Gross-Neveu-Yukawa models at criticality

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Research Training Group
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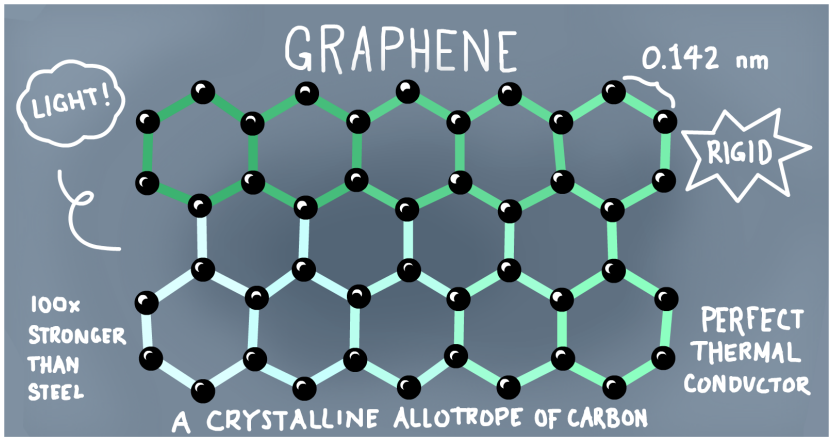
Gross-Neveu model

- classical GN model: asymptotically free fermionic theory in 2 dimensions
- action:

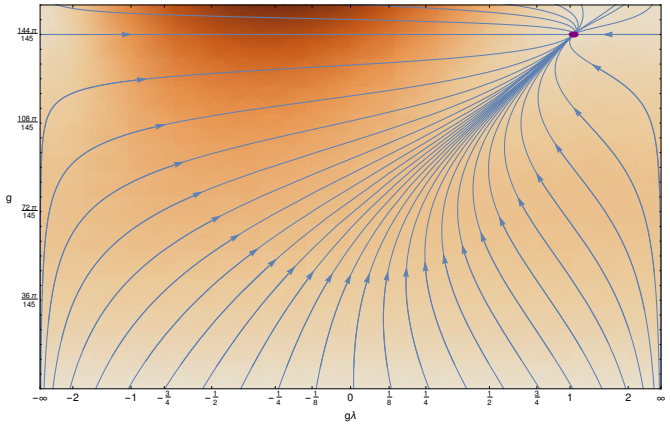
$$S^{\mathrm{GN}} = \int \mathrm{d}^2x \left[\overline{\psi} \partial\!\!\!/ \psi + \frac{\overline{g}}{2N_f} \left(\overline{\psi}\psi \right)^2 \right]$$

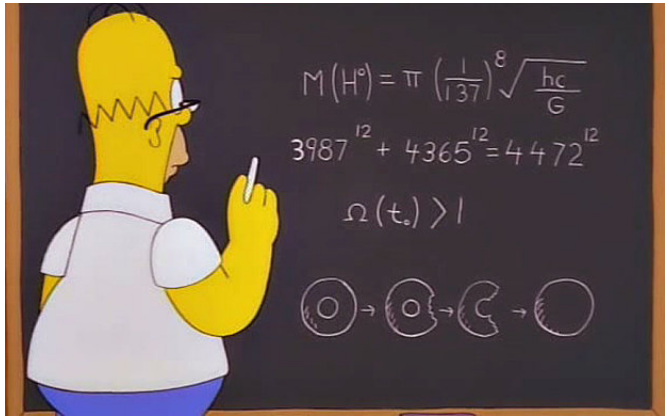
- Gross, Neveu (1974): spontaneous breaking of chiral symmetry by bifermion condensate $\rightarrow$ toy model for confinement

Why bother?


GRAPHENE
0.142 nm
LIGHT!
RIGID
100x STRONGER THAN STEEL
PERFECT THERMAL CONDUCTOR
A CRYSTALLINE ALLOTROPE OF CARBON


anti-up quark
up quark
pi-0 meson


g
gλ


M(H°) = π (1/137)^8 √(hc/G)
3987^12 + 4365^12 = 4472^12
Ω(t₀) > 1

Why bother?

- $d = 3$: relativistic, effective low energy model for fermions, interesting e.g. in condensed matter (graphene)
- $d = 4$: Higgs-top sector of Standard Model, low energy description of QCD (quark-meson model)
- can be interpreted as asymptotically safe field theory

Here: focus on $d = 3$ and $N_f = 1, 2$ flavours of fermions in 4-dimensional reducible representation

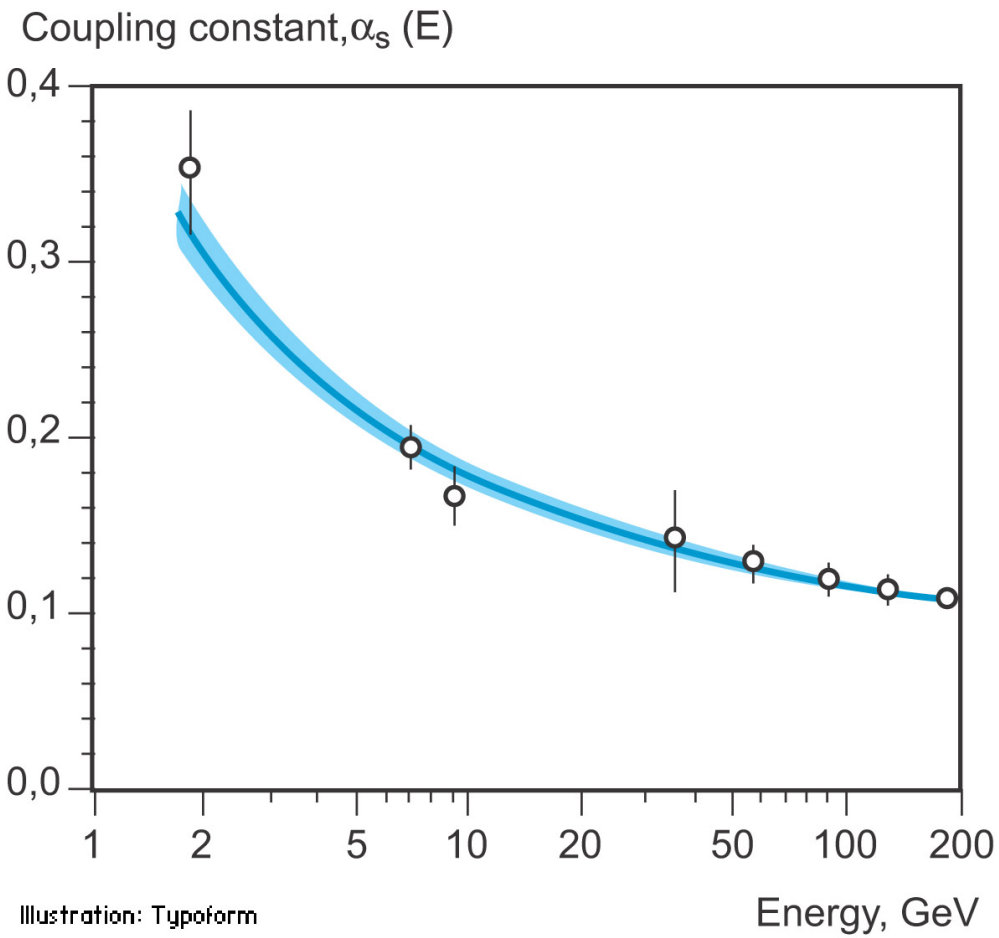
QFT, beta functions and the like

- calculating the path integral is very difficult[citation needed]:

$$Z = \int \mathcal{D}\Phi \, e^{iS[\Phi]/\hbar}$$

- study approximations within subspace of all possible interactions
- this makes observables depend on only a manageable amount of coupling constants
- coupling constants depend on energy scale $\Rightarrow$ encoded in beta functions
- criticality: correlation length diverges, beta functions vanish

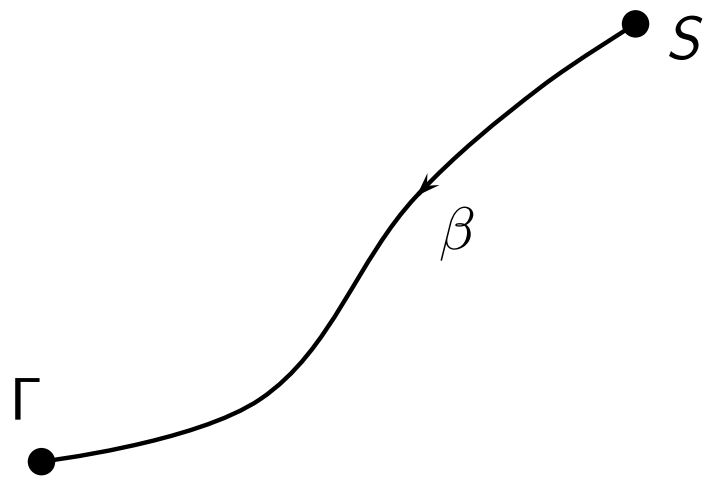
Example: Strong coupling constant



taken from nobelprize.org, popular information on '04 Nobel Prize in physics

Renormalisation

- beta functions can be calculated by renormalisation group techniques (either perturbatively or nonperturbatively)
- efficient description by effective action Γ (quantum analogue of classical action)



Partial bosonisation

$$S^{\mathrm{GN}} = \int \mathrm{d}^3x \left[\overline{\psi} \partial\!\!\!/ \psi + \frac{\overline{g}}{2N_f} \left(\overline{\psi}\psi \right)^2 \right]$$

- employ Hubbard-Stratonovich transformation to get rid of 4-fermi term ($\chi \sim \overline{\psi}\psi$):

$$S^{\mathrm{pbGN}} = \int \mathrm{d}^3x \left(\overline{\psi} \left(\partial\!\!\!/ + \overline{h}\,\chi \right) \psi - \frac{N_f}{2} \overline{m}^2 \chi^2 \right)$$

  Yukawa coupling $\overline{h}$ with $\overline{g} = \overline{h}^2 / \overline{m}^2$
- use functional RG methods to study this model at criticality in extended approximation

Approximation

$$
\begin{aligned}
\Gamma \approx \int \mathrm{d}^3x \Big[& \frac{1}{2} Z_\chi(\rho)(\partial_\mu \chi)^2 - V(\rho) + g_\chi(\rho)\chi\overline{\psi}\psi \\
& + \frac{1}{2} Z_\psi(\rho) \left[\overline{\psi}(\mathbb{1}_{N_f} \otimes \partial\!\!\!/)\psi - (\partial^\mu \overline{\psi})(\mathbb{1}_{N_f} \otimes \gamma_\mu)\psi \right] \\
& + \mathbf{i}\, J_\psi(\rho)(\partial^\mu \rho)\overline{\psi}(\mathbb{1}_{N_f} \otimes \gamma_\mu)\psi + X_1(\rho)\chi(\partial_\mu \overline{\psi})(\partial^\mu \psi) \\
& + \frac{\mathbf{i}}{2} X_2(\rho)(\partial^\mu \chi) \left(\overline{\psi}\partial_\mu \psi - (\partial_\mu \overline{\psi})\psi \right) + X_3(\rho)(\partial^2 \chi)\overline{\psi}\psi \\
& + \frac{1}{2} X_4(\rho)(\partial^\mu \chi) \left(\overline{\psi}(\mathbb{1}_{N_f} \otimes \Sigma_{\mu\nu})\partial^\nu \psi - (\partial^\nu \overline{\psi})(\mathbb{1}_{N_f} \otimes \Sigma_{\mu\nu})\psi \right) \\
& + \frac{1}{2}(X_5(\rho) + 2X_3'(\rho))(\partial_\mu \chi)^2 \chi\overline{\psi}\psi \Big]
\end{aligned}
$$

Technicalities

- by-hand derivation of beta functions practically impossible, computer algebra needed
- use xAct to do the job (tensor algebra package for gravity)
- still have to solve set of 10 ODEs $\rightarrow$ pseudo-spectral methods:

$$f(\rho) \approx \sum_{i=0}^{N_{\max}} c_n T_n \left(\frac{2\rho - \rho_{\max}}{\rho_{\max}} \right)$$

N_f=2

	θ_1	η_χ	η_ψ
FRG (this work)	0.994(2)	0.7765	0.0276
FRG [95]	0.996	0.789	0.031
Monte Carlo [125]	1.00(4)	0.754(8)	
Large N_f [41,126]	0.962	0.776	0.044
$(2+\epsilon)$ third order [127–129]	0.764	0.602	0.081
$(2+\epsilon)$ fourth order resummed [130]	0.931	0.745	0.082
$(4-\epsilon)$ second order [131]	1.055	0.695	0.065
Two-sided Padé of ϵ expansions [132]	0.948	0.739	0.041

$N_f = 1$

	θ_1	η_χ	η_ψ
FRG (this work)	1.075(4)	0.5506	0.0645
FRG [95]	1.077	0.602	0.069
Monte Carlo [134]	1.25(3)	0.302(7)	
Large N_f [41,126,135]	1.361	0.635	0.105
$(4-\epsilon)$ second order [131]	1.160	0.502	0.110
Two-sided Padé of ϵ expansions [132]	0.852	0.506	0.096

Summary & Outlook

- technical progress: extended approximations can be studied
- $N_f = 2$: results appear to be converged, good agreement
- $N_f = 1$: substantial disagreement of different methods, further studies necessary
- future: study other types of condensates: $\phi^a = \left[\overline{\psi}(\sigma^a \otimes \mathbb{1}_4)\psi\right]$