

The experimental results on V_{CKM} can be summarized in the Wolfenstein parametrization:

$$V_{CKM} \approx \begin{pmatrix} -\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & -\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

good approximation

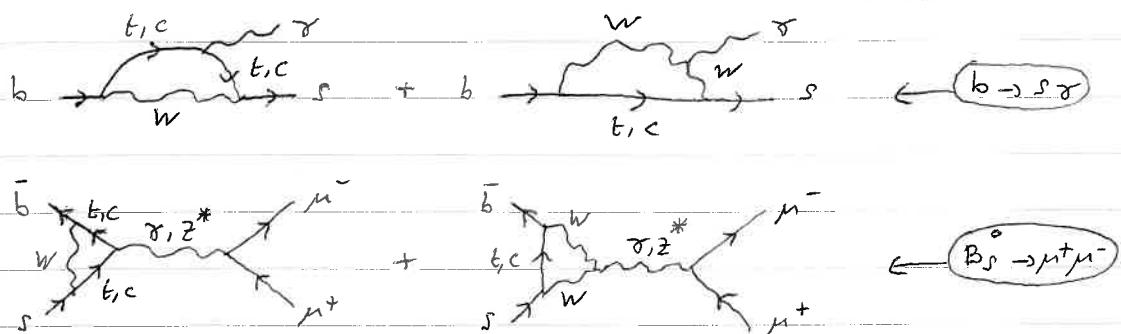
in fact: $|V_{CKM}| \approx \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 1.00 \end{pmatrix}_{AB}$

with $\lambda = 0.226 \pm 0.001$, $A = 0.81 \pm 0.02$, $\rho = 0.14 \pm 0.02$, $\eta = 0.35 \pm 0.02$.

\hookrightarrow pronounced hierarchy among components!

The mixing in the quark sector gives rise to some interesting phenomena.

* FCNC occur in higher loop order, inducing rare decays such as $b \rightarrow s\gamma$ and $B_s^0 \rightarrow \mu^+ \mu^-$:



The associated amplitudes are small due to the GIM mechanism (Glashow-Iliopoulos-Maiani, 1970):

$$\text{shorthand for } V_{CKM} \quad \text{spinors}$$

$$i \nu \xrightarrow{\text{W}} j \xrightarrow{\text{p}_1, m_j} k \xrightarrow{\mu, \kappa} \bar{\nu} \quad \propto \sum_j V_{kj} (V_{ji}^\dagger u_i) \bar{\nu} \gamma^\mu (I_4 - \gamma^5) \frac{p_1 + m_j}{p_1^2 - m_j^2} \bar{\nu} \gamma^\mu \frac{p_k + m_j}{p_k^2 - m_j^2} \bar{\nu} \gamma^\mu (I_4 - \gamma^5) u_i$$

$$V_{ji} \quad V_{jk}^* = V_{kj}^\dagger \quad \sum_j \frac{2 V_{kj} V_{ji}^\dagger}{(p_1^2 - m_j^2)(p_k^2 - m_j^2)} \bar{\nu} \gamma_\mu [p_1 \bar{\nu} p_k + m_j^2] \bar{\nu} \gamma^\mu (I_4 - \gamma^5) u_i$$

\Rightarrow suppression factors $V_{kj}^\dagger V_{ji}^\dagger$ for $k \neq i$, $g^2 m_j^2 / m_W^2$ if j light, and $g^2 m_i^2 / m_W^2$ if $i = \ell$.

needed: additional j dependence in order to avoid that $\sum_j V_{kj} V_{ji}^\dagger = \delta_{ki}$ (which would mean no FCNC)

Such FCNC processes put severe constraints on physics beyond the SM: models beyond the SM might give rise to additional generational mixing, involving new non-SM particles that feature in the FCNC loop diagrams

\Rightarrow constraints on the associated mixing matrices and/or the masses of the new particles of the model, in order not to exceed the amount of FCNC effects that we observe in experiment!

* Meson mixing effects + CP violation: let's consider the mixing effects in the $\bar{K}^0 - K^0$ (kaons) system, which give rise to energy eigenstates K_S, K_L .

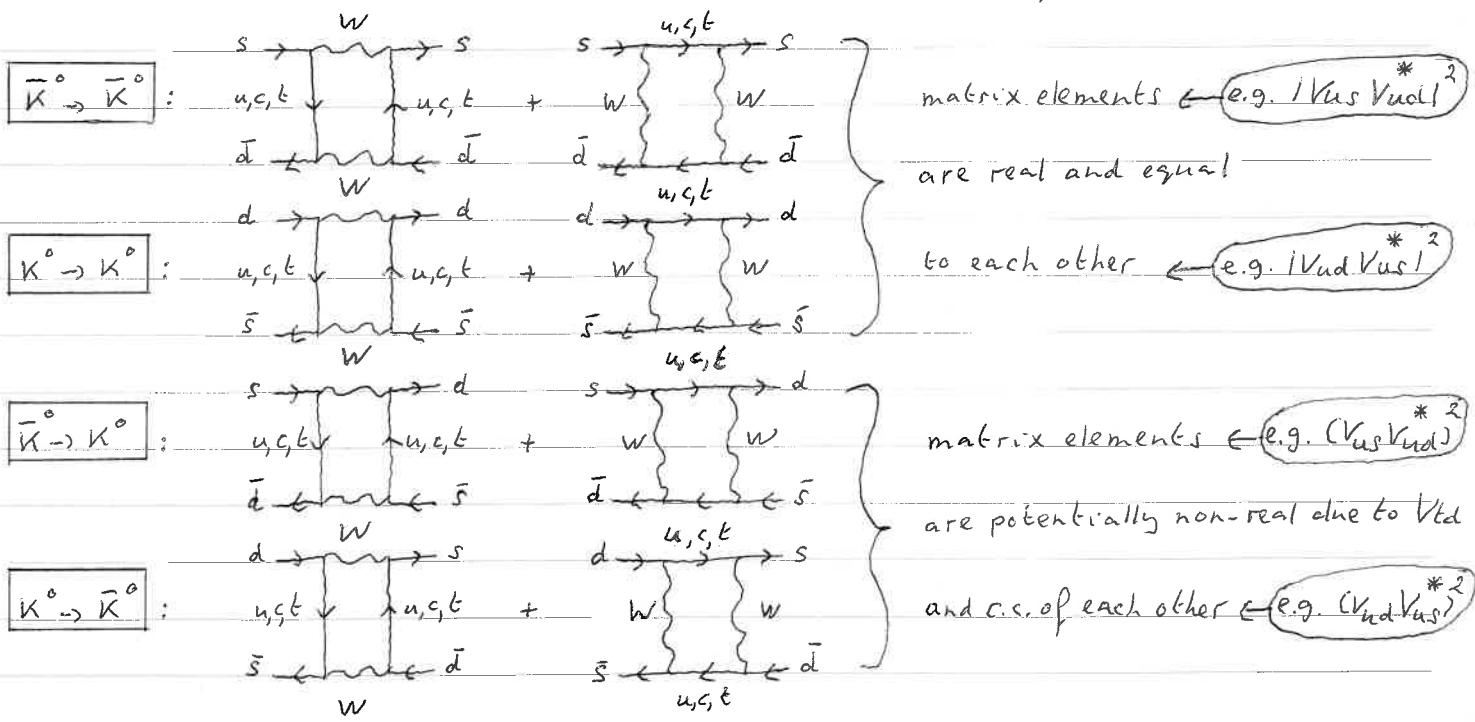
$\uparrow \bar{s}d \downarrow d\bar{s}$

kaons, pions are P-odd

short/long lived

under a CP transformation $|K\rangle \xleftrightarrow{\hat{CP}} |\bar{K}\rangle$. Imposing CP symmetry would imply $[\hat{CP}, \hat{H}] = 0 \Rightarrow$ a simultaneous set of eigenfunctions of \hat{H} and \hat{CP} .

Consequence: $K_S = \text{CP-even} \Rightarrow$ can decay into CP-even $\pi^0 \pi^0, \pi^+ \pi^-$ final states,
 $K_L = \text{CP-odd} \Rightarrow$ cannot decay into CP-even $\pi^0 \pi^0, \pi^+ \pi^-$ final states,
the CP-odd $\pi^0 \pi^0 \pi^0, \pi^0 \pi^+ \pi^-$ final states are possible!



The mixing matrix $\begin{pmatrix} A & B \\ B^* & A \end{pmatrix} = \begin{pmatrix} A & IB|e^{-i\varphi_B} \\ IB|e^{-i\varphi_B} & A \end{pmatrix}$ has eigenvalues $A \pm IB|$ and

$$\text{eigenvectors } \frac{1}{\sqrt{2}} \left(\pm e^{-i\varphi_B} \right) = \frac{1}{\sqrt{2}} \left(|K^0\rangle \pm e^{-i\varphi_B} |K^0\rangle \right).$$

Scenario 1: $B \in \mathbb{R}, \varphi_B = 0 \Rightarrow$ eigenstates $K_L = \frac{1}{\sqrt{2}} (\overbrace{|K^0\rangle + |K^0\rangle}^{CP\text{-odd}}) \equiv K^+$, $K_S = \frac{1}{\sqrt{2}} (\overbrace{|K^0\rangle - |K^0\rangle}^{CP\text{-even}}) \equiv K^-$
 \Rightarrow CP symmetry, $K_L \not\rightarrow \pi^0 \pi^0, \pi^+ \pi^-$.

CP-odd

CP-even

almost CP-odd

Scenario 2: $B \notin \mathbb{R}, \varphi_B \neq 0$ (small) \Rightarrow eigenstates $K_L \approx (1 - \frac{i}{2}\varphi_B) K^+ + \frac{i}{2}\varphi_B K^-$,
 $K_S \approx \frac{i}{2}\varphi_B K^+ + (1 - \frac{i}{2}\varphi_B) K^-$ ← almost CP-even
 \Rightarrow no CP symmetry, suppressed $K_L \rightarrow \pi^0 \pi^0, \pi^+ \pi^-$ allowed.

Scenario 2 observed in 1964! consequence: $V_{cb} \notin \mathbb{R}$ $\xrightarrow{\text{P.27}}$ the SM should involve at least three generations of quarks!

§3.2 The lepton sector: neutrino masses and mixing

Standard-Model style neutrino masser: as already indicated on p. 25-27, an explicit Yukawa interaction - $(\bar{\nu}_{\text{re}})_{AB} \bar{\nu}'_{A_L} \bar{\Phi}_{B_R} + \text{h.c.}$ can be added to the Standard Model in order to obtain neutrino mass terms $-\frac{v}{\sqrt{2}} (\bar{\nu}_{\text{re}})_{AB} \bar{\nu}'_{A_L} \bar{\nu}'_{B_R} + \text{h.c.}$. Here pure singlet right-handed neutrino modes have been added, which are not subject to electroweak or strong interactions. In analogy with the quark sector, gauge and mass eigenstates need not coincide in the lepton sector as well. This gives rise to lepton mixing in the charged-current interactions (Pontecorvo-Maki-Nakagawa-Sakata [PMNS, 1962]):

$$\bar{\nu}'_{A_L} \gamma_\mu e'_{A_L} = \bar{\nu}_{A_L} \gamma_\mu (S^+_{\nu} S_e)_{AB} e_{B_L} \equiv \bar{\nu}_{A_L} \gamma_\mu (U_{\text{PMNS}}^+)_{AB} e_{B_L},$$

and for three generations of fermions

$$U_{\text{PMNS}} = \begin{pmatrix} u_{e1} & u_{e2} & u_{e3} \\ u_{\mu 1} & u_{\mu 2} & u_{\mu 3} \\ u_{\tau 1} & u_{\tau 2} & u_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

with $s_{ij} \equiv \sin(\theta_{ij})$, $c_{ij} \equiv \cos(\theta_{ij})$.

This unitary PMNS-matrix describes lepton mixing. It involves 3 mixing angles and 1 CP-violating phase, just as argued for the CKM-matrix
↑ may be important for matter-antimatter asymmetry

* If all neutrino masses would be equal (e.g. all 0), then mixing would be irrelevant as each linear combination of mass eigenstates would be a mass eigenstate \Rightarrow gauge eigenstate = mass eigenstate!

* Lepton mixing induces rare decays such as $\mu^- \rightarrow \bar{\nu}_i e^-$: $\mu^- \rightarrow \bar{\nu}_{1,2,3} e^-$
extremely rare, due to a more pronounced GIM mechanism

The similarities with the quark sector stop here, in fact there are also clear differences!

* $|U_{\text{PMNS}}|_{AB} \approx \begin{pmatrix} 0.8 & 0.5 & 0.1 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \Rightarrow$ contrary to VCKM, U_{PMNS} effectively shows no sign of a component hierarchy!

*) Neutrinos can only be produced and detected through the weak interactions. Actually the neutrinos are detected by observing the associated lepton mass eigenstates! In most cases also a particular lepton mass eigenstate is involved in the production of neutrinos through weak CC interactions, such as in proton-proton fusion reactions in the sun ($p + p \rightarrow ^3D + (\bar{e}^+ + \nu_e)$), charged-meson decays (e.g. $\pi^+ \rightarrow (\bar{\mu}^+ + \nu_\mu)$, $\pi^- \rightarrow (\bar{\mu}^- + \bar{\nu}_\mu)$) or muon decays ($(\mu^+ + \bar{\nu}_\mu) \rightarrow (\bar{e}^+ + \bar{\nu}_e)$, $(\mu^- + \nu_\mu) \rightarrow (e^- + \bar{\nu}_e)$). Therefore U_{PMNS} is usually assigned to the neutrino mass eigenstates to form so-called neutrino flavour eigenstates:

$$\begin{aligned}\bar{\nu}_{A_L} &= \bar{\nu}_{\gamma_L} (U_{PMNS})_{\bar{\nu}_A} \Rightarrow \bar{\nu}_{A_L} \gamma_\mu (U_{PMNS})_{AB} e_{B_L} = \bar{\nu}_{\gamma_L} \gamma_\mu \underbrace{(U_{PMNS})_{\bar{\nu}_A}}_{I_3} e_{B_L} \\ &= \bar{\nu}_{\gamma_L} \gamma_\mu e_{B_L} \quad [\text{with } \tau = e, \mu, \tau \text{ labeling the flavours}],\end{aligned}$$

which reflects the intimate link between neutrino flavours and specific lepton mass eigenstates!

*) A non-unit lepton mixing matrix U_{PMNS} implies that the $\nu_{e,\mu,\tau}$ neutrinos, which are produced in weak-interaction processes, are linear superpositions of the neutrino mass eigenstates $\gamma_{1,2,3}$. Quantum mechanically these three mass-eigenstate components evolve differently with time. As a result, the $\nu_{e,\mu,\tau}$ neutrinos can oscillate into each other while traveling a certain distance!

See
Ex. 10

I (neutrinos are effectively stable)

This can be observed by measuring the neutrinos as flavour-eigenstates (through weak interactions in the detector). Neutrino oscillations were observed in 1998 @ SuperKamiokande and without realizing it could have been inferred from the solar ν_e deficit at the Homestake experiment (1960's) & solar neutrino problem
 $\Rightarrow (\nu_{e,\mu,\tau}) \neq (\nu_{1,2,3}) \Rightarrow$ massive neutrinos exist !

Question still to be answered: is the mass hierarchy the same as in the quark/charged-lepton sector?

Reason: no direct access to $m_{\nu_{1,2,3}}$ in oscillation experiments.

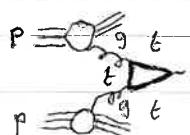


Opening the door to physics beyond the Standard Model: alternative mass terms.

A generic mass term for fermions takes the form $-m(\bar{\psi}_{iL}\psi_{jR} + \bar{\psi}_{iR}\psi_{jL} + \text{h.c.})$ for any combination of two spinor fields $\psi_{i,jL}$. Since electroweak symmetry breaking leaves the U(1)_{E.M.} symmetry intact, in order to guarantee that the photon remains massless, the fields $\psi_{i,jL}$ are required to describe particles with the same electrostatic charge! We exploited this fact in the discussion of lepton mixing on p.30. However, neutrinos are neutral and therefore do not possess quantum numbers that would allow us to differentiate between particles and anti-particles experimentally. We only know that a neutral fermion field should be combined with a charged lepton field to form an $SU(2)_L$ doublet under the weak interactions. The experimental evidence for this construction was based solely on helicity (chirality) considerations --- at no point did we have to demand that the doublet partners of the charged leptons should refer to particles rather than anti-particles. This opens up a new avenue of possibilities: ψ_{jR} could either be a pure singlet right-handed neutrino mode (as on p.30) or a charge-conjugated version of the left-handed neutrino mode that features in an $SU(2)_L$ doublet*.

- ↳ the latter option goes beyond the realm of the Standard Model
- \Rightarrow in various texts on the Standard Model non-zero neutrino masses are interpreted as a sign of beyond the Standard Model physics (although that is not strictly necessary) ☺

§ 3.3 No fourth Standard-Model style fermion generation

Higgs production @ LHC :  followed by subsequent Higgs-boson decay.

The corresponding matrix element becomes independent of m_t for $2m_t \gg m_H$ (heavy-mass limit) \Rightarrow adding heavier quarks from a fourth fermion generation would give rise to a three times larger matrix element and therefore a nine times larger Higgs production cross section, which is not observed experimentally ☹

(part of doublet)

pure singlet

* Typically one could add alternative mass terms $-\frac{m_L}{2} [(\bar{\nu}_L)\nu_L + \text{h.c.}] - \frac{m_R}{2} [(\bar{\nu}_R)\nu_R + \text{h.c.}]$, involving charge-conjugated $(\bar{\nu}_{LR})^c = \bar{\nu}^c \nu_{LR}$ fields. For $m_L \neq 0$ and/or $m_R \neq 0$ lepton number is violated, since ν and ν^c have opposite lepton number (just like ν and $\bar{\nu}$) \Rightarrow lepton-number violating processes would be a smoking gun for this!

Epilogue: status of the Standard Model

Over the past three decades the Standard Model has proven extremely successful, passing all high-precision tests that collider experiments (such as LEP1, SLC, LEP2, Tevatron, LHC) have thrown at it. It has truly established itself as the standard theoretical framework for describing strong-interaction and electroweak physics. Also the non-Abelian nature of the strong and weak interactions has been confirmed conclusively (e.g. the existence of the $w^+w^-/w^+w^- \pi\pi$ TGC in $e^+e^- \rightarrow w^+w^-$ at LEP2). With the discovery of the Higgs boson in 2012 at the LHC, the Standard Model's full predicted particle content has been confirmed!

- SM strengths → (The good)
- passed all high-precision collider tests;
 - complete particle content confirmed;
 - minimal particle, gauge and interaction content;
 - one Higgs doublet does all;
 - all properties of the Higgs particle are fixed
... except its mass (λ).

$\sigma(10 \text{ GeV})$ $\sigma(100 \text{ GeV})$
 \downarrow \downarrow

- SM weaknesses ... what it lacks → (The bads)
- gravity and a reason why $\Lambda_{\text{Planck}} \gg \Lambda_{\text{EW}}$;
 - a dark-matter candidate;
 - an explanation for the matter-antimatter asymmetry in the universe;
 - full-fledged massive ν 's (including all options).

- SM weaknesses ... what came out of the blue → (The ugly)
- the family structure, the magic quantum numbers, the parameters, the mass hierarchy;
 - charge quantization;
 - the three gauge couplings;
 - the Higgs potential;
 - the left-right asymmetry.

