Start-up Quantum Field Theory

Step 1: read Ch. 1 of Peskin & Schroeder + xix - xxii (conventions etc.). These conventions involve the use of so-called natural units $(\hbar = c = m = e = 1)$ by absorbing these constants in the relevant fields/quantities so a single scale remains: mass

For example: $E \rightarrow E \cdot \frac{1}{c} \quad (\text{cf. } m^2 \rightarrow m)$
$p \rightarrow p \cdot \frac{1}{c} \quad (\text{cf. } m \rightarrow m)$
$t \rightarrow t \cdot \frac{c^2}{\hbar} \quad (\text{cf. } \text{hartree} \times c^2 = \hbar m^2 \rightarrow \hbar m)$
$\gamma \rightarrow \gamma \cdot \frac{c}{\hbar} \quad (\text{cf. } \text{hartree} \times c = \hbar m^2 \rightarrow \hbar m)$

Step 2: refresh your knowledge about
- SRT (Appendix D of QM3)
- Klein-Gordon equation (§3.1 + 5.1 of QM3)
- complex contour integrations

Optimal way to follow the course: lecture notes + corresponding text in Peskin & Schroeder + don't fall behind!

Outline of the course:

Part 1 (Ch. 2 P.R.S.): QFT for free scalar & Klein-Gordon fields
- Why field theory?
- Wave equations as equations of motion for the fields (wave functions)
  - by use Lagrangian formalism for continuous systems
  - particularly suitable for discussing symmetries
  - make sure that the associated action is Poincare-invariant (rel. principle)
  - Noether's theorem: continuous symmetries and conserved currents/charges
    - fundamentally unobservable quantities
    - energy, momentum and angular momentum in field theories:
      needed for quantization (Hamiltonian) and particle interpretation
    - conserved charges (like electromagnetic charge):
      needed for describing fundamental interactions in nature
Quantization of the free KG theory:
- canonical quantization (like $\hat{\phi}$? $\hat{\pi}$?)
- energy spectrum bounded from below?
- causality $\Rightarrow$ antiparticles
- bosonic commutation relations
- the vacuum and particle states
- Green's functions (conversion of KG eqs.)
- needed for performing calculations

Part 2 (Ch.4 + part of Ch.5 P.FS.): Interacting fields and Feynman diagrams
- Weakly coupled field theories
  - $\phi^4$ theory (part of the Higgs model), scalar Yukawa theory
    (resembles the theory that describes the interactions between
    fermions and scalars)
- Perturbation theory for interacting QFTs
  - scattering matrix ($S$-matrix)
  - Wick's theorem
  - diagrammatic notation $\Rightarrow$ Feynman diagrams + Feynman rules
  - Green's functions (correlation functions);
    scattering amplitudes, cross sections, decay widths
- Energy eigenstates in the interacting theory:
  - Källén-Lehmann spectral representation
  - dressed states (or quasi-particle states in QFT)
  - field-strength renormalization (i-particle states among multi-particle states)
  - LSE reduction formula: Green's functions $\Rightarrow$ scattering amplitudes (rewritten)
  - perturbative (loop) corrections: tricks, analytical structure, optical theorem
  - Renormalization: dealing with infinities in loop corrections
    - quark/gluon (regularizing) infinities
      - renormalization and renormalization-group eqns.: running parameters, using the right unperturbed theory as a perturbative starting point
      - power counting and renormalizability: when infinities have no effect on the predictive power of the theory considered
Part 3 (Ch. 4, P. 85): The Dirac field (needed to describe matter particles)
- Part 1 repeated for spin-1/2 particles
- Representations of the Lorentz group (like representations of the rotation group)
- Dirac's trick
- Reducibility of the Dirac representation: Weyl spinors, chirality

Dirac field currents: building blocks for describing the non-gravitational fundamental interactions in nature
- Discrete symmetries (like parity and charge conjugations): the same
- Part 2 repeated for spin-1/2 particles = extra Feynman rules, trace technology

Part 4 (Extra material, not in this form in P. 85): QED (gauge bosons as force carriers)
- The electromagnetic theory
  - gauge freedom, charge conservation
  - charged Dirac particles in an electromagnetic field:
    - minimal substitution, Quantum Electrodynamics (QED)
  - QED from local gauge invariance: the gauge principle, fundamental postulate for describing the other non-gravitational fundamental interactions in nature in a similar way
- Quantization of the free electromagnetic field: part 1 repeated for spin-1 photons

Also covered (parts of Ch. 4, 5, 7, P. 85): Calculating with QED
- Part 2 repeated for QED = extra Feynman rules, more trace technology
- Ward - Tannhauser identity in QED: gauge invariance as seen in Green's functions and scattering amplitudes
- Charge screening: the concept of "running couplings"
- Gauge invariance and regularization = dimensional regularization
- Renormalization in QED