Lecture 12: The neutrino sector (neutrino masses and mixing)

There is more than one way to implement neutrino masses.

Standard-Model style neutrino masses: as already indicated on p. 29+30, an explicit Yukawa interaction $-(f_\nu)_{AB}\, \bar{l}'_{A_L}\, \tilde{\Phi}\, \nu'_{B_R}$ + h.c. can be added to the Standard Model in order to obtain neutrino mass terms $-\frac{v}{\sqrt{2}}(f_\nu)_{AB}\, \bar{\nu}'_{A_L}\, \nu'_{B_R}$ + h.c. . Here pure singlet right-handed neutrino modes have been added, which are not subject to electroweak or strong interactions. In analogy with the quark sector, gauge and mass eigenstates need not coincide in the lepton sector as well. This gives rise to lepton mixing in the charged-current interactions (Pontecorvo-Maki-Nakagawa-Sakata):

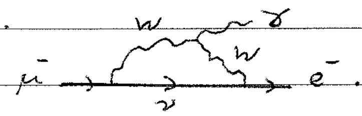
$$\bar{\nu}'_{A_L}\gamma_\mu e'_{A_L} = \bar{\nu}_{A_L}\gamma_\mu (S_\nu^\dagger S_e)_{AB}\, e_{B_L} \equiv \bar{\nu}_{A_L}\gamma_\mu (U^\dagger_{PMNS})_{AB}\, e_{B_L}$$

$$\text{and } U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

with $s_{ij} \equiv \sin(\theta_{ij})$, $c_{ij} \equiv \cos(\theta_{ij})$.

This unitary PMNS-matrix describes lepton mixing (being assigned to the neutrinos rather than the charged leptons). It involves 3 mixing angles and 1 CP-violating phase, just as argued for the CKM-matrix.

*) If all neutrino masses would be equal (e.g. all 0), then mixing would be irrelevant as each linear combination of mass eigenstates would be a mass eigenstate $\Rightarrow$ gauge eigenstate = mass eigenstate.

*) Neutrino mixing induces rare decays such as $\mu^- \to e^-\gamma$: 

*) A non-unit lepton mixing matrix U_{PMNS} implies that the $\nu_{e,\mu,\tau}$ neutrinos, which are produced in weak-interaction processes, are linear superpositions of the neutrino mass eigenstates $\nu_{1,2,3}$. Quantum mechanically these three mass-eigenstate components evolve differently with time. As a result, the $\nu_{e,\mu,\tau}$ neutrinos can oscillate into each other while traveling a certain distance!

(neutrinos are stable)

This can be observed by measuring the neutrinos as gauge eigenstates (through weak int.). Neutrino oscillations were observed in 1998 @ Superkamiokande

$\Rightarrow$ $(\nu_{e,\mu,\tau}) \neq (\nu_{1,2,3})$ $\Rightarrow$ massive neutrinos exist!!

(in the detector)

$\hookrightarrow$ Question: is the mass hierarchy the same as in the quark/charged-lepton sector?

Alternative mass terms: a generic mass term for fermions takes the form $-m(\bar{\psi}_{1_L}\psi_{2_R} + \bar{\psi}_{1_R}\psi_{2_L} + h.c.)$ for any combination of two spinor fields $\psi_{1,2}$. Since electroweak symmetry breaking leaves the $U(1)_{e.m.}$ symmetry intact, in order to guarantee that the photon remains massless, the fields $\psi_{1,2}$ are required to describe particles with the same electrostatic charge. We exploited this fact in the discussion of lepton mixing on p.34. However, neutrinos are neutral and therefore do not possess quantum numbers that would allow us to differentiate between particles and antiparticles experimentally. We only know that a neutral fermion field should be combined with a charged lepton field to form a $SU(2)_L$ doublet under the weak interactions. The experimental evidence for this construction was based solely on helicity (chirality) considerations --- at no point did we have to demand that the doublet partners of the charged leptons should refer to particles rather than antiparticles. This opens up a new avenue of possibilities: ψ_{2_R} could either be a pure singlet right-handed neutrino mode (as on p.34) or a charge-conjugated version of the left-handed neutrino mode that features in a $SU(2)_L$ doublet. Before we go into the details of this additional option, we first collect some properties of charge conjugation of spinor fields.

Intermezzo on charge conjugation of spinor fields:

$$\psi \xrightarrow[\text{conj.}]{\text{charge}} \psi^c \equiv C\bar{\psi}^T, \quad \text{with } C^\dagger = C^{-1},\ C^T = -C \text{ and } C^{-1}\gamma^\mu C = -(\gamma^\mu)^T.$$

$$\hookrightarrow C^{-1}\gamma^5 C = (\gamma^5)^T$$

Hence, $\bar{\psi}^c \equiv (C\bar{\psi}^T)^\dagger\gamma^0 = (\psi^\dagger\gamma^0{}^T)^\dagger C^\dagger\gamma^0 = \psi^T(\gamma^0)^T C^{-1}\gamma^0 = -\psi^T C^{-1}\underbrace{\gamma^0 C C^{-1}\gamma^0}_{I} = -\psi^T C^{-1}.$

$L/R \xrightarrow{c} R/L$:

$$\boxed{\begin{aligned} (\psi_{L/R})^c &= C(\bar{\psi}_{L/R})^T = C P_{R/L}^T \bar{\psi}^T = C[C^{-1}P_{R/L}C]\bar{\psi}^T = P_{R/L}\psi^c, \\ (\bar{\psi}_{L/R})^c &= -(\psi_{L/R})^T C^{-1} = -\psi^T P_{L/R}^T C^{-1} = -\psi^T[C^{-1}P_{L/R}C]C^{-1} = \bar{\psi}^c P_{L/R} \end{aligned}}.$$

By means of charge conjugation a left-handed fermion field is turned into a right-handed fermion field. Useful identities for the following discussion will be (see exercise):

$$\boxed{\begin{aligned} \bar{\psi}_R\psi_L &= (\bar{\psi}_L)^c(\psi_R)^c \\ \bar{\psi}_L\psi_R &= (\bar{\psi}_R)^c(\psi_L)^c \\ (\psi^c)^c &= \psi \end{aligned}}$$

bearing in mind that ψ and $\bar{\psi}$ are in fact fermionic operators.

Most general Lagrangian for the free neutrino: ($\nu_{L/R}(x)$ = chiral field)

$$\mathcal{L}_\nu^{free} = \bar\nu i\gamma^\mu\partial_\mu\nu - m_D(\bar\nu_L\nu_R + h.c.) - \frac{m_L}{2}\big[\overbrace{(\bar\nu_L)^c}^{\text{part of doublet}}\nu_L + h.c.\big] - \frac{m_R}{2}\big[\overbrace{(\bar\nu_R)^c}^{\text{pure singlet}}\nu_R + h.c.\big].$$

Next we introduce two so-called Majorana fields $N_1 \equiv \frac{\overset{L}{\nu_L} + \overset{R}{(\nu_L)^c}}{\sqrt2}$ and $N_2 \equiv \frac{\overset{R}{\nu_R} + \overset{L}{(\nu_R)^c}}{\sqrt2}$ comprised entirely of ν_L or ν_R. These fields satisfy the Majorana condition $N_i = N_i^c$ $(i=1,2)$ and thus have half the d.o.f. of a Dirac field.

Relations (see exercise): $\bar N_1 N_1 = \frac12[(\bar\nu_L)^c\nu_L + h.c.]$,

$$\bar N_2 N_2 = \frac12[(\bar\nu_R)^c\nu_R + h.c.],$$

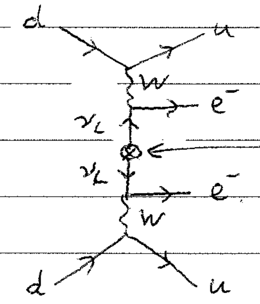
$$\bar N_1 N_2 + \bar N_2 N_1 = \frac12[\bar\nu_L\nu_R + (\bar\nu_L)^c(\nu_R)^c + h.c.] = \bar\nu_L\nu_R + h.c.$$

$$\Rightarrow \mathcal{L}_\nu^{free} = \bar N_1 i\gamma^\mu\partial_\mu N_1 + \bar N_2 i\gamma^\mu\partial_\mu N_2 - \overbrace{m_D}^{\text{Dirac mass}}(\bar N_1 N_2 + \bar N_2 N_1) - \overbrace{m_L \bar N_1 N_1 - m_R \bar N_2 N_2}^{\text{Majorana masses}}$$

$$= \bar N_1 i\gamma^\mu\partial_\mu N_1 + \bar N_2 i\gamma^\mu\partial_\mu N_2 - (\bar N_1, \bar N_2)\underbrace{\begin{pmatrix} m_L & m_D \\ m_D & m_R\end{pmatrix}}_{\equiv [M^\nu] \text{ neutrino mass matrix}}\begin{pmatrix} N_1 \\ N_2\end{pmatrix}.$$

Lepton-number violation: for $m_L \neq 0$ and/or $m_R \neq 0$ lepton number (fermion number for leptons) is violated, since $\underset{(L=1)}{\nu}$ and $\underset{(L=-1)}{\nu^c}$ have opposite lepton number. The Majorana mass term $\propto m_L$ violates lepton number by two units, since $L(\nu_L) = L([\bar\nu_L]^c) = +1$. In view of the fact that ν_L participates in weak interactions, this induces the possibility of neutrinoless double-β decay, such as $\underset{(\bar u s)}{K^-} \to \underset{(u\bar d)}{\pi^+} e^- e^-$ or nucleus$(A,Z) \to$ nucleus$'(A, Z+2)\, e^- e^-$:

e.g.



Majorana-mass insertion

This would be the smoking gun for Majorana fermions !

Negative mass eigenvalues: a negative value for the mass in a fermionic mass term can be straightforwardly turned into a positive value by writing

$$\psi = \gamma^5\psi', \quad \bar\psi = (\gamma^5\psi')^\dagger\gamma^0 = \psi'^\dagger\gamma^5\gamma^0 = -\bar\psi'\gamma^5 \Rightarrow \bar\psi\psi = -\bar\psi'\psi'.$$

Special cases: 1) $m_{L,R} = 0$, $m_D \neq 0$ ⇒ Dirac-mass case (like in Standard Model), with the Dirac field written as two Majorana fields with identical mass: $\nu = \nu_L + \nu_R = \sqrt{2}(P_L N_1 + P_R N_2) = \underbrace{\frac{N_1+N_2}{\sqrt{2}}}_{\text{mass } m_D} - \underbrace{\gamma^5 \frac{N_1 - N_2}{\sqrt{2}}}_{\text{mass } -m_D \to +m_D}$.

cf. complex number written as two real numbers

2) $m_R = m_D = 0$, $m_L \neq 0$ ⇒ the neutrino mass is obtained by solely resorting to the ν_L field and lepton number is violated by the mass term $-m_L \bar{N}_1 N_1$. The massive neutrino is a Majorana neutrino in this case!

3) $m_L = 0$, $m_{R,D} \neq 0$ ⇒ in this case ν_R is needed for all mass terms. The mass term $-m_R \bar{N}_2 N_2$ violates lepton number, just as in case 2). The mass eigenvalues equal $\frac{1}{2}\{ m_R \pm \sqrt{m_R^2 + 4m_D^2} \}$, which opens up a theoretically very interesting possibility:

if $m_R \gg m_D$, then M^ν has one very small and one very large eigenvalue

seesaw mechanism

$$\begin{cases} \boxed{-m_\nu \approx -m_D^2/m_R} \to \text{eigenstate } \nu' = \gamma^5(N_1 - N_2 m_D/m_R) \\ m_N \approx m_R \to \text{eigenstate } N = N_2 + N_1 m_D/m_R \end{cases}$$

↳ ν' dominated by "observable" ν_L mode and naturally light ($m_\nu \ll m_D$)!,

N dominated by unobservable ν_R mode and very heavy.

two Majorana fields with different mass instead of one Dirac field ☺

Remarks: *) Majorana fields are not capable of absorbing phases ⇒ more sources of CP violation in cases 2) and 3).

*) The mass m_R in case 3) could be linked to the scale of new (BSM) physics, e.g. caused by a vev of a scalar SM singlet (as typically present in unification theories)

⇒ possible scenario: three very light Majorana states that are dominated by the three ν_L modes (active neutrinos) and three substantially heavier Majorana states that are dominated by the sterile ν_R modes ⇒ U_{PMNS} → 6x6 matrix (not unitary anymore)

called: νMSM

- $\mathcal{O}$(10 keV) Majorana neutrinos might be viable DM candidates!

  (grav. int. + m_D/m_R suppressed weak int. (ν_L component in N))

- Majorana neutrinos in principle decay democratically into leptons and antileptons (being their own antiparticles). CP violation in the lepton sector can tilt the balance slightly towards the matter side ⇒ matter-antimatter asymmetry from $\gtrsim \mathcal{O}(10^8$ GeV) neutrinos!