ bound state})$

This: $\pi^0 \rightarrow u, d \rightarrow u, d + \text{reversed loop, with } \Gamma_{\pi^0 \rightarrow 2\gamma} \propto N_c^2$

colour neutral \Rightarrow no preference for colour d.o.f.)

The weak interactions: the matter particles that are subject to weak interactions are grouped into isospin doublets and the associated interactions are governed by an $SU(2)$ gauge theory. However, each Dirac fermion has one spin-related mode that is subject to weak interactions (left-handed part) and one mode that is not (right-handed part). The latter modes are therefore supposed to not transform under $SU(2)$ gauge transformations, i.e. they form singlets under $SU(2)$.

e.g. $(\begin{matrix} \nu_L \\ e_L \end{matrix})^L$: isospin $I = 1/2$

$L = \text{left-handed chirality}$: matter doublets (left-handed quarks/leptons), matter singlets (right-handed quarks/leptons) and gauge-boson triplets.

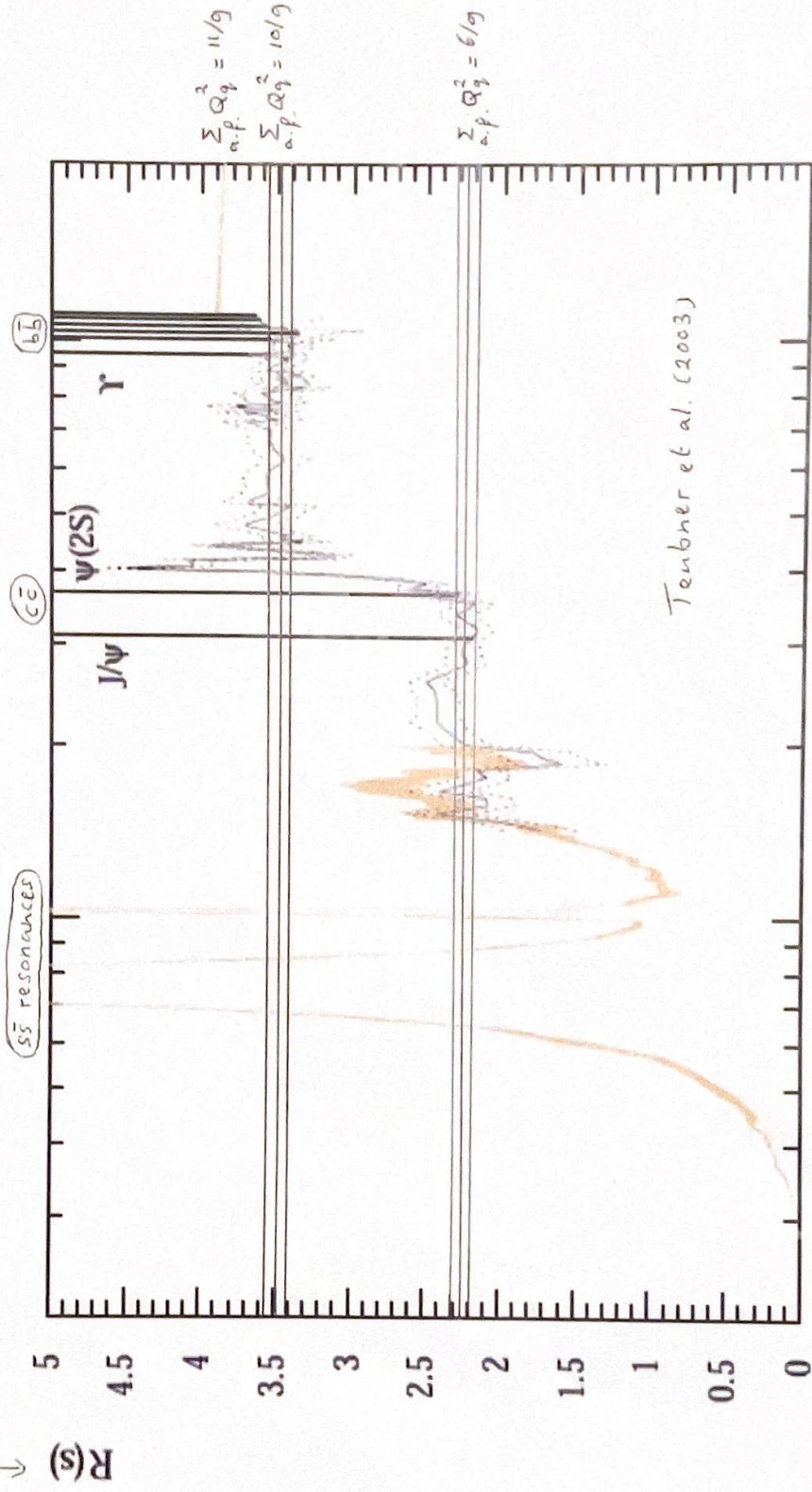
W_μ^a ($a = 1, 2, 3$)

e.g. ν_R, e_R : isospin $I = 0$

Gauge coupling: g = weak interaction strength.

using that $R(s) = N_c \sum_{\text{act.}} Q_q^2 \left\{ 1 + \frac{\gamma_F(s)}{s} + \dots \right\}$ from perturbative QCD

\uparrow active flavours for which $m_q \ll E$



$$R \equiv \frac{\sigma_{e^- e^- \rightarrow \text{hadrons}}^{\text{unpol.}}}{\sigma_{e^+ e^- \rightarrow \mu^+ \mu^-}^{\text{unpol.}}} : \frac{e^+ e^- \rightarrow \bar{p} p}{e^+ e^- \rightarrow \bar{\mu} \mu} + \text{higher orders,}$$

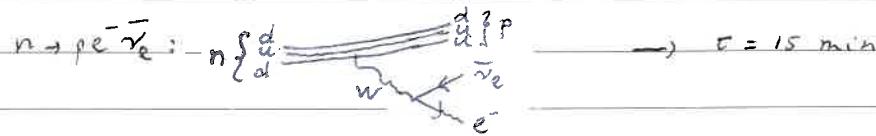
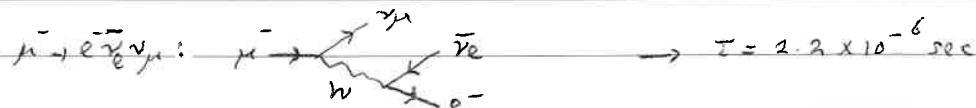
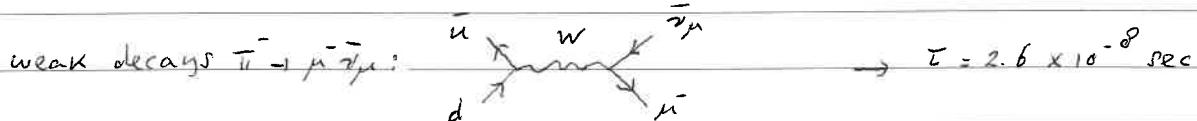
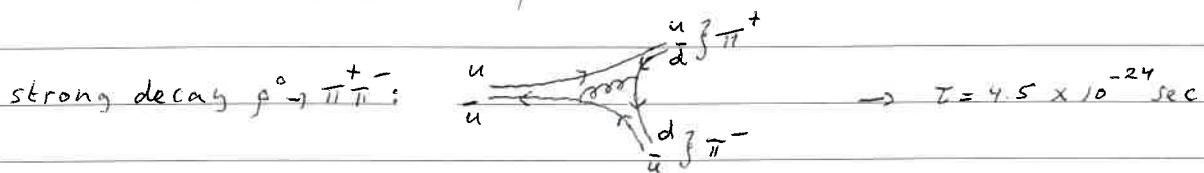
$$\sigma_{e^+ e^- \rightarrow \bar{p} p}^{\text{unpol.}} \approx \frac{\pi^2 Q_F^2}{9 E^2} \sqrt{1 - \frac{m_F^2}{E^2}} \left[1 + \frac{m_F^2}{2 E^2} \right] \Theta(E - m_F) \xrightarrow{E \gg m_F} \frac{\pi^2 Q_F^2}{3 E^2} \quad \text{for each type of fermion } F$$

$E \ll M_Z$

Let's go through the experimental evidence:

- * The existence of slow decays like $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$, $\mu^- \rightarrow e^- \bar{\nu}_e$, and p -decay $n \rightarrow p \bar{\nu}_e$ with lifetimes $\tau > 10^{-8}$ sec hints at a new type of interaction, bearing in mind that strong interaction decays have $\tau \sim 10^{-23}$ sec and electromagnetic decays have $\tau \sim 10^{-16}$ sec \Rightarrow weak interactions (much weaker than the electromagnetic interactions at low energies)!

e.m. decay $\pi^0 \rightarrow 2\gamma$ (see before) $\rightarrow \tau = 8.4 \times 10^{-17}$ sec



- * Most weak interactions appear to be charge-current interactions, which couple fermions of non-opposite charge (unlike e.m. interactions)

\Rightarrow hints at doublet structure!

($e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$, $n \bar{p}$, ...)

in fact: $d \bar{u}$

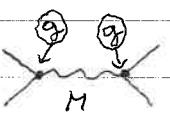
- * Fixed interaction strength G_F at low energies between 4 fermions:
 - short-range interactions (unlike e.m. and gluon-mediated int.)!

concept: if $E = 1/\lambda_{\text{FB}} \ll 1/a$ (a = range int.), only the net effect of 1/mass force carrier

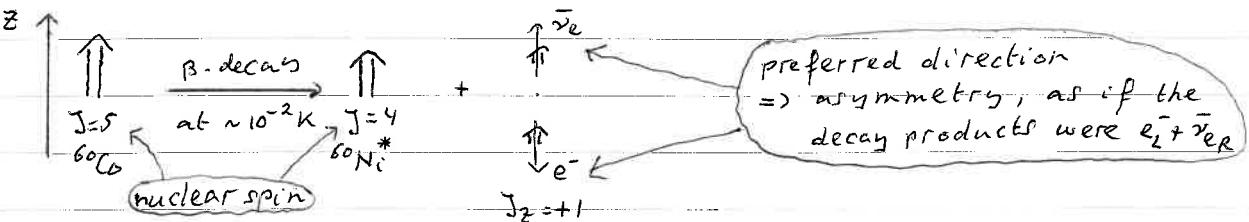
The interaction potential is felt (i.e. not enough energy for resolving details of the interaction potential)

\Rightarrow effectively constant interaction strength [unlike QED where $a \rightarrow \infty$ in view of $m_\gamma = 0 \Rightarrow$ in that case there will always be sensitivity to the detailed form of the interaction potential].

Fermi-coupling $G_F \approx 10^{-5} \text{ GeV}^{-2} = \mathcal{O}(e^2/M^2)$ with $M = \mathcal{O}(100 \text{ GeV})$ the mass of the force carrier (gauge boson)!

Concept:  $\propto \frac{i g^2}{q^2 - M^2} \xrightarrow{q^2 \ll M} \frac{-i g^2}{M^2}$ ~~\propto~~ effective coupling with mass dimension -2.

* The decay $^{60}\text{Co} \rightarrow ^{60}\text{Ni}^* + e^- + \bar{\nu}_e$ displays a preferred decay direction in the presence of an external magnetic field along the z-axis



If the interaction current would be like in QED/QCD [current $\propto \bar{\psi} \gamma^\mu \psi$], then we would expect an equal number of e^- to be emitted parallel and antiparallel to the magnetic field.

similar experiments: in charged-current weak interactions only $\bar{\nu}_L e_R$, e_L^+ and $\bar{\nu}_R e_R^+$ feature [corresponding to Dirac fields with left-handed chirality: $\psi_L = \frac{1}{2}(I_4 - \sigma^5)\psi$, $\bar{\psi}_L = (\psi^\dagger \frac{1}{2}(I_4 - \sigma^5))\bar{\psi} = \bar{\psi}^\dagger \frac{1}{2}(I_4 + \sigma^5)$] and no $\bar{\nu}_R$ or $\bar{\nu}_L$ are "observed"!

\Rightarrow hints at a so-called chiral gauge theory, which treats L/R modes differently! This implies the following for weak interactions:

- parity is violated, since $L/R \xrightarrow{P} R/L$.

Indeed: $\Gamma(\pi^+ \rightarrow \mu_R^+ \bar{\nu}_{\mu L}) \neq \Gamma(\pi^+ \rightarrow \mu_L^+ \bar{\nu}_{\mu R}) = 0$ (P violated maximally).

- charge-conjugation invariance is violated, since $\nu_L \xrightarrow{C} \bar{\nu}_L$.

Indeed: $\Gamma(\pi^+ \rightarrow \mu_R^+ \bar{\nu}_{\mu L}) \neq \Gamma(\pi^- \rightarrow \mu_R^- \bar{\nu}_{\mu L}) = 0$ (C violated maximally).

- CP could still be conserved.

Indeed: $\Gamma(\pi^+ \rightarrow \mu_R^+ \bar{\nu}_{\mu L}) = \Gamma(\pi^- \rightarrow \mu_L^- \bar{\nu}_{\mu R})$ (CP conserved).

Difference between vector (QED/QCD-like) theories and chiral theories:

* vector theories are democratic w.r.t. the two chiral modes and therefore conserve parity (which interchanges chiral modes).

Proof: $P_L = \frac{1}{2}(I_2 - \gamma^5) = P_L^2$, $P_R = \frac{1}{2}(I_2 + \gamma^5) = P_R^2$, $P_L + P_R = I_2$ with $(\gamma^5)^2 = I_2$, $\gamma^\mu \gamma^5 \gamma^\nu = 0$ and $(\gamma^5)^\dagger = \gamma^5$

$P_{L/R}$ project on L/R chiral modes:

$$\psi_{L/R} = P_{L/R} \psi, \bar{\psi}_{L/R} = (P_{L/R} \psi)^\dagger = \psi^\dagger P_{L/R} \gamma^0 = \bar{\psi} P_{L/R}$$

$$\Rightarrow \bar{\psi} \gamma_\mu \psi = \bar{\psi} \gamma_\mu (P_L^2 + P_R^2) \psi = \bar{\psi} P_R \gamma_\mu P_L \psi + \bar{\psi} P_L \gamma_\mu P_R \psi$$

$\xrightarrow{\text{vector current}}$ $= \bar{\psi}_L \gamma_\mu \psi_L + \bar{\psi}_R \gamma_\mu \psi_R$ (conserving parity).

under the corresponding local gauge transformations chiral states transform in the same way, i.e. $\psi_{L/R} \rightarrow \psi'_{L/R} = u(x) \psi_{L/R}$.

*) chiral theories treat different chiral modes differently and thereby implicitly violate parity. under the corresponding local gauge transformations chiral states transform differently.

Example: under $SU(2)_L$, $\psi_L \rightarrow \psi'_L = u(x) \psi_L$ and $\psi_R \rightarrow \psi'_R = \psi_R$.

A chiral $SU(2)_L$ gauge theory for the weak interactions requires three massive gauge bosons, two responsible for the charge-current (CC) weak interactions [w^\pm corresponding to the generators $T^\pm = T^1 \pm i T^2 = \frac{1}{2}(\sigma^1 \pm i \sigma^2)$] and one predicted to be responsible for the neutral-current (NC) weak interactions [w^0 corresponding to the generator $T^3 = \sigma^3/2$]. The existence of this third massive vector boson was confirmed experimentally by the not photon, since $SU(2)_L$ is chiral and U(1)e.m. is not!

*) observation of weak NC effects in 1973 @ CERN:

$$\text{e.g. } \bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-, \bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X, \bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X,$$

$\xrightarrow{\text{nucleon}}$ $\xrightarrow{\text{anything}}$

↓
neutral!

*) discovery of the massive gauge bosons w^\pm and Z in 1983 at the CERN $p\bar{p}$ experiments UA1 and UA2.

Including electromagnetic interactions: the most economic gauge group for the electroweak sector (= weak + e.m. interactions) would therefore be $SU(2)_L \times U(1)_Y$. The generator T_Y belonging to this U(1) group has to commute with the $SU(2)$ generators T^+, T^- and $T^3 \Rightarrow [T_Y, T^\pm] = 0$, i.e. the Y quantum number is not affected by raising/lowering \Rightarrow each $SU(2)$ multiplet has one Y quantum number, i.e. $U(1)_Y$ does not refer to the electromagnetic gauge group as the neutrino and electron have different charges!