ART

SET 4

4.1

(i) $g^{\mu\nu} g_{\mu\lambda} = \delta^{\nu}{}_{\lambda}$

$$\delta g^{\mu\nu} g_{\mu\lambda} + g^{\mu\nu} \delta g_{\mu\lambda} = 0$$

$$\delta g^{\mu\nu} g_{\mu\lambda} = - g^{\mu\nu} \delta g_{\mu\lambda}$$

$$\delta g^{\mu\nu} g_{\mu\lambda} g^{\kappa\lambda} = - g^{\mu\nu} \delta g_{\mu\lambda} g^{\kappa\lambda}$$

$$\delta g^{\mu\nu} \delta_{\mu}{}^{\kappa} = - g^{\mu\nu} \delta g_{\mu\lambda} g^{\kappa\lambda}$$

$$\delta g^{\kappa\nu} = - g^{\mu\nu} \delta g_{\mu\lambda} g^{\kappa\lambda}$$

ii) from 1.5 of Ronald's notes:

$$\delta g = g\, g^{\alpha\beta} \delta g_{\alpha\beta}$$

Divide by $\sqrt{g}$

$$\frac{\delta g}{\sqrt{g}} = \sqrt{g}\, g^{\alpha\beta} \delta g_{\alpha\beta}$$

$$2 \cdot \delta \sqrt{g} = \sqrt{g}\, g^{\alpha\beta} \delta g_{\alpha\beta}$$

$$\delta \sqrt{g} = \frac{1}{2} \sqrt{g}\, g^{\alpha\beta} \delta g_{\alpha\beta}$$

SET ART 4

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4.3

$$G_N = 6,67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$$

$$c = 3 \cdot 10^8 \frac{m}{s}$$

$$\hbar = 1 \cdot 10^{-34} \frac{kg \cdot m^2}{s}$$

$$\Rightarrow \quad \hbar \cdot G_N \rightarrow \frac{\cancel{kg} \cdot m^2}{s} \cdot \frac{m^3}{\cancel{kg} \cdot s^2} \sim \frac{m^5}{s^3}$$

$$\Rightarrow \quad \frac{\hbar G_N}{c^3} \sim \frac{m^5}{s^3} \frac{s^3}{m^3} \sim m^2$$

$$\Rightarrow \quad L_P = \sqrt{\frac{\hbar G_N}{c^3}} \simeq 1,6 \times 10^{-35} m$$

and $\quad \Lambda \sim \frac{1}{L_P^2} \sim 10^{70} m \rightarrow$ rather problematic

4.4

Use the hint: $(p^\mu p_\mu)^{1/2} = \rho\sqrt{|g|}$

Then $I = \int d^4x \sqrt{|g|}\,\rho = \int d^4x\,(p^\mu p_\mu)^{1/2}$

$$\delta I = \int d^4x \frac{1}{2} \frac{1}{(p\cdot p)^{1/2}} p^\mu p^\nu \delta g_{\mu\nu} =$$

$$= \int d^4x \frac{1}{2\rho\sqrt{|g|}} \rho^2 u^\mu u^\nu |g| \delta g_{\mu\nu} =$$

$$= \int d^4x \sqrt{|g|} \frac{\rho}{2} u^\mu u^\nu \delta g_{\mu\nu}$$

All together:

$$I_3 = \int d^4x \sqrt{|g|} \left(R - 2\Lambda + \frac{16\pi G}{c^2}\rho\right)$$