

Sol. 8 From the kinetic part of the Higgs Lagrangian $(\partial_\mu \Phi)^T (\partial^\mu \Phi)$ we obtain the gauge-boson mass terms by inserting the vev $\langle \Phi \rangle_0 = (v/\sqrt{2})$.

(a) $\mathcal{L}_{\text{mass}}^{\text{GB}} = \frac{v^2}{2} (0, 1) \left[-\frac{i}{2} g \vec{\sigma} \cdot \vec{w}_\mu - \frac{i}{2} g' B_\mu \right] T \left[\frac{i}{2} g \vec{\sigma} \cdot \vec{w}^\mu + \frac{i}{2} g' B^\mu \right] (0)$

$\xrightarrow{\text{only } \sigma^3 \text{ contributes}}$

$$\begin{aligned} &= \frac{v^2}{8} \left[g^2 \vec{w}_\mu \cdot \vec{w}^\mu + g'^2 B_\mu B^\mu + gg' B_\mu (0, 1) \vec{\sigma} \cdot \vec{w}^\mu (0) + gg' (0, 1) \vec{\sigma} \cdot \vec{w}_\mu (0) B^\mu \right] \\ &= \frac{1}{8} v^2 g^2 (w_\mu^1 w^{1\mu} + w_\mu^2 w^{2\mu}) + (w_\mu^3, B_\mu) \frac{v^2}{8} \underbrace{\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}}_{\equiv M} \begin{pmatrix} w^{3\mu} \\ B^\mu \end{pmatrix}. \end{aligned}$$

(b) $M = M^T$ real matrix \Rightarrow $O^T M O = O^T M O = M_{\text{diag}}$ real diagonal matrix.
real orth. O eigenvalues M_i

$$\begin{aligned} 0 = \det(M) = \det(O^T M O) &= \prod_i M_i \\ \frac{1}{8} v^2 (g^2 + g'^2) = \text{Tr}(M) &= \text{Tr}(O^T M O) = \sum_i M_i \end{aligned} \quad \Rightarrow \text{eigenvalues } \frac{1}{8} v^2 (g^2 + g'^2), 0.$$

Hence, $\mathcal{L}_{\text{mass}}^{\text{GB}} = \frac{1}{2} \sum_{a=1}^4 M_a^2 w_\mu^a w^{a\mu}$ with $M_a^2 = (\frac{1}{8} v^2 g^2, \frac{1}{8} v^2 g^2, \frac{1}{8} v^2 (g^2 + g'^2), 0)$.

(c) Eigenvalues $M_w^2 = \frac{1}{4} v^2 g^2$: have any linear combination of w_μ^a as eigenvectors, e.g. $w_\mu^\pm \equiv \frac{1}{\sqrt{2}} (w_1 \mp i w_2) \leftarrow w^\pm \text{ bosons}$.

These w bosons are charged, since they correspond to raising/lowering ($T^+/-T^-$).

Eigenvalues $M_Z^2 = \frac{1}{4} v^2 (g^2 + g'^2)$, 0: have as corresponding eigenvectors (w_μ^3, B_μ)

$$(w_\mu^3, B_\mu) \xrightarrow{\text{rotation}} O = (w_\mu^3, B_\mu) \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} = (\underbrace{c_W w_\mu^3 - s_W B_\mu}_{\equiv Z^0}, \underbrace{s_W w_\mu^3 + c_W B_\mu}_{\equiv A^0}).$$

These two gauge bosons are neutral, since they correspond to the diagonal generators T^3 and T_Y .

(d) We could also use instead of Φ another (conjugate) doublet $\tilde{\Phi}$, with opposite hypercharge and vev $\langle \tilde{\Phi} \rangle_0 = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \Rightarrow g' \rightarrow -g'$ and due to $(1, 0) [-\dots] [+\dots] (0)$ this time the upper left component of σ^3 is picked up, so that effectively also $g \rightarrow -g$.
Hence, nothing would change in part (b).

(e) Extra condition on $c_W = \cos \theta_W$ and $s_W = \sin \theta_W$ (θ_W = weak mixing angle):

$$M \begin{pmatrix} s_W \\ c_W \end{pmatrix} = \begin{pmatrix} s_W g^2 - g g' c_W \\ -s_W g g' + c_W g'^2 \end{pmatrix} \xrightarrow{M \tilde{g} = 0} \begin{pmatrix} s_W \\ c_W \end{pmatrix} = \tan \theta_W = g'/g.$$

photon

Sol. 9 Electroweak gauge interactions among quark mass eigenstates.

We start from the expressions in terms of gauge eigenstates:

e.w. int.

$$\delta_{\text{quark}} = \sum_A \bar{Q}_{AL}^i \gamma^\mu [\frac{i}{2} g \vec{\sigma} \cdot \vec{W}_\mu + \frac{i}{2} g' Y(Q_L) B_\mu] Q_{AL}^i$$

$$+ \sum_A (\bar{u}_{AR}^i \gamma^\mu [\frac{i}{2} g' Y(u_R) B_\mu] u_{AR}^i + \bar{d}_{AR}^i \gamma^\mu [\frac{i}{2} g' Y(d_R) B_\mu] d_{AR}^i),$$

with A labeling the various generations of quarks.

(a) Next we switch to the mass eigenstates. For NC interactions, which involve diagonal components of the generators only, we obtain the same expression for both gauge and mass eigenstates!

$$\begin{aligned} \delta_{\text{quark}}^{\text{NC}} = & - \sum_{\text{quarks } q} \bar{\psi}_q \gamma^\mu (P_L \underbrace{[\frac{1}{2} (\sigma_1^1)_{qq} q w_\mu^1 + \frac{1}{2} (\sigma_2^1)_{qq} q w_\mu^2 + \frac{1}{2} (\sigma_3^1)_{qq} q w_\mu^3]}_{Q_q - I_3(q)} \\ & + P_L q \underbrace{[\frac{i}{2} Y(q_L) B_\mu]}_{Q_q - I_3(q)} + P_R q \underbrace{[\frac{i}{2} Y(q_R) B_\mu]}_{Q_q - I_3(q)}) \psi_q \\ & + \underbrace{c_w Z_\mu + s_w A_\mu}_{L \text{ w.r.t. } \delta Z_\mu + \delta A_\mu} \end{aligned}$$

$$\Rightarrow \bar{q}_1 q_1 Z \text{ int. : } - \frac{g}{c_w} \sum_q \bar{\psi}_q \gamma^\mu Z_\mu \left[\underbrace{[\frac{1}{2} c_w I_3(q_1) + \frac{1}{2} s_w c_w \frac{q^1}{q_1} I_3(q_1) (1 - \delta^5) - s_w c_w \frac{q^1}{q_1} Q_2]}_{\frac{1}{2} I_3(q_1)} \right] \psi_q - s_w^2 Q_2$$

$$\bar{q}_1 q_1 \gamma \text{ int. : } - \underbrace{s_w g \sum_q \bar{\psi}_q \gamma^\mu A_\mu}_{\text{1st term}} \left[\underbrace{[\frac{1}{2} I_3(q_1) - \frac{c_w q^1}{2 s_w q_1} I_3(q_1) (1 - \delta^5) + \frac{c_w q^1}{s_w q_1} Q_2]}_0 \right] \psi_q$$

(apparently $q_1 = 1e1/s_w$)

The NC interactions for leptons are carbon copies of this.

(b) For CC interactions the CKM matrix enters the expressions for mass eigenstates. This time the off-diagonal generator components feature:

$$\begin{aligned} \delta_{\text{quark}}^{\text{CC}} = & - \sum_{q_1, q_2} \bar{\psi}_{q_1} \gamma^\mu P_L (V_{CKM})_{q_2 1} \left[\underbrace{[\frac{1}{2} (\sigma_1^1)_{q_1 q_2} q w_\mu^1 + \frac{1}{2} (\sigma_2^1)_{q_1 q_2} q w_\mu^2 + \frac{1}{2} (\sigma_3^1)_{q_1 q_2} q w_\mu^3]}_{\frac{g}{\sqrt{2}} w_\mu^+} \right] \psi_{q_2} \\ & \text{up-type} \quad \text{down-type} \end{aligned}$$

$$\xrightarrow{\text{h.c. of 1st term}} - \sum_{q_1, q_2} \bar{\psi}_{q_1} \gamma^\mu P_L (V_{CKM})_{q_2 1}^* \frac{g}{\sqrt{2}} w_\mu^- \psi_{q_2}.$$

The leptonic case will be treated in a slightly different way
(see Ex. 10).