Exercise 18.) $\Psi(x)$ contains the $\psi^-$ terms, $\bar{\Psi}(x)$ the $\bar{\psi}^+$ terms:

$\psi^-$ contains the $\bar{\psi}^-$ terms, $\bar{\psi}^+$ the $\bar{\psi}^+$ terms.

As all anti-commutators vanish with the exception of

\[ \{ \bar{\psi}^+, \bar{\psi}^- \} \text{ and } \{ \psi^+, \psi^- \}. \]

\[ \text{Spinor indices.} \]

\[ \begin{align*}
N^a(b) &= (\psi^a(x), \psi^b(x))(\bar{\psi}^a(x), \bar{\psi}^b(x)), \\
N(\psi^a(x), \bar{\psi}^b(x)) &= \psi^a(x) \bar{\psi}^b(x) + \bar{\psi}^b(x) \psi^a(x) - \bar{\psi}^b(x) \psi^a(x).
\end{align*} \]

\[ T(\psi^a(x), \bar{\psi}^b(x)) = -T(\bar{\psi}^b(x), \psi^a(x)). \]

\[ N(\psi^a(x), \bar{\psi}^b(x)) = N(\bar{\psi}^b(x), \psi^a(x)) = -N^a(b) = -N\tau_{(a+b)}(x) (\psi^a(x), \psi^b(x)) + \bar{\psi}^b(x) \psi^a(x).
\]

\[ T(\psi^a(x), \bar{\psi}^b(x)) = T(\bar{\psi}^b(x), \psi^a(x)). \]

\[ \text{Interchanging } \psi^a(x) \text{ and } \bar{\psi}^b(x) \text{ give } a-b \text{ sign.} \]

Since $\psi^a(x), \bar{\psi}^b(x), \bar{\psi}^b(x), \psi^a(x)$ are one finds $\psi^a(x) \bar{\psi}^b(x) = \bar{\psi}^b(x) \psi^a(x) = 0$.

The proof of Wick's theorem follows the steps outlined on p. 374 of the lecture notes, with $\hat{\phi}_0$ representing a fermionic field at the space-time point $x_0$ (i.e. either $\hat{\phi}_0 = \hat{\psi}_0(x_0)$ or $\hat{\phi}_0 = \hat{\bar{\psi}}_0(x_0)$). The differences with the scalar case are:

\[ \hat{\phi}_0, N(\hat{\phi}_3, \ldots, \hat{\phi}_m) = N(\hat{\phi}_0, \hat{\phi}_3, \ldots, \hat{\phi}_m + \hat{\phi}_0, \hat{\phi}_3, \ldots, \hat{\phi}_m + \ldots + (\hat{\phi}_0, \hat{\phi}_3, \ldots, \hat{\phi}_m) + N(\hat{\phi}_0, \hat{\phi}_3, \ldots, \hat{\phi}_m) + \ldots, \]

interchanging two fields generates a minus sign.

\[ T(\psi(x), \bar{\psi}(x), \bar{\psi}(x), \psi(x)) = \left( N(\psi(x), \bar{\psi}(x)), \psi(x) \bar{\psi}(x) + \bar{\psi}(x) \psi(x) + \ldots \right) \]

\[ \left( N(\bar{\psi}(x), \psi(x)), \bar{\psi}(x) \psi(x) + \psi(x) \bar{\psi}(x) + \ldots \right). \]

\[ \text{Fermi Statistics.} \]

\[ \text{Only fully contracted terms contribute.} \]