Baryogenesis with Leptons

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- Introduction: baryogenesis requirements
- Baryon number violation in the Standard Model
- Baryogenesis via leptogenesis
- Neutrino experiments

Baryogenesis: generalities

The big question on the early (t < 1 s) Universe:

- in our "standard" picture, the Big Bang started out as a symmetric situation: there were equal amounts of matter and anti-matter
- but the present Universe is entirely dominated by matter
- how did this transition happen?

But let's be more precise. Is this a large asymmetry?

- ► the temperature of the present Universe is sufficiently low that all available matter and anti-matter must have annihilated, leading to radiation ⇒ need to compare matter and radiation densities
- photon density follows simply from CMBR spectrum:

$$n_{\gamma} = 2\zeta(3)T^3/\pi^2 \stackrel{T=2.7\text{K}}{\longrightarrow} 420/\text{cm}^3$$

Big Bang Nucleosynthesis

BBN (t > 1 s): a (successful) quantitative explanation for the abundance (at $t \sim 180$ s) of light elements starting from available protons & neutrons.

- all starts with formation of deuterium: efficient if $T < \Delta = 2.2$ MeV binding energy
- more precisely, $\eta^{-1} e^{-\Delta/T} < 0.1,$ $\eta \equiv n_{\rm B}/n_{\gamma}$
- also other processes depend on the ratio η

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Result (at 95% CL):

$$\eta = (4.7 - 6.5) \cdot 10^{-10}$$



The Sakharov Conditions

So all of the visible matter is due to a tiny effect: at freeze-out ($T\sim 1$ GeV), we have $n_q/n_{\bar{q}}=1+\eta$

Can the SM somehow be responsible for this?

Conditions for the generation of a baryon asymmetry formulated by Sakharov (1967):

- 1. there must be baryon number violating interactions
- 2. there must be CP-violating interactions
- 3. these interactions must be out of thermal equilibrium

We have seen (previous lecture) that the SM does provide (small amounts of) CP violation; and the rapid expansion of the early Universe may well have led to an out-of-equilibium situation. But how about *B*-violation? (And what does baryon number mean in detail?)

The Standard Model Lagrangian

Consider the parts of the SM Lagrangian involving fermions:

$$\mathcal{L} = h_{ij}^{\ell} \overline{\ell}_{i,L} \phi \ell_{j,R} + h_{ij}^{\ell\dagger} \overline{\ell}_{i,R} \phi^{\dagger} \ell_{j,L}$$

$$+ \frac{1}{2} \overline{\ell}_{i,L} \left(g \vec{\sigma} \cdot \vec{W} + g' \beta \right) \ell_{i,L} + \overline{\ell}_{i,R} g' \beta \ell_{i,R}$$

$$+ h_{ij}^{q,d} \overline{q}_{i,L} \phi q_{j,R} + h_{ij}^{q,d\dagger} \overline{q}_{i,R} \phi^{\dagger} q_{j,L} + h_{ij}^{q,u} \overline{q}_{i,L} \widetilde{\phi} q_{j,R} + h_{ij}^{q,u\dagger} \overline{q}_{i,R} \widetilde{\phi}^{\dagger} q_{j,L}$$

$$+ \frac{1}{2} \overline{q}_{i,L} \left(g \vec{\sigma} \cdot \vec{W} + \frac{1}{3} g' \beta \right) q_{i,L} - \frac{1}{3} \overline{d}_{i,R} g' \beta d_{i,R} + \frac{2}{3} \overline{u}_{i,R} g' \beta u_{i,R}$$

Here:

- $\tilde{\phi} \equiv i\sigma_2 \phi^*$ (required in order to give mass to the up-type quarks)
- *l*_{L,R} : left-handed lepton doublet, right-handed (charged) lepton singlet

▶ q_L, u_R, d_R: left-handed quark doublet, right-handed up- and down-type quark singlets

Baryon and Lepton Number

The SM Lagrangian features the $SU(3)_C \otimes SU(2)_W \otimes SU(1)_Y$ gauge symmetry.

However, it also features (additional) global symmetries:

According to the Noether Theorem, these additional symmetries can be associated with conserved currents (like for the gauge symmetries):

$$\begin{aligned} \partial_{\mu}J_{\mathsf{B}}^{\mu} &= 0, \qquad J_{\mathsf{B}}^{\mu} \equiv \frac{1}{3}\sum_{i}\bar{q}_{i}\gamma^{\mu}q_{i} = \frac{1}{3}\sum_{i}\bar{q}_{i,\mathsf{L}}\gamma^{\mu}q_{i,\mathsf{L}} + \bar{q}_{i,\mathsf{R}}\gamma^{\mu}q_{i,\mathsf{R}} \\ \partial_{\mu}J_{\mathsf{L}}^{\mu} &= 0, \qquad J_{\mathsf{L}}^{\mu} \equiv \frac{1}{3}\sum_{i}\bar{\ell}_{i}\gamma^{\mu}\ell_{i} = \frac{1}{3}\sum_{i}\bar{\ell}_{i,\mathsf{L}}\gamma^{\mu}\ell_{i,\mathsf{L}} + \bar{\ell}_{i,\mathsf{R}}\gamma^{\mu}\ell_{i,\mathsf{R}} \end{aligned}$$

The (conserved) baryon and lepton number are defined as

$$B \equiv \int d^3x J_{\rm B}^0(x), \qquad L \equiv \int d^3x J_{\rm L}^0(x)$$

Quantum Corrections

Although these currents are conserved at the classical level, this is no longer true when quantum corrections are applied! This is a consequence of the Adler-Jackiw-Bell anomaly (a.k.a. triangle anomaly)



Upon applying these quantum corrections (with gauge bosons $F^a_{\mu\nu}$), we obtain

$$\begin{split} \partial_{\mu} \bar{f}_{\rm L} \gamma^{\mu} f_{\rm L} &= -c_{\rm L} \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}, \\ \partial_{\mu} \bar{f}_{\rm R} \gamma^{\mu} f_{\rm R} &= +c_{\rm R} \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \end{split}$$

with the dual tensor $\tilde{F}^{a\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} F^a_{\alpha\beta}/2$. Here the constants $c_{L,R}$ depend on the group representation of the $f_{L,R}$.

Quantum Corrections (cont'd)

Consider first the left-handed representations only (with $n_F = 3$ the number of fermion families):

$$\partial_{\mu}J^{\mu}_{\mathsf{B}} = \partial_{\mu}J^{\mu}_{\mathsf{L}} = \frac{n_{\mathsf{F}}}{32\pi^{2}}\left(-g^{2}W^{a}_{\mu\nu}\tilde{W}^{a\mu\nu} + g'^{2}B_{\mu\nu}\tilde{B}^{\mu\nu}\right) = n_{\mathsf{F}}\partial_{\mu}K^{\mu}$$

with

$$\mathcal{K}^{\mu} = -\frac{g^{2}}{32\pi^{2}} 2\epsilon^{\mu\nu\alpha\beta} W^{a}_{\nu} \left(\partial_{\alpha} W^{a}_{\beta} + \frac{g}{3} W^{b}_{\alpha} W^{c}_{\beta} \epsilon^{abc} \right) + \frac{g^{\prime 2}}{32\pi^{2}} \epsilon^{\mu\nu\alpha\beta} B_{\nu} B_{\alpha\beta}$$

Such total derivatives can usually be neglected because field strengths $F^a_{\mu\nu}$ vanish at infinity. However, this is not the case for the gauge fields themselves! Hence processes can have

$$\Delta B = \Delta L = n_{\rm F} \Delta N_{\rm CS}, \quad N_{\rm CS} = \frac{g^3}{96\pi^2} \int {\rm d}^3 x \epsilon_{ijk} \epsilon^{abc} W^{ai} W^{bj} W^{ck}$$

Note that always $\Delta B = \Delta L$: (B - L) is conserved! Also, a nonzero result is due only to the non-Abelian symmetries \Rightarrow forget about right-handed contributions

The Sphaleron

Some more detail on such baryon number violating processes:

- ► N_{CS} is the Chern-Simons number, denoting the topological charge of the classical (weak) gauge fields
 - ▶ non-Abelian symmetry group ⇒ "nontrivial" classical structure (infinite number of ground states with different N_{CS})
- Baryon (and lepton) number is always violated by a multiple of 3 units. Closer inspection: ΔL_e = ΔL_μ = ΔL_τ = ΔB/3 = ΔN_{CS}

The resulting processes can be represented by a pseudo-particle called the sphaleron.

Possible scattering processes:

$$u + d \rightarrow \bar{d} + 2\bar{s} + 2\bar{b} + \bar{t} + \bar{\nu}_e + \bar{\nu}_\mu + \bar{\nu}_\tau$$



Sphaleron Process Rates

How important are the rates for such processes?



Note: also involves classical solutions for Higgs fields (not so strange, given the Higgs potential's effect on the vacuum).

• "potential barrier": $E_{sph}(T) \sim \frac{4\pi}{g} v(T) \Rightarrow E_{sph}(T) \sim 8-13$ TeV

▶ here, $v(T) \equiv \langle 0 | \phi | 0 \rangle_T$: effective Higgs potential T dependent

• $T \approx$ 0: only through quantum tunneling: $\sigma \sim e^{-1/g^2} \sim 10^{-164}$

► high
$$T < T_{\text{EW}}$$
: $\Gamma^{\text{sph}} \sim \left(\frac{m_{\text{W}}}{\alpha_w T}\right)^3 m_{\text{W}}^4 e^{-E_{\text{sph}}/T}$, with $m_{\text{W}} = gv(T)/2$

The Effective Higgs Potential

How about yet higher temperatures?

- ▶ naively, for $T > E_{\rm sph}$, expect rates unsuppressed by factors $e^{-E_{\rm sph}/T}$
- ► but this does not account for the fact that for $T > T_{EW}$, the effective Higgs potential yields an unbroken state! For these temperatures, the rate is rather expected to be $\Gamma^{\text{sph}} \sim \alpha_w^5 T^4$ (on dimensional grounds)

Result: $\Gamma^{\rm sph} \gg H$ for $T_{\rm EW} < T < 10^{12}~{\rm GeV}$ This implies that thermal equilibrium is maintained for these

temperatures. In turn, this means that any net pre-existing B + L will be washed out by sphaleron processes.

The Standard Model alone is not sufficient!

More precisely, this involves the fact that the electroweak phase transition (from unbroken to broken state) is too slow to switch off sphaleron transitions in time: electroweak baryogenesis does not work for $m_{\rm H} > 70$ GeV.

Majorana Neutrinos

At this point, we digress to discuss how neutrino masses can be generated. In the Weyl representation, "ordinary" Dirac fields ψ , and their charge conjugate fields ψ^c , can be written as

$$\psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \quad \psi^{c} \equiv i\gamma^{2}\psi^{\dagger} = \begin{pmatrix} i\sigma_{2}\eta^{\dagger} \\ -i\sigma_{2}\xi^{\dagger} \end{pmatrix}.$$

In this representation, $\gamma^5 = \text{diag}(1, -1)$. Hence the top (bottom) components represent right-handed (left-handed) fields. As such, we can write

$$\psi_{\mathsf{L}} = \begin{pmatrix} 0 \\ \eta \end{pmatrix}, \quad \psi_{\mathsf{L}}^{\mathsf{c}} \equiv (\psi_{\mathsf{L}})^{\mathsf{c}} = \begin{pmatrix} i\sigma_2\eta^{\dagger} \\ 0 \end{pmatrix}$$

and likewise for ψ_{R} , $\psi_{\mathsf{R}}^{\mathsf{c}}$.

Starting from these projections, one can construct Majorana neutrinos:

$$\psi_1 = \psi_{\mathsf{L}} + \psi_{\mathsf{L}}^{\mathsf{c}}, \quad \psi_1 = \psi_{\mathsf{R}} + \psi_{\mathsf{R}}^{\mathsf{c}}$$

which satisfy $\psi_{1,2}^c = \psi_{1,2}$: they are their own anti-particles!

Majorana Mass Terms

These Majorana neutrinos may give rise to Majorana mass terms in the Lagrangian:

$$\mathcal{L}_{\mathsf{M}} = -\frac{m_1}{2}\overline{\psi_1}\psi_1 = -\frac{m_1}{2}\overline{\psi_{\mathsf{L}}^{\mathsf{c}}}\psi_{\mathsf{L}}^{\mathsf{c}} + \mathsf{h.c.}$$
 etc.

This construction is possible only for neutrinos! Quantum numbers carried by ψ in such a mass term are not conserved. This is true, in particular, for the Lepton number!

A general Lagrangian can now be constructed from both Dirac and Majorana mass terms (changing slightly the notation):

$$-\mathcal{L} = \frac{m_{\rm L}}{2} \overline{\nu_{\rm L}^{\rm c}} \nu_{\rm L} + \frac{m_{\rm R}}{2} \overline{\nu_{\rm R}^{\rm c}} \nu_{\rm R} + m_{\rm D} \overline{\nu_{\rm R}} \nu_{\rm L} + \text{h.c.}$$
$$= \frac{1}{2} \left(\overline{\psi_1}, \overline{\psi_2} \right) \begin{pmatrix} m_{\rm L} & m_{\rm D} \\ m_{\rm D} & m_{\rm R} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

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The See-saw Mechanism

Next, suppose that $m_R \gg m_D \gg m_L$, and neglect m_L . The mass matrix can be diagonalized, with the result

$$\begin{aligned} -\mathcal{L} &= -\frac{m_{\nu}}{2}\bar{\nu}\nu + \frac{m_{N}}{2}\bar{N}N, \\ m_{N,\nu} &= \frac{1}{2}\left(\sqrt{m_{\mathrm{R}}^{2} + 4m_{\mathrm{D}}^{2}} \pm m_{\mathrm{R}}\right) \end{aligned}$$

and $\nu \approx \psi_1$, $N \approx \psi_2$. (The minus sign in front of m_{ν} can be got rid of by defining $\nu \rightarrow i\nu$) In particular, $m_{\nu} \approx m_D^2/m_R$.

This basic see-saw mechanism can be embedded in a slightly extended 3-family SM Lagrangian (3×3 complex Yukawa couplings only):

$$-\mathcal{L} = h_e \bar{\ell}_{\mathsf{L}} \phi e_{\mathsf{R}} + h_\nu \bar{\ell}_{\mathsf{L}} \tilde{\phi} \nu_{\mathsf{R}} + \frac{1}{2} M_{\mathsf{R}} \overline{\nu_{\mathsf{R}}^c} \nu_{\mathsf{R}} + \text{h.c.}$$

After symmetry breaking, these yield mass matrices $m_{e,\nu} = h_{e,\nu}v/\sqrt{2}$, and the see-saw mechanism proceeds as before.

Heavy Neutrino Decays

So why is all of this relevant? The same heavy Majorana neutrinos N may cause CP violation! Consider their decays $N \rightarrow \ell \phi$, $\bar{\ell} \phi^*$:



Total decay rates are dominated by lowest order diagrams:

$$\Gamma_i = \Gamma(N_i \to \ell \phi) + \Gamma(N_i \to \bar{\ell} \phi^*) \sim \frac{(M_{\rm D}^{\dagger} M_{\rm D})_{ii}}{v^2} M_i$$

The numerator term in the corresponding asymmetry

$$\epsilon_i \equiv \frac{\Gamma(N_i \to \ell \phi) - \Gamma(N_i \to \bar{\ell} \phi^*)}{\Gamma(N_i \to \ell \phi) + \Gamma(N_i \to \bar{\ell} \phi^*)}$$

arises entirely from interferences with the higher order corrections.

CP Violation in Heavy Majorana Neutrino Decays

When computing these interferences, two crucial ingredients need to be accounted for:

- ► the Yukawa couplings (and hence the M_D) are intrinsically complex ⇒ additional CP-violating phases
- in the higher order corrections, when particles in the loop can propagate on shell, this leads to a complex phase in the loop phase space integral (this is exactly analogous to what happens in the case of direct CP violation in the quark sector, discussed in the previous lecture)

Neglecting the masses of all but the N, and when $M_{2,3} \gg M_1$, one obtains

$$\epsilon_1 \approx -\frac{3}{4\pi v^2} \sum_{k=2,3} \mathcal{I}_{k1} \frac{M_1}{M_k}, \quad \mathcal{I}_{ki} = \frac{\Im[(M_D^{\dagger} M_D)_{ik}^2]}{(M_D^{\dagger} M_D)_{ii}}$$

These decays therefore provide both lepton (!) number violation and CP violation.

Baryogenesis via Leptogenesis

The final ingredients for a "successful" baryogenesis:

► even after the N have decayed (and produced a nonzero lepton number), L = 0 could be re-established by processes mediated by N exchange, e.g. \(\bar{l}\phi^* → l\phi)\):



The rate for these $\Delta L = 2$ processes should be smaller than the Hubble rate:

$$\Gamma(T) \sim \frac{T^3}{v^4} \sum_{i=\mathrm{e},\mu,\tau} m_i^2 < H(T) \Rightarrow \sqrt{\sum_i m_i^2} < 0.2 \text{ eV} \cdot \left(\frac{10^{12} \text{ GeV}}{T}\right)$$

So this implies a constraint on the light neutrino masses! For "typical" leptogenesis temperatures $T \sim 10^{10}$ GeV, $m_{\nu} < 1$ eV. Nevertheless, a suppression factor $\kappa \sim 10^{-1} - 10^{-3}$ results.

Baryogenesis via Leptogenesis (cont'd)

• the sphaleron transitions we encountered earlier then transfer part of the lepton asymmetry into a baryon asymmetry: $\eta_B = C\eta_{B-L}$ with *C* depending on which processes are in thermodynamic equilibrium (this depends on the precise particle physics model, but $C \sim O(1)$).



With $\eta_{\rm B}(\text{now}) \approx 0.07 \eta_{\rm B}(T = M_1)$ (photons now make up a larger fraction of the Universe's entropy) we need

$$\eta_{\sf B}({\sf now}) = 0.07 \cdot \kappa \cdot \epsilon_1 \Rightarrow \ \epsilon_1 \sim 10^{-6}$$

so only a tiny amount of CP violation is required!

Experimental Consequences

From the preceding, it follows that the presently observable light neutrinos should satisfy the following:

- they should be light, $m_{\nu} < 1$ eV (even $\ll 1$ eV ?)
- they should be of Majorana type

Note that these hypotheses are difficult to verify, because neutrinos only experience the weak interaction! Nevertheless, experiments have been done or are being done to try and pin down neutrino properties:

- 1. direct m_{ν} measurements
- 2. ν oscillation measurements
- 3. attempts to observe $\beta\beta0\nu$ (neutrinoless double β decays)

Direct Neutrino Mass Measurement

Since the neutrino itself is only extremely rarely observed, use its effect on decay kinematics. Present status: $m_{\nu} < 2 \text{ eV}$



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Neutrino Oscillations

The phenomenon of neutrino oscillations tells us that neutrinos (those that participate in the weak interaction) *do* have mass – although it does not directly tell us what these masses are!

In this case, the CKM matrix has a leptonic equivalent: weak eigenstates ν_{α} ($\alpha = e, \mu\tau$) and mass eigenstates ν_i (i = 1, 2, 3) are related by

$$|
u_i\rangle = \sum_{lpha} U_{lpha i} |
u_{lpha}
angle$$

Here, the Maki-Nakagawa-Sakata or MNS matrix U can be defined as

$$U_{\rm D} = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{array} \right) \cdot \left(\begin{array}{ccc} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{array} \right) \cdot \left(\begin{array}{ccc} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{array} \right),$$

with $c_{23} \equiv \cos \theta_{23}$, $s_{23} \equiv \sin \theta_{23}$ etc.; $U_{M} = U_{D} \cdot \text{diag}(e^{i\alpha_{1}/2}, e^{i\alpha_{2}/2}, 1)$.

 while U_D (relevant for Dirac ν) has only one CP-violating phase δ, extra phases α_{1,2} appear for Majorana ν

Neutrino Oscillations (cont'd)

What are neutrino oscillations?

It is possible for a neutrino produced as one weak eigenstate ν_{α} to be detected (through its conversion to a charged lepton) after a time t as a different weak eigenstate ν_{β} .

Applying the standard Schrödinger equation to relativistic freely propagating mass eigenstates ν_i :

$$i rac{\mathsf{d}}{\mathsf{d} au} |
u_i(au)
angle = m_i |
u_i(au)
angle \Rightarrow |
u_i(au)
angle = e^{-im_i au} |
u_i(0)
angle$$

Using $m_i \tau_i \approx E(t - L) + m_i^2 L/2E$, the transition amplitude $\mathcal{A}(\nu_{\alpha} \rightarrow \nu_{\beta})$ becomes (apart from an overall phase)

$$\mathcal{A}(\nu_{lpha}
ightarrow \nu_{eta}) = \sum_{i} U^*_{lpha i} e^{-im_i^2 L/2E} U_{eta i}$$

and the transition probability is simply the square of the amplitude.

► it can be seen straight away that the Majorana phases drop out of the equation ⇒ no sensitivity to the precise nature of neutrinos

Neutrino Oscillations Simplified

For most purposes, a description featuring only two neutrinos suffices:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

In that case, also the oscillation formula becomes very simple:

$$\begin{array}{lll} P(\nu_{\alpha} \rightarrow \nu_{\beta}) &=& \sin^2 2\theta \sin^2(\Delta m^2 L/4E) \ \ (\text{natural units}) \\ &=& \sin^2 2\theta \sin^2\left(\frac{1.27L(\text{km})\Delta m^2(\text{ eV}^2)}{E(\text{ GeV})}\right) \end{array}$$

So ν oscillations are sensitive only to differences between (squared) masses

Atmospheric Neutrino Oscillation Measurements

In decays of (light) hadrons in cosmic rays (with energies in the GeV regime, or higher), twice as many ν_{μ} as ν_{e} are expected:

 $\begin{array}{rccc} \pi^+ & \to & \mu^+ \nu_\mu, \\ \mu^+ & \to & \mathrm{e}^+ \bar{\nu}_\mu \nu_\mathrm{e} \end{array}$

(and likewise for π^- decays).

This is not what was observed! Evidence for ν oscillations from the Super-Kamiokande experiment in 1998

- a huge (50 kton) water Cherenkov detector
- underground to shield against ubiquitous backgrounds



Detection of ultrarelativistic charged particles using Cherenkov radiation:

Cherenkov radiation directional

- different patterns for μ^{\pm} , e[±]
- also energy measurement

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Different angles θ correspond to different path lengths *L* traveled by the neutrinos

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A clear oscillation pattern:

- u_{μ} partially converted to u_{τ}
- $\nu_{\rm e}$ signal only slightly affected

Solar Neutrino Oscillations

A well-known problem dating from the 60's: observed ν_e flux ($E \sim a$ few MeV) inconsistent with (only $\sim 1/3$ of) predictions from the Standard Solar Model. But firm conclusions could not be drawn directly:

- the physics underlying the Standard Solar Model isn't simple!
- only v_e disappearance experiments could be carried out: v_e energy too low to lead to the creation of charged leptons other than electrons

In 2002, the Sudbury Neutrino Observatory provided the answer



Apart from Charged Current interactions ($\nu_e \rightarrow e$ conversions), this experiment was sensitive also to Neutral Current interactions

- these are ν flavour blind and therefore measure the total ν flux emanating from the Sun
- possible by using heavy water D₂O rather than normal water

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It clearly established that part of the solar $\nu_{\rm e}$ convert to $\nu_{\mu,\tau}$

► Elastic Scattering involves only electrons: $\nu_e e^- \rightarrow \nu_e e^-$. It occurs for all ν types but with a different cross section for ν_e than for $\nu_{\mu,\tau}$

Neutrino Oscillation Parameters

A consistent picture is now emerging:

- ▶ from solar ν measurements: $\Delta m_{12}^2 \equiv |m_1^2 m_2^2| \approx 8 \cdot 10^{-5} \text{ eV}^2$, $\theta_{12} \approx 34^\circ$
- ▶ from atmospheric ν measurements: $\Delta m^2_{23} \approx 2.4 \cdot 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta_{23} > 0.90$
- ► only an upper limit exists for θ₁₃: sin² 2θ₁₃ < 0.1, from the non-observation of a ν
 _e disappearance signal in short baseline (L = 1km) ν
 _e from nuclear reactors
- ▶ nothing is known about the CP-violating phase δ as yet. The chances of observing CP violation in the MNS matrix depend on the value of θ_{13}

It is not yet known whether a normal hierarchy $(m_1 < m_2 < m_3)$ or an inverted hierarchy $(m_3 < m_1 < m_2)$ applies. But it appears to be very reasonable to assume that neutrinos do have masses below O(1 eV)!

Neutrinoless Double Beta Decay

Unambiguous evidence for their Majorana nature would be provided by the observation of neutrinoless double beta decay:

 $(A,Z) \rightarrow (A,Z+2) + 2e^{-}$



- ▶ particle identical to its own antiparticle ⇒ Majorana nature
- ► "mixing" between left-handed and right-handed particles ⇒ amplitude proportional to

$$m_{\beta\beta} \equiv |\sum_i m_i U_{ei}^2|$$

Unfortunately, there are also decays that look very similar but that also yield two neutrinos (possible with even-even nuclei) \Rightarrow requires

- very precise energy measurement
- excellent knowledge/control of backgrounds

The next generation of experiments will likely tell!

Further Reading

- B.D. Fields and S. Sarkar, "Big-Bang Nucleosynthesis", in The Review of Particle Physics, W.-M. Yao *et al.*, J. Phys. G33 (2006) 1
- ▶ W. Bernreuther, "CP Violation and Baryogenesis", Lect.Notes Phys. 591 (2002) 237-293 [hep-ph/0205279]
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