**PHYSICS**

# The Self-Organizing QUANTUM

## A new approach to the decades-old problem of quantum gravity goes back to basics and shows how the building blocks of space and time pull themselves together

### By Jan Ambjørn, Jerzy Jurkiewicz and Renate Loll

## KEY CONCEPTS

- Quantum theory and Einstein's general theory of relativity are famously at loggerheads. Physicists have long tried to reconcile them in a theory of quantum gravity—with only limited success.
- A new approach introduces no exotic components but rather provides a novel way to apply existing laws to individual motes of spacetime. The motes fall into place of their own accord, like molecules in a crystal.
- This approach shows how four-dimensional spacetime as we know it can emerge dynamically from more basic ingredients. It also suggests that spacetime shades from a smooth arena to a funky fractal on small scales.

*—The Editors*

How did space and time come about? How did they form the smooth four-dimensional emptiness that serves as a backdrop for our physical world? What do they look like at the very tiniest distances? Questions such as these lie at the outer boundary of modern science and are driving the search for a theory of quantum gravity—the long-sought unification of Einstein's general theory of relativity with quantum theory. Relativity theory describes how spacetime on large scales can take on countless different shapes, producing what we perceive as the force of gravity. In contrast, quantum theory describes the laws of physics at atomic and subatomic scales, ignoring gravitational effects altogether. A theory of quantum gravity aims to describe the nature of spacetime on the very smallest scales—the voids in between the smallest known elementary particles—by quantum laws and possibly explain it in terms of some fundamental constituents.

Superstring theory is often described as the leading candidate to fill this role, but it has not yet provided an answer to any of these pressing questions. Instead, following its own inner logic, it has uncovered ever more complex layers of new, exotic ingredients and relations among them, leading to a bewildering variety of possible outcomes.

Over the past few years our collaboration has developed a promising alternative to this much traveled superhighway of theoretical physics. It follows a recipe that is almost embarrassingly simple: take a few very basic ingredients, assem-

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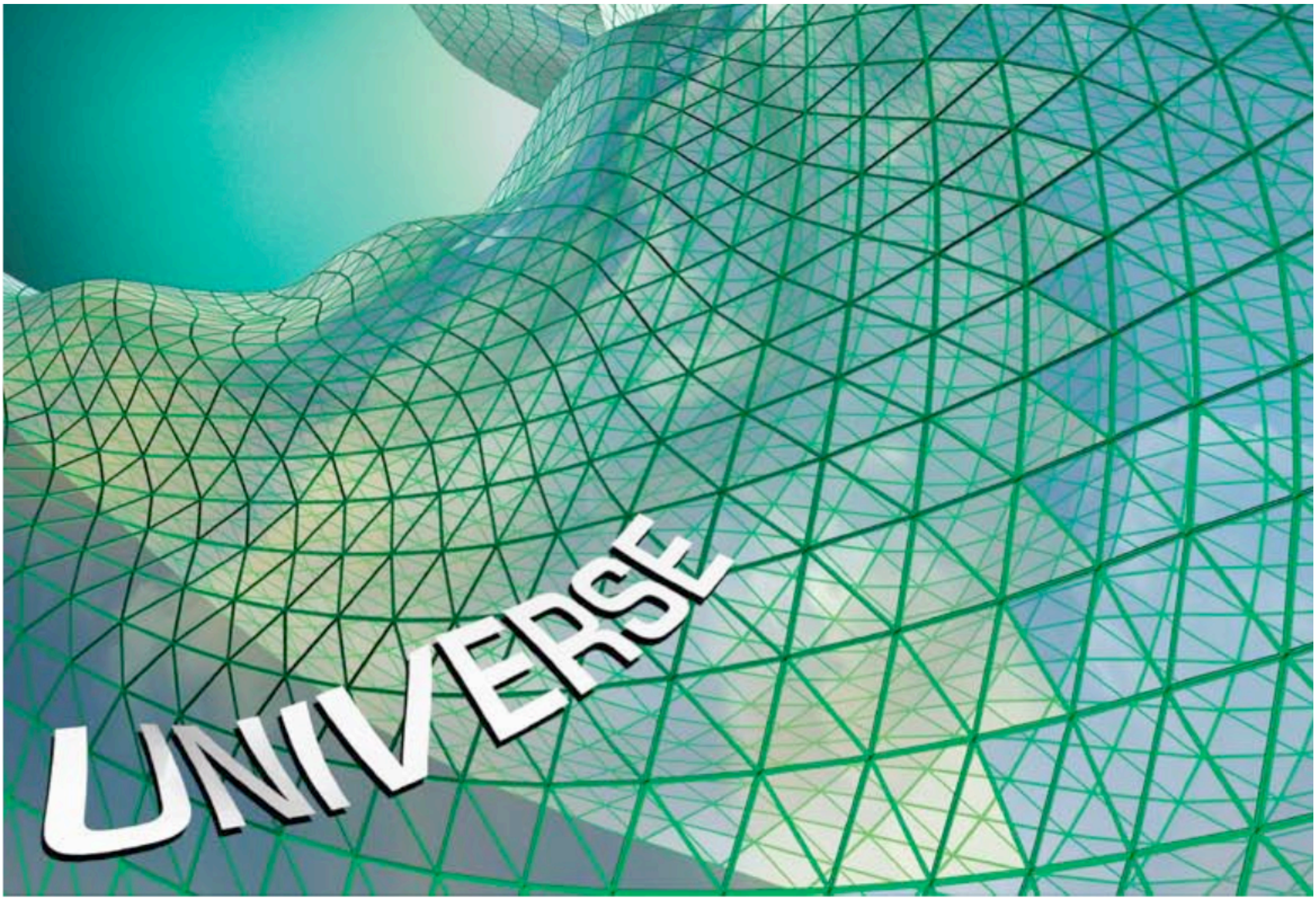

ble them according to well-known quantum principles (nothing exotic), stir well, let settle—and you have created quantum spacetime. The process is straightforward enough to simulate on a laptop.

To put it differently, if we think of empty spacetime as some immaterial substance, consisting of a very large number of minute, structureless pieces, and if we then let these microscopic building blocks interact with one another according to simple rules dictated by gravity and quantum theory, they will spontaneously arrange themselves into a whole that in many ways looks like the observed universe. It is similar to the way that molecules assemble themselves into crystalline or amorphous solids.

Spacetime, then, might be more like a simple stir fry than an elaborate wedding cake. Moreover, unlike other approaches to quantum gravity our recipe is very robust. When we vary the details in our simulations, the result hardly changes. This robustness gives reason to believe we are on the right track. If the outcome were sensitive to where we put down each piece of this enormous ensemble, we could generate an enormous number of baroque shapes, each a priori equally likely to occur—so we would lose all explanatory power for why the universe turned out as it did.

Similar mechanisms of self-assembly and self-organization occur across physics, biology and other fields of science. A beautiful example is the behavior of large flocks of birds, such as European starlings. Individual birds interact only with a small number of nearby birds; no leader tells them what to do. Yet the flock still forms and moves as a whole. The flock possesses collective, or emergent, properties that are not obvious in each bird's behavior.

## A Brief History of Quantum Gravity

Past attempts to explain the quantum structure of spacetime as a process of emergence had only limited success. They were rooted in Euclidean quantum gravity, a research program initiated at the end of the 1970s and popularized by physicist Stephen Hawking's best-selling book *A Brief History of Time*. It is based on a fundamental principle from quantum mechanics: superposition. Any object, whether a classical or quantum one, is in a certain state—characterizing its position and velocity, say. But whereas the state of a classical object can be described by a unique set of numbers, the state of a quan-

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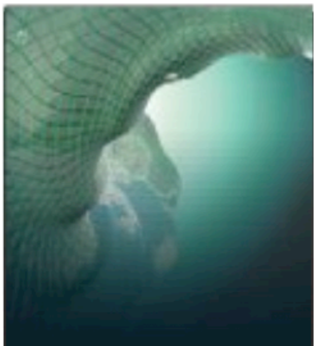

## THEORIES OF QUANTUM GRAVITY

**STRING THEORY**
The approach favored by most theoretical physicists, it is a theory not just of quantum gravity but of all matter and forces. It is based on the idea that particles (including the hypothetical ones that transmit gravity) are vibrating strings.

**LOOP QUANTUM GRAVITY**
The main alternative to string theory, it invokes a new technique for applying quantum rules to Einstein's general theory of relativity. Space is divided into discrete "atoms" of volume.

**EUCLIDEAN QUANTUM GRAVITY**
Made famous by physicist Stephen Hawking, this approach supposes that spacetime emerges from a grand quantum average of all possible shapes. It puts time on the same footing as space.

**CAUSAL DYNAMICAL TRIANGULATIONS**
This approach, the subject of this article, is a modern version of the Euclidean approach. It approximates spacetime as a mosaic of triangles, which have a built-in distinction between space and time. On small scales, spacetime takes on a fractal shape.

tum object is far richer. It is the sum, or superposition, of all possible classical states.

For instance, a classical billiard ball moves along a single trajectory with a precise position and velocity at all times. That would not be a good description for how the much smaller electron moves. Its motion is described by quantum laws, which imply that it can exist simultaneously in a wide range of positions and velocities. When an electron travels from point A to point B in the absence of any external forces, it does not just take the straight line between A and B but all available routes simultaneously. This qualitative picture of all possible electron paths conspiring together translates into the precise mathematical prescription of a quantum superposition, formulated by Nobel laureate Richard Feynman, which is a weighted average of all these distinct possibilities.

With this prescription, one can compute the probability of finding the electron in any particular range of positions and velocities away from the straight path that we would expect if the electrons followed the laws of classical mechanics. What makes the particles' behavior distinctly quantum mechanical are the deviations from a single sharp trajectory, called quantum fluctuations. The smaller the size of a physical system one considers, the more important the quantum fluctuations become.

Euclidean quantum gravity applies the superposition principle to the entire universe. In this case, the superposition consists not of different particle paths but of different ways the entire universe could evolve in time—in particular, the various possible shapes of spacetime. To make the problem tractable, physicists typically consider only the general shape and size of spacetime, rather than every single one of its conceivable contortions [see "Quantum Cosmology and the Creation of the Universe," by Jonathan J. Halliwell; Scientific American, December 1991].

Euclidean quantum gravity took a big technical leap during the 1980s and 1990s with the development of powerful computer simulations. These models represent curved spacetime geometries using tiny building blocks, which, for convenience, are taken to be triangular. Triangle meshes can efficiently approximate curved surfaces, which is why they are frequently used in computer animations. For spacetime, the elementary building blocks are four-dimensional generalizations of triangles, called four-simplices. Just as gluing together triangles at their edges creates a two-dimensional curved surface, gluing four-simplices along their "faces" (which are actually three-dimensional tetrahedra) can produce a four-dimensional spacetime.

[SLALOMING THROUGH SPACE]

### Space: The Final Frontier

Although we usually think of space as mere void, both it and time have an invisible structure that guides how we move—much as the moguls (bumps) on a slope guide a skier. We perceive this structure as the force of gravity. Explaining the detailed shape of spacetime is the main goal of a theory of quantum gravity.

The tiny building blocks themselves have no direct physical meaning. If one could examine real spacetime with an ultrapowerful microscope, one would not see small triangles. They are merely approximations. The only physically relevant information comes from the collective behavior of the building blocks imagining that each one is shrunk down to zero size. In this limit, nothing depends on whether the blocks were triangular, cubic, pentagonal or any mixture thereof to start with.

The insensitivity to a variety of small-scale details also goes under the name of "universality." It is a well-known phenomenon in statistical mechanics, the study of molecular motion in gases and fluids; these substances behave much the same whatever their detailed composition is. Universality is associated with properties of systems of many interacting parts and shows up on a scale much larger than that of the individual constituents. The analogous statement for a flock of starlings is that the color, size, wingspan and age of individual birds are completely

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irrelevant in determining the flying behavior of the flock as a whole. Only a few microscopic details filter through to macroscopic scales.

## Shriveling Up

With these computer simulations, quantum gravity theorists began to explore the effects of superposing spacetime shapes that classical relativity cannot handle—specifically, ones that are highly curved on very small distance scales. This so-called nonperturbative regime is precisely what physicists are most interested in but is largely inaccessible with the usual pen-and-paper calculations.

Unfortunately, these simulations revealed that Euclidean quantum gravity is clearly missing an important ingredient somewhere along the line. They found that nonperturbative superpositions of four-dimensional universes are inherently unstable. The quantum fluctuations of curvature on short scales, which characterize the different superposed universes contributing to the average, do not cancel one another out to produce a smooth, classical universe on large scales. Instead they typically reinforce one another to make the entire space crumple up into a tiny ball with an infinite number of dimensions. In such a space, arbitrary pairs of points are never more than a tiny distance apart, even if the space has an enormous volume. In some instances, space goes to the other extreme and becomes maximally thin and extended, like a chemical polymer with many branches. Neither of these possibilities remotely resembles our own universe.

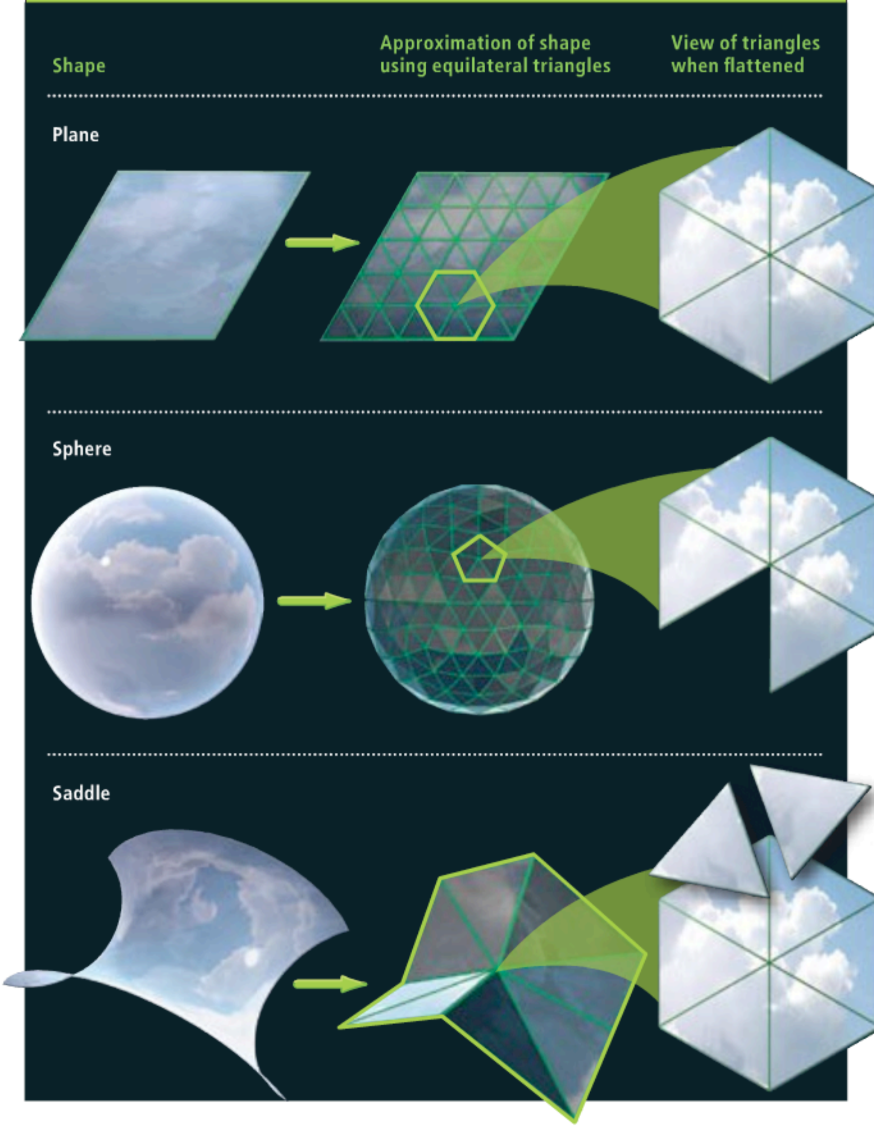
Before we reexamine the assumptions that led physicists down this dead-end street, let us pause to consider an odd aspect of this result. The building blocks are four-dimensional, yet they collectively give rise to a space having an infinite number of dimensions (the crumpled universe) or two dimensions (the polymer universe). Once the genie is let out of the bottle by allowing large quantum fluctuations of empty space, even a very basic notion such as dimension becomes changeable. This outcome could not possibly have been anticipated from the classical theory of gravity, in which the number of dimensions is always taken as a given.

One implication may come as a bit of a disappointment to science-fiction aficionados. Science-fiction stories commonly make use of wormholes—thin handles attached to the universe that provide a shortcut between regions that would otherwise be far apart. What makes wormholes so exciting is their promise of time travel and faster-than-light transmission of signals. Although such phenomena have never been observed, physicists have speculated that wormholes might find a justification within the still unknown theory of quantum gravity. In view of the negative results from the computer simulations of Euclidean quantum gravity, the viability of wormholes now seems exceedingly unlikely. Wormholes come in such a huge variety that they tend to dominate the superposition and destabilize it, and so the quantum universe never gets to grow beyond a

**[DESCRIBING THE SHAPE OF SPACE]**

## A Mosaic of Triangles

To determine how space sculpts itself, physicists first need a way to describe its shape. They do so using triangles and their higher-dimensional analogues, a mosaic of which can readily approximate a curved shape. The curvature at a point is reflected in the total angle subtended by the triangles that surround it. For a flat surface, the angle is exactly 360 degrees, but for curved surfaces it can be less or more.



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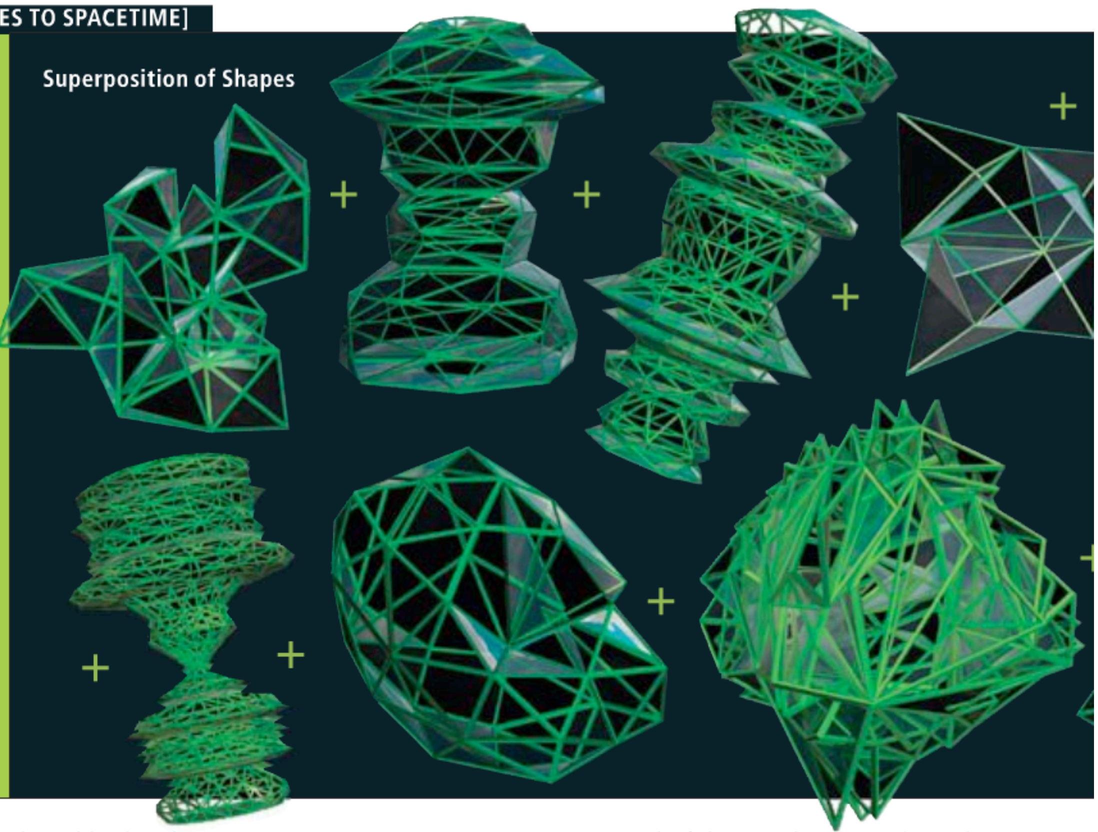
[APPLYING QUANTUM RULES TO SPACETIME]

## Taking the Average

Spacetime can take on a huge number of possible shapes. According to quantum theory, the shape we are most likely to observe is a superposition, or weighted average, of all these possibilities. When constructing shapes from triangles, theorists weight each shape depending on how exactly they glue together triangles to form it. The authors have discovered that the triangles must follow certain rules for the average to match what we observe. In particular, the triangles must have a built-in arrow of time.



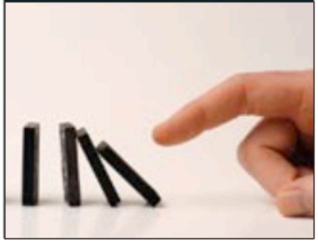
small but highly interconnected neighborhood.

What could the trouble be? In our search for loopholes and loose ends in the Euclidean approach, we finally hit on the crucial idea, the one ingredient absolutely necessary to make the stir fry come out right: the universe must encode what physicists call causality. Causality means that empty spacetime has a structure that allows us to distinguish unambiguously between cause and effect. It is an integral part of the classical theories of special and general relativity.

Euclidean quantum gravity does not build in a notion of causality. The term "Euclidean" indicates that space and time are treated equally. The universes that enter the Euclidean superposition have four spatial directions instead of the usual one of time and three of space. Because Euclidean universes have no distinct notion of time, they have no structure to put events into a specific order; people living in these universes would not have the words "cause" or "effect" in their vocabulary. Hawking and others taking this approach have said that "time is imaginary," in both a mathematical sense and a colloquial one. Their hope was that causality would emerge as a large-scale property from microscopic quantum fluctuations that individually carry no imprint of a causal structure. But the computer simulations dashed that hope.

### WHAT IS CAUSALITY?

Causality is the principle that events occur in a specific temporal sequence of cause and effect, rather than as a haphazard jumble. In the authors' approach to quantum gravity, the distinction between cause and effect is fundamental to nature, rather than a derived property.



Instead of disregarding causality when assembling individual universes and hoping for it to reappear through the collective wisdom of the superposition, we decided to incorporate the causal structure at a much earlier stage. The technical term for our method is causal dynamical triangulations. In it, we first assign each simplex an arrow of time pointing from the past to the future. Then we enforce causal gluing rules: two simplices must be glued together to keep their arrows pointing in the same direction. The simplices must share a notion of time, which unfolds steadily in the direction of these arrows and never stands still or runs backward. Space keeps its overall form as time advances; it cannot break up into disconnected pieces or create wormholes.

After we formulated this strategy in 1998, we demonstrated in highly simplified models that causal gluing rules lead to a large-scale shape different from that of Euclidean quantum gravity. That was encouraging but not yet the same as showing that these rules are enough to stabilize a full four-dimensional universe. Thus, we held our breath in 2004 when our computer was about to give us the first calculations of a large causal superposition of four-simplices. Did this spacetime really behave on large distances like a four-dimensional, extended object and not like a crumpled ball or polymer?

JEAN-FRANCOIS PODEVIN; IMAGESHOP *Corbis* (*dominoes*)

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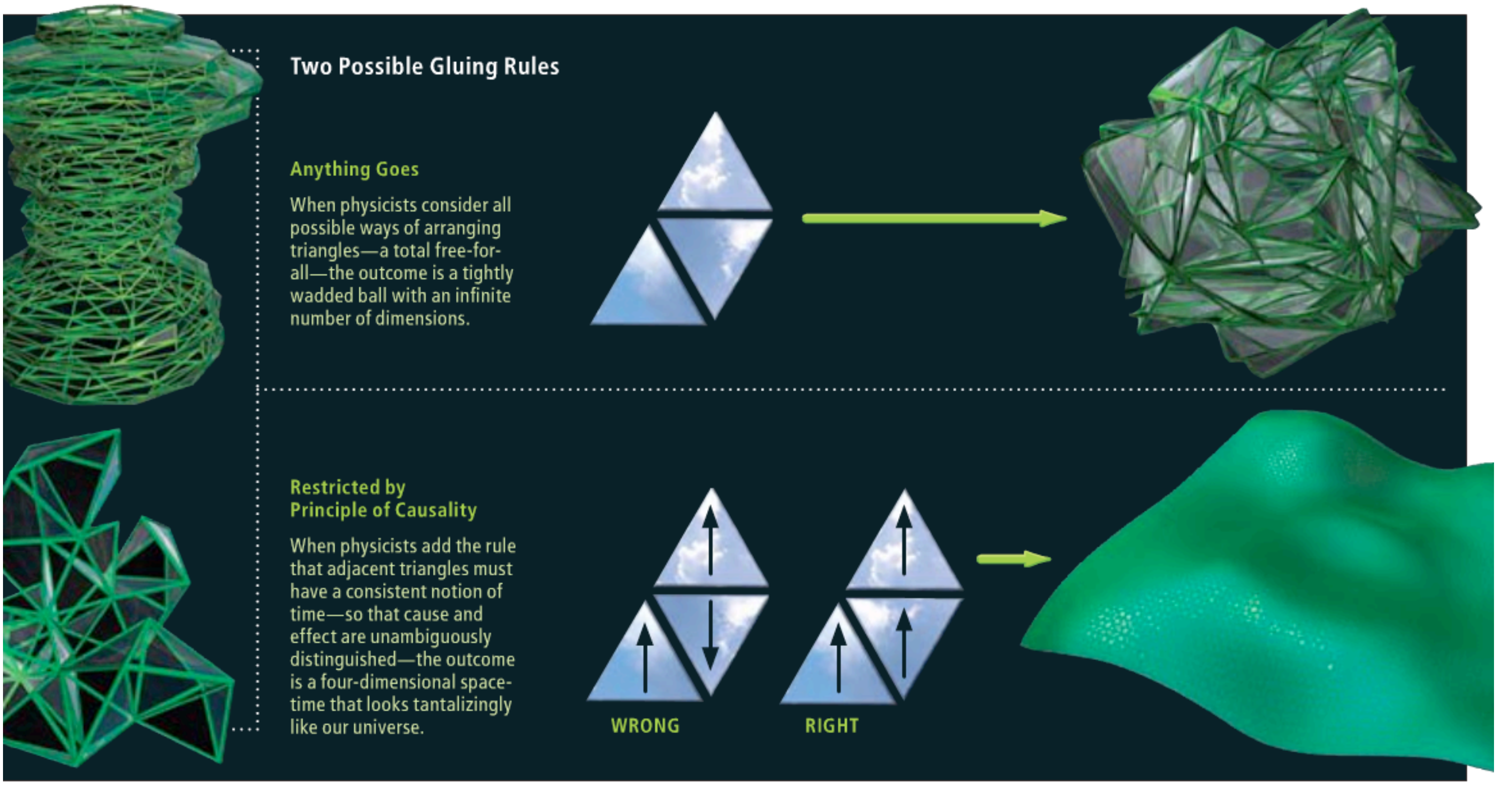
**Two Possible Gluing Rules**

**Anything Goes**

When physicists consider all possible ways of arranging triangles—a total free-for-all—the outcome is a tightly wadded ball with an infinite number of dimensions.

**Restricted by Principle of Causality**

When physicists add the rule that adjacent triangles must have a consistent notion of time—so that cause and effect are unambiguously distinguished—the outcome is a four-dimensional spacetime that looks tantalizingly like our universe.

Imagine our elation when the number of dimensions came out as four (more precisely, as 4.02 ± 0.1). It was the first time anyone had ever derived the observed number of dimensions from first principles. To this day, putting causality back into quantum-gravitational models is the only known cure for the instabilities of superposed spacetime geometries.

## Spacetime at Large

This simulation was the first in an ongoing series of computational experiments whereby we have attempted to extract the physical and geometric properties of quantum spacetime from the computer simulations. Our next step was to study the shape of spacetime over large distances and to verify that it agrees with reality—that is, with the predictions of general relativity. This test is very challenging in nonperturbative models of quantum gravity, which do not presume a particular default shape for spacetime. In fact, it is so difficult that most approaches to quantum gravity—including string theory, except for special cases—are not sufficiently advanced to accomplish it.

It turned out that for our model to work we needed to include from the outset a so-called cosmological constant, an invisible and immaterial substance that space contains even in the complete absence of other forms of matter and energy. This requirement is good news, because cosmologists have found observational evidence for such energy. What is more, the emergent spacetime has what physicists call a de Sitter geometry, which is exactly the solution to Einstein's equations for a universe that contains nothing but the cosmological constant. It is truly remarkable that by assembling microscopic building blocks in an essentially random manner—without regard to any symmetry or preferred geometric structure—we end up with a spacetime that on large scales has the highly symmetric shape of the de Sitter universe.

This dynamical emergence of a four-dimensional universe of essentially the correct physical shape from first principles is the central achievement of our approach. Whether this remarkable outcome can be understood in terms of the interactions of some yet to be identified fundamental "atoms" of spacetime is the subject of ongoing research.

Having convinced ourselves that our quantum-gravity model passed a number of classical tests, it was time to turn to another kind of experiment, one that probes the distinctively quantum structure of spacetime that Einstein's classical theory fails to capture. One of the simulations we have performed is a diffusion process—that is, we let a suitable analogue of an ink drop fall into the superposition of universes

**[THE AUTHORS]**

**Jan Ambjørn, Jerzy Jurkiewicz** and **Renate Loll** developed their approach to quantum gravity in 1998. Ambjørn is a member of the Royal Danish Academy and a professor at the Niels Bohr Institute in Copenhagen and at Utrecht University in the Netherlands. He has a reputation as an accomplished Thai cook, a claim that the editors look forward to evaluating firsthand.

Jurkiewicz is head of the department of the theory of complex systems at the Institute of Physics at the Jagiellonian University in Kraków. His many past positions include one at the Niels Bohr Institute in Copenhagen, along whose shores he was introduced to the beauty of sailing.

Loll is a professor at Utrecht University, where she heads one of the largest groups for quantum gravity research in Europe. Previously she worked at the Max Planck Institute for Gravitational Physics in Golm, Germany, where she held a Heisenberg Fellowship. In her rare spare time, Loll enjoys playing chamber music.

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[WHAT IS A DIMENSION, ANYWAY?]

# A Whole New Dimension to Space

In everyday life the number of dimensions refers to the minimum number of measurements required to specify the position of an object, such as latitude, longitude and altitude. Implicit in this definition is that space is smooth and obeys the laws of classical physics.

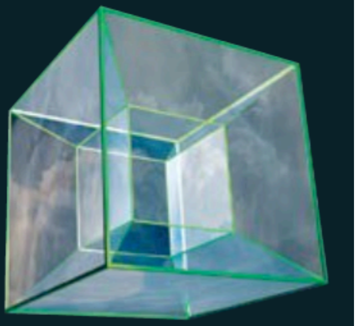
But what if space is not so well behaved? What if its shape is determined by quantum processes in which everyday notions cannot be taken for granted? For these cases, physicists and mathematicians must develop more sophisticated notions of dimensionality. The number of dimensions need not even be an integer, as in the case of fractals—patterns that look the same on all scales.

## Integer Dimensions ▼

**1 Dimension**

**2 Dimensions**

**3 Dimensions**



**4 Dimensions**

## Fractal Dimensions ▼

**Cantor Set**
Take a line, chop out the middle third and repeat ad infinitum. The resulting fractal is larger than a solitary point but smaller than a continuous line. Its Hausdorff dimension [*see below*] is **0.6309**.

**Sierpiński Gasket**
A triangle from which ever smaller subtriangles have been cut, this figure is intermediate between a one-dimensional line and a 2-D surface. Its Hausdorff dimension is **1.5850**.

**Menger Sponge**
A cube from which subcubes have been cut, this fractal is a surface that partially spans a volume. Its Hausdorff dimension is **2.7268**, similar to that of the human brain.

## Generalized Definitions Of Dimensions

### Hausdorff Dimension ▼

Formulated by the early 20th-century German mathematician Felix Hausdorff, this definition is based on how the volume, $V$, of a region depends on its linear size, $r$. For ordinary three-dimensional space, $V$ is proportional to $r^3$. The exponent gives the number of dimensions. "Volume" can also refer to other measures of total size, such as area. For the Sierpiński gasket, $V$ is proportional to $r^{1.5850}$, reflecting the fact that this figure does not even fully cover an area.

### Spectral Dimension ▼

This definition describes how things spread through a medium over time, be it an ink drop in a tank of water or a disease in a population. Each molecule of water or individual in the population has a certain number of closest neighbors, which determines the rate at which the ink or disease diffuses. In a three-dimensional medium, a cloud of ink grows in size as time to the $^{3}/_{2}$ power. In the Sierpiński gasket, ink must ooze through a twisty shape, so it spreads more slowly—as time to the 0.6826 power, corresponding to a spectral dimension of 1.3652.

## Applying the Definitions

In general, different ways to calculate the number of dimensions give different numbers, because they probe different aspects of the geometry. For some geometric figures, the number of dimensions is not fixed. For instance, diffusion may be a more complicated function than time to a certain power.

Quantum-gravity simulations focus on the spectral dimension. They imagine dropping a tiny being into one building block in the quantum spacetime. From there the being walks around at random. The total number of spacetime building blocks it touches over a given period reveals the spectral dimension.

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and watch how it spreads and is tossed around by the quantum fluctuations. Measuring the size of the ink cloud after a certain time allows us to determine the number of dimensions in space [*see box on opposite page*].

The outcome is pretty mind-boggling: the number of dimensions depends on the scale. In other words, if we let the diffusion go on for just a short while, spacetime appears to have a different number of dimensions than when we let it run for a long time. Even those of us who specialize in quantum gravity can scarcely imagine how spacetime could smoothly change its dimension depending on the resolution of one's microscope. Evidently, a small object experiences spacetime in a profoundly different way than a large object does. To that object, the universe has something akin to a fractal structure. A fractal is a bizarre kind of space where the concept of size simply does not exist. It is self-similar, which means that it looks the same on all scales. This implies there are no rulers and no other objects of a characteristic size that can serve as a yardstick.

How small is "small"? Down to a size of about $10^{-34}$ meter, the quantum universe at large is well described by the classical, four-dimensional de Sitter geometry, although quantum fluctuations become increasingly significant. That one can trust the classical approximation to such short distances is rather astonishing. It has important implications for the universe both very early in its history and very far into its future. At both these extremes the universe is effectively empty. Early on, gravitational quantum fluctuations may have been so enormous that matter barely registered; it was a tiny raft tossed on a roiling ocean. Billions of years from now, because of the universe's rapid expansion, matter will be so diluted that it likewise will play little or no role. Our technique may explain the shape of space in both cases.

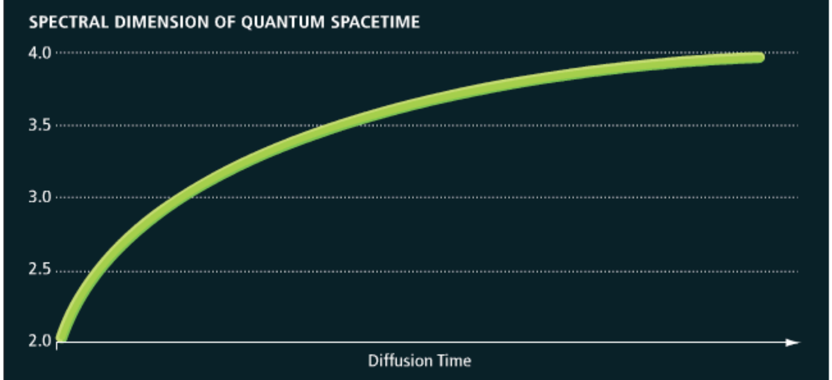
On still shorter scales, quantum fluctuations of spacetime become so strong that classical, intuitive notions of geometry break down altogether. The number of dimensions drops from the classical four to a value of about two. Nevertheless, as far as we can tell, spacetime is still continuous and does not have any wormholes. It is not as wild as a burbling spacetime foam, as the late physicist John Wheeler and many others imagined. The geometry of spacetime obeys nonstandard and nonclassical rules, but the concept of distance still applies. We are now in the process of probing even finer scales. One possibility is that the universe becomes self-similar and looks the same on all scales below a certain threshold. If so, spacetime does not consist of strings or atoms of spacetime, but a region of infinite boredom: the structure found just below the threshold will simply repeat itself on every smaller scale, ad infinitum.

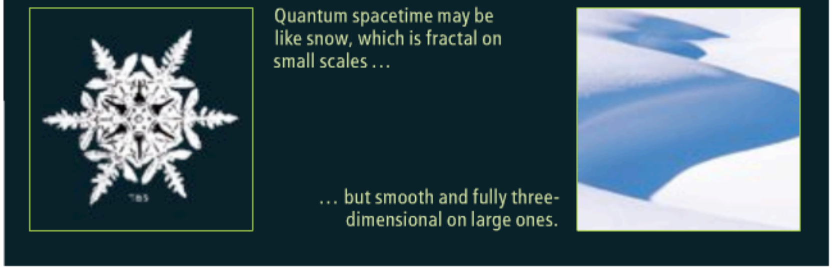
It is difficult to imagine how physicists could get away with fewer ingredients and technical tools than we have used to create a quantum universe with realistic properties. We still need to perform many tests and experiments—for example, to understand how matter behaves in the universe and how matter in turn influences the universe's overall shape. The holy grail, as with any candidate theory for quantum gravity, is the prediction of observable consequences derived from the microscopic quantum structure. That will be the ultimate criterion for deciding whether our model really is the correct theory of quantum gravity. ■

**[SIMULATION RESULTS]**

## Zooming In on Spacetime

By the authors' calculations, the spectral dimension of spacetime shades from four (on large scales) to two (on small scales), and spacetime breaks up from a smooth continuum into a gnarled fractal. Physicists are still puzzling over whether this conclusion means that spacetime ultimately consists of localized "atoms" or is built up out of intricate patterns only very loosely related to our usual concepts of geometry.





Quantum spacetime may be like snow, which is fractal on small scales …

… but smooth and fully three-dimensional on large ones.

LUCY READING-IKKANDA; NOAA/PHOTO RESEARCHERS, INC. (*snowflake*); TERRY W. EGGERS *Corbis* (*snowdrift*)

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**Deriving Dimensions.** Adrian Cho in *Physical Review Focus;* September 28, 2004. **http://focus.aps.org/story/v14/st13**

**Planckian Birth of a Quantum de Sitter Universe.** J. Ambjørn, A. Görlich, J. Jurkiewicz and R. Loll n *Physical Review Letters,* Vol. 100, Article No. 091304; March 7, 2008. Preprint at **arxiv.org/abs/0712.2485**

**The Complete Idiot's Guide to String Theory.** George Musser. Alpha, 2008.

**The Emergence of Spacetime, or, Quantum Gravity on Your Desktop.** R. Loll in *Classical and Quantum Gravity,* Vol. 25, No. 11, Article No. 114006; June 7, 2008. **arxiv.org/abs/0711.0273**

Renate Loll's Web site is **www.phys.uu.nl/~loll**

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