Gross-Neveu-Yukawa models at criticality

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Gross-Neveu model

- classical GN model: asymptotically free fermionic theory in 2 dimensions
- action:
  \[ S^{GN} = \int d^2x \left[ \bar{\psi} \gamma \psi + \frac{\bar{g}}{2N_f} (\bar{\psi} \psi)^2 \right] \]
- Gross, Neveu (1974): spontaneous breaking of chiral symmetry by bifermion condensate → toy model for confinement
Why bother?
Why bother?

- $d = 3$: relativistic, effective low energy model for fermions, interesting e.g. in condensed matter (graphene)
- $d = 4$: Higgs-top sector of Standard Model, low energy description of QCD (quark-meson model)
- can be interpreted as asymptotically safe field theory

Here: focus on $d = 3$ and $N_f = 1, 2$ flavours of fermions in 4-dimensional reducible representation
QFT, beta functions and the like

- calculating the path integral is very difficult:\footnote{citation needed}:

\[ Z = \int \mathcal{D}\Phi \, e^{iS[\Phi]}/\hbar \]

- study approximations within subspace of all possible interactions
- this makes observables depend on only a manageable amount of coupling constants
- coupling constants depend on energy scale \( \Rightarrow \) encoded in beta functions
- criticality: correlation length diverges, beta functions vanish
Example: Strong coupling constant

![Graph showing coupling constant vs energy](image)

- Coupling constant, $\alpha_s (E)$
- Energy, GeV

Illustration: Typoform

taken from nobelprize.org, popular information on '04 Nobel Prize in physics
Renormalisation

- beta functions can be calculated by renormalisation group techniques (either perturbatively or nonperturbatively)
- efficient description by effective action $\Gamma$ (quantum analogue of classical action)
Partial bosonisation

\[ S^{GN} = \int d^3 x \left[ \bar{\psi} \phi \psi + \frac{g}{2N_f} \left( \bar{\psi} \psi \right)^2 \right] \]

- employ Hubbard-Stratonovich transformation to get rid of 4-fermi term (\( \chi \sim \bar{\psi} \psi \)):

\[ S^{pbGN} = \int d^3 x \left( \bar{\psi} \left( \phi + \bar{h} \chi \right) \psi - \frac{N_f}{2} \bar{m}^2 \chi^2 \right) \]

Yukawa coupling \( \bar{h} \) with \( \bar{g} = \bar{h}^2 / \bar{m}^2 \)

- use functional RG methods to study this model at criticality in extended approximation
Approximation

\[ \Gamma \approx \int d^3x \left[ \frac{1}{2} Z_\chi(\rho)(\partial_\mu \chi)^2 - V(\rho) + g_\chi(\rho)\chi \bar{\psi}\psi \\
+ \frac{1}{2} Z_\psi(\rho) \left[ \bar{\psi}(\mathbb{1}_{N_f} \otimes \bar{\phi})\psi - (\partial_\mu \bar{\psi})(\mathbb{1}_{N_f} \otimes \gamma_\mu)\psi \right] \\
+ i J_\psi(\rho)(\partial^\mu \rho)\bar{\psi}(\mathbb{1}_{N_f} \otimes \gamma_\mu)\psi + X_1(\rho)\chi(\partial_\mu \bar{\psi})(\partial^\mu \psi) \\
+ \frac{i}{2} X_2(\rho)(\partial^\mu \chi) \left( \bar{\psi} \partial_\mu \psi - (\partial_\mu \bar{\psi})\psi \right) + X_3(\rho)(\partial^2 \chi)\bar{\psi}\psi \\
+ \frac{1}{2} X_4(\rho)(\partial^\mu \chi) \left( \bar{\psi}(\mathbb{1}_{N_f} \otimes \Sigma_{\mu\nu})^{\partial_\nu} \psi - (\partial_\nu \bar{\psi})(\mathbb{1}_{N_f} \otimes \Sigma_{\mu\nu})\psi \right) \\
+ \frac{1}{2} (X_5(\rho) + 2X'_3(\rho))(\partial_\mu \chi)^2 \chi \bar{\psi}\psi \right] \]
Technicalities

- by-hand derivation of beta functions practically impossible, computer algebra needed
- use xAct to do the job (tensor algebra package for gravity)
- still have to solve set of 10 ODEs → pseudo-spectral methods:

\[ f(\rho) \approx \sum_{i=0}^{N_{\text{max}}} c_n T_n \left( \frac{2\rho - \rho_{\text{max}}}{\rho_{\text{max}}} \right) \]
$N_f = 2$

<table>
<thead>
<tr>
<th></th>
<th>$\theta_1$</th>
<th>$\eta_\chi$</th>
<th>$\eta_\psi$</th>
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</thead>
<tbody>
<tr>
<td>FRG (this work)</td>
<td>0.994(2)</td>
<td>0.7765</td>
<td>0.0276</td>
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<tr>
<td>FRG [95]</td>
<td>0.996</td>
<td>0.789</td>
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<td>Monte Carlo [125]</td>
<td>1.00(4)</td>
<td>0.754(8)</td>
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<tr>
<td>Large $N_f$ [41,126]</td>
<td>0.962</td>
<td>0.776</td>
<td>0.044</td>
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<tr>
<td>(2 + $\epsilon$) third order [127–129]</td>
<td>0.764</td>
<td>0.602</td>
<td>0.081</td>
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<tr>
<td>(2 + $\epsilon$) fourth order</td>
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<td>resummed [130]</td>
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<tr>
<td>(4 − $\epsilon$) second order [131]</td>
<td>1.055</td>
<td>0.695</td>
<td>0.065</td>
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<tr>
<td>Two-sided Padé of $\epsilon$ expansions [132]</td>
<td>0.948</td>
<td>0.739</td>
<td>0.041</td>
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</table>
\[ N_f = 1 \]

<table>
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<tr>
<th>Method</th>
<th>( \theta_1 )</th>
<th>( \eta_\chi )</th>
<th>( \eta_\psi )</th>
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<tr>
<td>FRG (this work)</td>
<td>1.075(4)</td>
<td>0.5506</td>
<td>0.0645</td>
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<tr>
<td>FRG [95]</td>
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<td>0.069</td>
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<td>Monte Carlo [134]</td>
<td>1.25(3)</td>
<td>0.302(7)</td>
<td></td>
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<tr>
<td>Large ( N_f ) [41,126,135]</td>
<td>1.361</td>
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<td>(4 ( - \epsilon )) second order</td>
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<td>Two-sided Padé of ( \epsilon ) expansions [132]</td>
<td>0.852</td>
<td>0.506</td>
<td>0.096</td>
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</table>
technical progress: extended approximations can be studied

$N_f = 2$: results appear to be converged, good agreement

$N_f = 1$: substantial disagreement of different methods, further studies necessary

future: study other types of condensates:

$$\phi^a = \left[ \overline{\psi} (\sigma^a \otimes 1_4) \psi \right]$$