Start-up Quantum Field Theory

Step 1: read Ch. 1 of Peskin & Schroeder + xix - xxi (conventions etc.).
These conventions involve the use of so-called natural units ($\hbar = c = \mu_0 = \epsilon_0 = 1$) by absorbing these constants in the relevant fields / quantities ⇒ a single scale remains: mass

For example: $E \to E * \frac{1}{c^2}$ (cf $mc^2 \to m$)

$p \to p * \frac{1}{c}$ (cf $mc \to m$)

$t \to t * \frac{c^2}{\hbar}$ (cf $\lambda_{compton}/c = \hbar/mc^2 \to 1/m$)

$r \to r * \frac{c}{\hbar}$ (cf $\lambda_{compton} = \hbar/mc \to 1/m$)

Step 2: refresh your knowledge about
- SRT (Appendix D of QM3)
- Klein-Gordon equation (§3.1 + 5.1 of QM3)
- complex contour integrations

Optimal way to follow the course: lecture notes + corresponding text in Peskin & Schroeder + don't fall behind!

Outline of the course:

Part 1 (Ch. 2 P.&S.) (~ 4 weeks): QFT for free scalar (Klein-Gordon) (KG) fields (spin-0)

① Why field theory? ⟷ Compton wavelength

② Wave equations as equations of motion for the fields (wave functions)

↳ ⓐ use Lagrangian formalism for continuous systems
  ↳ particularly suitable for discussing symmetries

ⓑ make sure that the associated action is Lorentz-invariant (rel. principle)

③ Noether's theorem: continuous symmetries and conserved currents/charges
  ↳ fundamentally unobservable quantities

↳ ⓐ energy, momentum and angular momentum in field theories: needed for quantization (Hamiltonian) and particle interpretation.

ⓑ conserved charges (like electromagnetic charge): needed for describing fundamental interactions in nature.

④ Quantization of the free KG theory : ⓐ canonical quantization (like $[\hat{x}, \hat{p}_x] = i\hbar \hat{1}$)

(to be repeated later for the other, more complicated higher-spin theories)

ⓑ energy spectrum bounded from below?

ⓒ causality → antiparticles

→ bosonic commutation relations

ⓓ the vacuum and particle states

ⓔ Green's functions (inversion of KG eqn.)

↑ needed for performing calculations

(~ 8 weeks)

Part 2 (Ch. 4 + parts of Ch. 7, 10 P. & S.) : interacting fields and Feynman diagrams

⑤ Weakly coupled field theories

↳ ⓐ dimensional analysis and smallness of (scalar) interactions

ⓑ effective field theories : integrating out physics, parametrizing ignorance

ⓒ ϕ^4-theory (part of the Higgs model),

scalar Yukawa theory (resembles the theory that describes the interactions between fermions and scalars)

⑥ Perturbation theory for interacting QFTs

↳ ⓐ scattering matrix (S-matrix)

ⓑ Wick's theorem

ⓒ diagrammatic notation for time-ordered vacuum expectation values of interaction-picture fields : Feynman diagrams + rules

⑦ Plane-wave amplitudes for decay processes and scattering reactions

↳ ⓐ fully connected Feynman diagrams

ⓑ amputation procedure

ⓒ Feynman rules for incoming / outgoing particles

ⓓ drawing conventions for particles and antiparticles

ⓔ non-relativistic limit : forces and force carriers

⑧ From amplitudes to probabilities

↳ ⓐ decay widths

ⓑ cross sections for scattering reactions (+ kinematics)

⑨ Dealing with energy eigenstates in the interacting theory :

ⓐ linking Green's functions and time-ordered vacuum expectation values of interaction-picture fields

ⓑ dressed states (Källén-Lehmann spectral representation, field-strength renormalization)

ⓒ perturbative (loop) corrections (tricks, analytical structure)

ⓓ LSZ reduction formula : Green's functions → scattering amplitudes (revisited)

ⓔ optical theorem

(10) Renormalization: dealing with infinities in loop corrections
↳ (a) quantifying (regularizing) infinities
(b) renormalization and renormalization-group eqns.: running parameters, using the correct unperturbed theory as a perturbative starting point
(c) power counting and renormalizability: when infinities have no effect on the predictive power of the theory considered

(~ 4 weeks)

Parts 3 + 4 (Ch. 3 + parts of Ch. 4, 7 P.&S.): dealing with matter fermions

(11) The Dirac field (spin-½)
(a) representations of the Lorentz group ⟺ representations of the rotation group
(b) Dirac's trick + algebra
(c) reducibility of the Dirac representation: Weyl spinors, chirality (= helicity if m=0)
(d) Dirac-field currents: building blocks for fundamental interactions
(e) Dirac eqn.: symmetries + solutions
(f) quantization of the Dirac theory (cf. part 1) → fermionic anticommutation relations
(g) particle interpretation and Feynman propagator
(h) discrete symmetries (e.g. parity and charge conjugation)
↳ fundamental interactions

(12) Part 2 repeated for spin-½ Dirac fermions
↳ (a) Wick's theorem revisited
(b) extra Feynman rules
(c) extended arrow convention
(d) trace technology

(~ 2 weeks)

Part 5 (extra material + parts of Ch. 4, 5, 7 P. & S.): QED (photons as force carriers → gauge bosons)

(13) The electromagnetic theory
↳ (a) gauge freedom, gauge fixing and charge conservation
(b) charged Dirac fermions in an electromagnetic field: minimal substitution, Quantum Electrodynamics (QED)
(c) QED from local gauge invariance: the gauge principle, fundamental postulate for describing the other non-gravitational fundamental interactions in nature in a similar way
(d) quantization of the free electromagnetic field: part 1 repeated for spin-1 photons and its complications

(14) Calculating with QED

↳ (a) Part 2 repeated for QED : extra Feynman rules, more trace technology

(b) protected masses in QED : - fermions protected by chiral symmetry.

- $m_{photon} = 0$ from gauge invariance

(c) Ward - Takahashi identity in QED : gauge invariance as seen in Green's functions and scattering amplitudes

(d) charge screening : the concept of "running couplings"

(e) gauge-invariance and regularization : dimensional regularization