Mid-term harvest

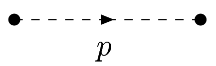
Already at this point in the lecture series we are able to draw some far-reaching conclusions. In order to do so we only need a modest generalization of the knowledge that we have acquired so far. The statement that we want to address is the following:

> **Forces in quantum field theory:** the forces between particles are caused (mediated) by the exchange of virtual particles! Interactions caused by spin-0 force carriers (such as the Yukawa interactions) are universally attractive, just like interactions due to the exchange of spin-2 particles (such as gravity). The exchange of spin-1 particles can result in both attractive and repulsive interactions, as we know from electromagnetism.

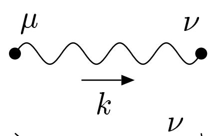
The relevant details of this statement were worked out in Ex. 11 for the scalar case and will now be discussed for the higher-spin case. This actually used to be an old exercise that was not part of the exam material, but the group-discussion format works better.

Exercise 10: **Non-relativistic interactions mediated by spin-1 and spin-2 force carriers (Coulomb potential and Newton's gravitational potential)**

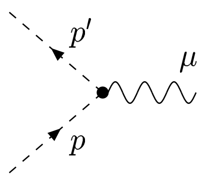
Part a: consider scalar QED, i.e. the theory that describes the interactions between complex scalar fields and real electromagnetic fields. The relevant momentum-space Feynman rules for propagators and interactions in this theory are given by



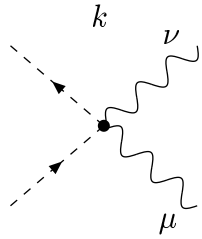
$$= \frac{i}{p^2 - m^2 + i\epsilon}$$



$$= \frac{-\,i\,\eta_{\mu\nu}}{k^2 + i\epsilon}$$



$$= \; -ie(p+p')^{\mu} \;\equiv\; -ie\,J^{\mu}(p,p')$$



$$= \; 2ie^2\eta^{\mu\nu}$$

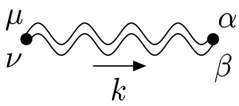
using dashed lines to indicate the massive scalar bosons and wiggly ones to indicate the massless spin-1 photons. The momenta of the particles are defined along the arrow of particle-number flow and the indices μ and ν are Minkowski indices with $\eta_{\mu\nu}$ representing the flat-spacetime metric.

> *Remark: as you can see from the Feynman rule involving one photon and two scalar particles, the photon couples to a conserved charge current eJ^{μ} with one Lorentz index μ and with e the charge of the scalar particles. Upon contraction with the photon momentum $(p-p')_{\mu}$ one indeed obtains $e\,(p^2 - p'^2) = 0$ for on-shell scalar particles, as required for a conserved current.*

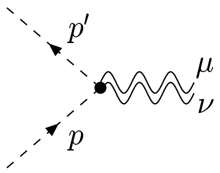
Just like in the case of the scalar Yukawa theory, we consider the lowest-order scattering reactions $\psi\psi \to \psi\psi$ and $\psi\bar{\psi} \to \psi\bar{\psi}$ for scalar particles (ψ) and antiparticles ($\bar{\psi}$).

To be discussed: in scalar QED the interaction between ψ-particles is repulsive, whereas the interaction between ψ-particles and $\bar{\psi}$-antiparticles is attractive.

Part b: next we consider the following Feynman rules for the propagator of the spin-2 graviton (double wiggly line) and the interaction between complex scalar fields and real graviton fields in linearized gravity:



$$= \frac{i}{k^2 + i\epsilon}\,\frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right)$$



$$= i\sqrt{8\pi G}\left[p^\mu p'^\nu + p^\nu p'^\mu + \eta^{\mu\nu}(m^2 - p\cdot p')\right] \equiv i\sqrt{8\pi G}\,T^{\mu\nu}(p,p')$$

where G is Newton's constant. This interaction originates from the coupling of the symmetric graviton field (tensor field) to the conserved energy-momentum tensor $T^{\mu\nu}$ for the scalar matter, which has two Lorentz indices μ and ν. As can be checked trivially, contracting the given Feynman rule by either $(p-p')_\mu$ or $(p-p')_\nu$ indeed yields 0 for on-shell scalar particles.

<u>Linearized gravity</u>: the interaction given above is obtained by expanding the full GR metric around the flat (Minkowskian) spacetime metric $\eta_{\mu\nu}$ according to

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \sqrt{32\pi G}\,h_{\mu\nu}(x)\ ,$$

where the second term involving the symmetric graviton field $h_{\mu\nu}(x) \in \mathbb{R}$ is treated as a small perturbation (weak-field limit). We can use this to expand the matter part of the GR action in powers of the graviton field:

$$\begin{aligned} S_M(h) &= S_M(h=0) + \int \mathrm{d}^4x\,\sqrt{32\pi G}\,h_{\mu\nu}(x)\left(\frac{\partial S_M}{\partial g_{\mu\nu}(x)}\right)_{h\,=\,0,\,x\,\text{fixed}} + \cdots \\ &\equiv S_M(h=0) + \int \mathrm{d}^4x\,\sqrt{32\pi G}\,h_{\mu\nu}(x)\,\frac{1}{2}\,T^{\mu\nu}(x) + \cdots \end{aligned}$$

At first order this gives rise to an interaction

$$\mathcal{L}_{int}(x) = \sqrt{8\pi G}\,h_{\mu\nu}(x)T^{\mu\nu}(x)$$

between the graviton field and the symmetrized energy-momentum tensor in flat spacetime, which is a conserved tensor.

To be discussed: the non-relativistic gravitational interaction between the scalar (anti)particles is always attractive and of the expected form.