Quantum Field Theory Exercises week 6

Exercise 7: Feynman rules for the scalar Yukawa theory + arrow convention

Take the Lagrangian for the scalar Yukawa theory:

$$\mathcal{L} = (\partial_\mu \psi^*)(\partial^\mu \psi) + \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - M^2 \psi^* \psi - \frac{1}{2} m^2 \phi^2 - g \psi^* \phi \psi,$$

with $\phi \in \mathbb{R}$ and $\psi \in \mathbb{C}$.

(a) Which term in the Lagrangian gives rise to an interaction?

(b) Calculate up to first order in $g$:

$$\langle 0 | T \left( \hat{\psi}^I_1(x_1) \hat{\phi} I(x_2) \hat{\psi} I(x_3) e^{-i \int d^4 x \hat{H} I(x)} \right) | 0 \rangle,$$

indicating the Feynman propagator for the $\psi$ field by

$$\langle 0 | T (\hat{\psi}^I_I(x)) \rangle | 0 \rangle = \frac{1}{i(M^2 - p^2 + i\epsilon)}$$

and the one for the $\phi$ field by $D_F(x - y; m^2)$.

(c) Write the answer in part (b) in terms of Feynman diagrams. Use a solid line for the $\psi$ propagators and a dotted line for the $\phi$ propagators. Use the following extra drawing convention: draw an arrow on $\psi$ propagators, representing the direction of particle-number flow. Hence, the $\psi$-particles are defined to flow along the arrow, whereas $\psi$-antiparticles flow against it!

Explain in this context why the operator $\hat{\psi} I(x)$ corresponds to an arrow that flows into the external/internal point $x$, whereas $\hat{\psi}^I_1(x)$ corresponds to an arrow that flows out of the external/internal point $x$.

(d) – Derive the Feynman rules in position space.

– Why do we not have to worry about symmetry factors?

(e) Translate these Feynman rules to the momentum representation.

Exercise 8: More on the arrow convention for particles and antiparticles

In exercises 5 and 7 you have seen various aspects of particle flow (arrows) in the scalar Yukawa theory, pertaining to internal/external points as well as propagators. Use these aspects to explain why the arrows in the associated Feynman diagrams link up to form a continuous flow.

What conservation law is actually causing this phenomenon?