Recent L3 results (and questions) on BEC at LEP

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**BEC Introduction**

\[
R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} = \frac{\rho_2(Q)}{\rho_0(Q)}
\]

Assuming particles produced incoherently with spatial source density \( S(x) \),

\[
R_2(Q) = 1 + \lambda |\tilde{S}(Q)|^2
\]

where \( \tilde{S}(Q) = \int dx \, e^{iQx} S(x) \) – Fourier transform of \( S(x) \)

\( \lambda = 1 \) — \( \lambda < 1 \) if production not completely incoherent

Assuming \( S(x) \) is a Gaussian with radius \( r \)

\[
R_2(Q) = 1 + \lambda e^{-Q^2r^2}
\]
Elongation Results

Results in LCMS frame:

\[
\frac{r_L}{r_{\text{side}}} \quad \text{L3} \quad 1.25 \pm 0.03^{+0.36}_{-0.05} \\
\quad \text{OPAL} \quad 1.19 \pm 0.03^{+0.08}_{-0.01}
\]

(ZEUS finds similar results in ep)

\sim 25\% \text{ elongation along thrust axis}

OPAL:

Elongation larger for narrower jets

- Conclusion: Elongation, but it is relatively small.
- So: Ignore it. — Assume spherical.
Results on $Q$ from $L_3 Z$ decay

$$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \delta Q)$$

- **Gaussian**
  $$G = \exp(- (rQ)^2)$$

- **Edgeworth expansion**
  $$G = \exp(- (rQ)^2) \cdot [1 + \frac{\kappa}{3!} H_3(rQ)]$$
  Gaussian if $\kappa = 0$
  $\kappa = 0.71 \pm 0.06$

- **symmetric Lévy**
  $$G = \exp(- |rQ|^\alpha)$$
  $0 < \alpha \leq 2$
  $\alpha = 1.34 \pm 0.04$

Poor $\chi^2$. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of $R_2$ in the region $0.6 < Q < 1.5 \text{ GeV}$
Transverse Mass dependence of $r$

$r_L$

$r_{\text{side}}$

$r_{\text{out}}$

Smirnova&Lörstad, 7th Int. Workshop on Correlations and Fluctuations (1996)

Van Dalen, 8th Int. Workshop on Correlations and Fluctuations (1998)

$r$ decreases with $m_t$ (or $k_t$) for all directions

Conclusions

- BEC depend (approximately) only on $Q$, not its components.
- BEC depend on $m_t$.

Turn now to a model providing such dependence.
The $\tau$-model


- Assume momentum is related to avg. production point:
  \[ \bar{x}^\mu(p^\mu) = a_\tau p^\mu \]
where for 2-jet events, \( a = 1/m_t \)
  \[ \tau = \sqrt{t^2 - r_z^2} \]
  is the “longitudinal” proper time
  and \( m_t = \sqrt{E^2 - p_z^2} \) is the “transverse” mass
- Let \( \delta_\Delta(x^\mu - \bar{x}^\mu) \) be dist. of prod. points about their mean,
  and \( H(\tau) \) the dist. of \( \tau \). Then the emission function is
  \[ S(x, p) = \int_0^\infty d\tau H(\tau) \delta_\Delta(x - a_\tau p) \rho_1(p) \]
- In the plane-wave approx.
  \[ \rho_2(p_1, p_2) = \int d^4x_1 d^4x_2 S(x_1, p_1) S(x_2, p_2) \left( 1 + \cos \left( [p_1 - p_2] [x_1 - x_2] \right) \right) \]
- Assume \( \delta_\Delta(x - a_\tau p) \) is very narrow — a \( \delta \)-function. Then
  \[ R_2(p_1, p_2) = 1 + \lambda \text{Re} \tilde{H} \left( \frac{a_1 Q^2}{2} \right) \tilde{H} \left( \frac{a_2 Q^2}{2} \right), \quad \tilde{H}(\omega) = \int d\tau H(\tau) \exp(i\omega \tau) \]
BEC in the $\tau$-model

- **Assume** a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction, $H(\tau)$ is one-sided.
  Characteristic function is
  $$\tilde{H}(\omega) = \exp \left[ -\frac{1}{2} (\Delta \tau |\omega|)^{\alpha} \left( 1 - i \, \text{sign}(\omega) \tan \left( \frac{\alpha \pi}{2} \right) \right) + i \omega \tau_0 \right], \quad \alpha \neq 1$$
  where
  - $\alpha$ is the index of stability
  - $\tau_0$ is the proper time of the onset of particle production
  - $\Delta \tau$ is a measure of the width of the dist.

- **Assume** $a_1 \approx a_2 \approx \bar{a}$

- Then,
  $$R_2(Q, \bar{a}) = \gamma \left[ 1 + \lambda \cos \left( \bar{a} \tau_0 Q^2 + \tan \left( \frac{\alpha \pi}{2} \right) \left( \frac{\bar{a} \Delta \tau Q^2}{2} \right)^{\alpha} \right) \exp \left( - \left( \frac{\bar{a} \Delta \tau Q^2}{2} \right)^{\alpha} \right) \right] \cdot (1 + \delta Q)$$
BEC in the \( \tau \)-model

\[
R_2(Q, a) = \gamma \left[ 1 + \lambda \cos \left( a \tau_0 Q^2 + \tan \left( \frac{\alpha \pi}{2} \right) \left( \frac{a \Delta \tau Q^2}{2} \right)^\alpha \right) \exp \left( - \left( \frac{a \Delta \tau Q^2}{2} \right)^\alpha \right) \right] \cdot (1 + \delta Q)
\]

Simplification:

- Particle production begins immediately, \( \tau_0 = 0 \)
- Effective radius, \( R = \sqrt{\frac{a \Delta \tau}{2}} \)

\[
R_2(Q) = \gamma \left[ 1 + \lambda \cos \left( (R_a Q)^{2\alpha} \right) \exp \left( - (R Q)^{2\alpha} \right) \right] (1 + \delta Q)
\]

where \( R_a^{2\alpha} = \tan \left( \frac{\alpha \pi}{2} \right) R^{2\alpha} \)

Compare to sym. Lévy parametrization:

\[
R_2(Q) = \gamma \left[ 1 + \lambda \exp \left( - |r Q|^{\alpha} \right) \right] (1 + \delta Q)
\]
2-jet Results on Simplified $\tau$-model from $L_3 Z$ decay

2-jet events $\text{Durham } y_{\text{cut}} = 0.006$

- symmetric Lévy does not describe dip or large $Q$
- $R_{a}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ good description
- $R_a$ free good description
- Effective $R$ works well

Simplified $\tau$-model works better than sym. dists.

\[\chi^2/\text{dof} = 148/95\]

\[\chi^2/\text{dof} = 97/95\]

\[\chi^2/\text{dof} = 97/94\]
3-jet Results on Simplified $\tau$-model from $L3 Z$ decay

3-jet events Durham $y_{\text{cut}} = 0.006$

- $R^2_{a} = \tan\left(\frac{\alpha \pi}{2}\right) R^2_{\alpha}$
  - poor description
- $R_a$ free
  - good description
- $R^2_{a} = \tan\left(\frac{\alpha \pi}{2}\right) R^2_{\alpha}$
  - pretty good description (CL=1%)
  - $\overline{a}_{\tau 0} > 0$
  - (VERY PRELIMINARY)
Summary of Simplified $\tau$-Model.

- Simplified $\tau$-model works well
  - For 2-jet events including $R_2^{\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$
  - For 3-jet events
    - only if $R_a$ a free parameter.
    - Possibly the assumption $a_1 \approx a_2 \approx \bar{a}$ is less valid.
    - or if $\tau_0 > 0$

Limit analysis to 2-jet events (Durham, $y_{cut} = 0.006$)
For 2-jet events, $a = 1/m_t$
Full $\tau$-model for 2-jet events

$$R_2(Q, \overline{m_t}) = \gamma \left[ 1 + \lambda \cos \left( \frac{\tau_0 Q^2}{m_t} + \tan \left( \frac{\alpha \pi}{2} \right) \left( \frac{\Delta\tau Q^2}{2m_t} \right)^{\alpha} \right) \exp \left( - \left( \frac{\Delta\tau Q^2}{2m_t} \right)^{\alpha} \right) \right] \cdot (1 + \delta Q)$$

- Parameters $\alpha, \Delta\tau, \tau_0$ are $\sim$ independent of $m_t$
- Note: $\Delta\tau$ indep. of $m_t$ equiv. to radius $r \propto 1/\sqrt{m_t}$
Emission Function of 2-jet Events.

In the $\tau$-model, the emission function in configuration space is

$$S(x) = \frac{d^4n}{d\tau d^3r} = \left(\frac{m_t}{\tau}\right)^3 H(\tau) \rho_1 \left( p = \frac{r m_t}{\tau} \right)$$

For simplicity, assume $S(r, z, t) = G(\eta) l(r) H(\tau)$

($\eta = $ space-time rapidity)

Strongly correlated $x, p \implies$

$$\eta = y \quad r = p_t \tau / m_t$$

$$G(\eta) = N_y(\eta) \quad l(r) = \left(\frac{m_t}{\tau}\right)^3 N_{p_t}(r m_t / \tau)$$

($N_y, N_{p_t}$ are inclusive single-particle distributions)

So, using experimental $N_y, N_{p_t}$ dists. and $H(\tau)$ from BEC fits, we can reconstruct $S$. 

$\alpha = 0.43$

$\Delta \tau = 1.8$ fm

$\tau_0 = 0$
Emission Function of 2-jet Events.

Integrating over $r$, Integrating over $z$,

$\tau = 0.05 \text{ fm}$  
$\tau = 0.10 \text{ fm}$  
$\tau = 0.15 \text{ fm}$  

$\tau = 0.20 \text{ fm}$  
$\tau = 0.25 \text{ fm}$  
$\tau = 0.30 \text{ fm}$  

$\tau = 0.50 \text{ fm}$  
$\tau = 1.00 \text{ fm}$  

"Boomerang" shape  
Expanding ring

Particle production is close to the light-cone
Inter-string BEC?

Recall $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$

- 2 strings, inter-string BEC?
- large syst. uncertainty on $M_W$
- No inter-string BEC found. BUT
  - low statistics
  - small overlap in $\vec{p}$
    exasperated by expt. sel.
    of 4 well-separated jets

Also 2 strings in $e^+e^- \rightarrow Z \rightarrow q\bar{q}g$

- high statistics
- larger overlap

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Signal of Inter-string BEC

Assuming $\lambda_{\text{inter-string}} = \lambda_{\text{intra-string}}$,

- momentum overlap, $f_{\text{str-1}}(\vec{p}) = f_{\text{str-2}}(\vec{p})$
- If no $p$ overlap, can not detect inter-string BEC
  
  no $p$ overlap $\implies \lambda_2 = \lambda_1$ and $r_2 = r_1$
- spatial overlap, $f_{\text{str-1}}(\vec{r}) = f_{\text{str-2}}(\vec{r})$
- Assuming full $p$ overlap and $\lambda_{\text{inter-string}} = \lambda_{\text{intra-string}}$,

<table>
<thead>
<tr>
<th></th>
<th>no spatial overlap</th>
<th>full spatial overlap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter-string BEC</td>
<td>$\lambda_2 = \lambda_1$ $r_2 &gt; r_1$</td>
<td>$\lambda_2 = \lambda_1$ $r_2 = r_1$ (HBT) $r_2 &gt; r_1$ (Lund)</td>
</tr>
<tr>
<td>No inter-str. BEC</td>
<td>$\lambda_2 &lt; \lambda_1$ $r_2 = r_1$</td>
<td>$\lambda_2 &lt; \lambda_1$ $r_2 = r_1$</td>
</tr>
</tbody>
</table>
2-jet / 3-jet
fit with Edgeworth expansion parametrization:

\[ R_2(Q) = \gamma (1 + \delta Q + \varepsilon Q^2) \left[ 1 + \lambda e^{-Q^2} \left( 1 + \frac{\kappa}{3!} H_3(rQ) \right) \right] \]
The $τ$-model Emission Function

Inter-string BEC

Summary

$q/g$

fit with Edgeworth expansion parametrization:

$$R_2(Q) = γ \left[ 1 + λe^{-r^2Q^2} \left( 1 + \frac{κ}{3!}H_3(rQ) \right) \right]$$

<table>
<thead>
<tr>
<th>Sample</th>
<th>g fract. (%)</th>
<th>$⟨E⟩$ (GeV)</th>
<th>$λ$</th>
<th>$r$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dusc 2-jet</td>
<td>0</td>
<td>43</td>
<td>0.93±0.05</td>
<td>0.75±0.03</td>
</tr>
<tr>
<td>b-tag Yg g</td>
<td>32</td>
<td>40</td>
<td>0.96±0.24</td>
<td>0.56±0.10</td>
</tr>
<tr>
<td>dusc Merc.</td>
<td>33</td>
<td>29</td>
<td>1.25±0.11</td>
<td>0.78±0.05</td>
</tr>
<tr>
<td>dusc Yq</td>
<td>50</td>
<td>24</td>
<td>1.13±0.09</td>
<td>0.78±0.04</td>
</tr>
<tr>
<td>b-tag Merc. g</td>
<td>74</td>
<td>25</td>
<td>1.31±0.50</td>
<td>0.96±0.21</td>
</tr>
<tr>
<td>b-tag Yq g</td>
<td>75</td>
<td>19</td>
<td>0.83±0.18</td>
<td>0.76±0.09</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td></td>
<td>1.01±0.04</td>
<td>0.76±0.02</td>
</tr>
</tbody>
</table>

CL = 6%  CL = 37%

No evidence for q/g differences – Inter-String BEC (HBT) ?
q/g: $x$, $y$ dependence

expect more spatial overlap in tip of g jet

- $\lambda \downarrow$, not const. with increasing $x$ or $y$, but equally for $q$, $g$.
- $\lambda_g$ of $E$-tag, b-tag inconsistent – systematics?
- $r$ const., not decreasing with $x$ and $y$ – no inter-str. BEC?
Summary 2

- 2-jet/3-jet: inconclusive.
- No evidence for different BEC in $q$, $g$.
  - No inter-string BEC?
  - or Inter-string BEC, but HBT rather than Lund?
  - or no sensitivity? - difficult to assess without a model

Inter-string BEC remains an open question.

Summary 1

- $R_2(Q)$, not $R_2(\bar{Q})$ is a reasonably good approximation
- But sym. Gaussian, Edgeworth, Lévy $R_2(Q)$ do not fit well
- $\tau$-model with a one-sided Lévy proper-time distribution
  - Simplified, it provides a new parametrization of $R_2$:
    - Works well with eff. $R$, $R_a$ for all events;
    - with only eff. $R$ for 2-jet events.
  - $R_2(Q, m_t)$ successfully fits $R_2$ for 2-jet events
    - both $Q$- and $m_t$-dependence described correctly
    - Note: we found $\Delta \tau$ to be independent of $m_t$
      $\Delta \tau$ enters $R_2$ as $\Delta \tau Q^2/m_t$
      In Gaussian parametrization, $r$ enters $R_2$ as $r^2 Q^2$
      Thus $\Delta \tau$ independent of $m_t$ corresponds to $r \propto 1/\sqrt{m_t}$

- Emission function shaped like a boomerang in $z$-$t$
  and an expanding ring in $x$-$y$
  Particle production is close to the light-cone

L3 Data

- $e^+e^- \rightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about $10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\text{cut}} = 0.006$
- use mixed events for reference sample in $\tau$-model studies
- use mixed events or MC for reference sample in inter-string BEC studies