lecture notes BSM — Paul de Jong

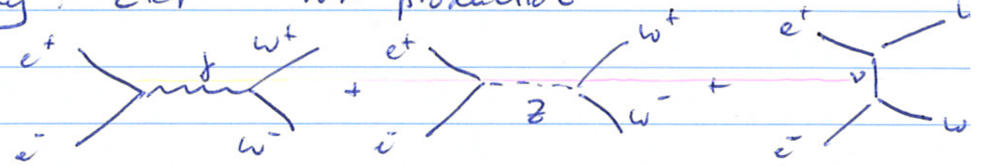
Yang-Mills interactions between three or four gauge bosons

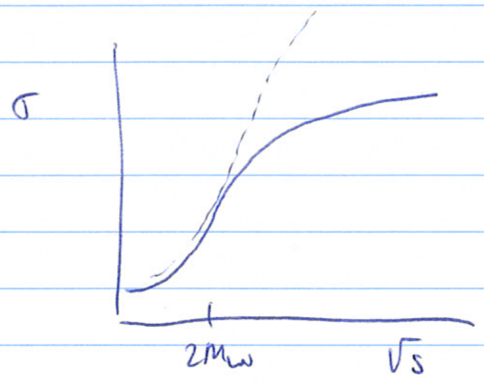
$$+ g\,(\partial_\mu W_\nu^a)\, W^{b,\mu} W^{c,\nu} f^{abc} \qquad \text{TGC's}$$

$$- \frac{g^2}{4} f^{abc} f^{cde} W_\mu^a W_\nu^b W^{c,\mu} W^{d,\nu} \qquad [T^a, T^b] = i f^{abc}$$

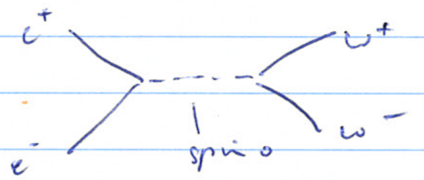
Experimental study: LEP WW production

$e^+e^- \to WW$





However: still a component missing

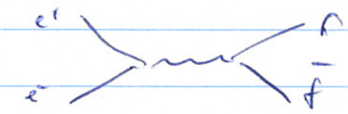


Deviations? Different gauge structure, or new particle

Strong interactions

Colour as a quantum number Δ^{++} spin 3/2

Number of colours $e^+e^- \to f\bar{f}$



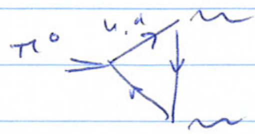
$$\sigma \sim \frac{\cancel{\pi\alpha^2} Q_f^2}{3E^2} \sqrt{1 - \frac{m_f^2}{E^2}}\; \Theta(E - m_f)$$

threshold

$$E \gg m_f: \quad = \frac{\pi\alpha^2}{3E^2} Q_f^2$$

$$R = \frac{\sigma(\text{hadrons})}{\sigma(\mu^+\mu^-)} \simeq N_c \sum_{\substack{\text{active}\\ \text{flavours}}} Q_q^2$$

Also $\pi^0 \to 2\gamma$

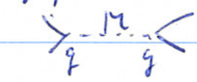


$\Gamma \sim N_c^2$

Weak interactions

- Strange particles K, Λ long lifetimes
  weak decay vs strong decay
  $\pi^\pm, \mu^\pm$ decay $\qquad\qquad \rho \to \pi^+\pi^-$

- Fermi approach: 

low-energy approximation of 

$G_F \approx 10^{-5}\ \text{GeV}^{-2}$

$$\frac{ig^2}{q^2 - M^2}$$

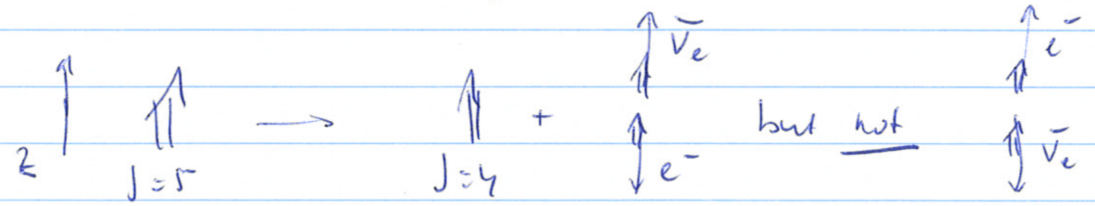
low energies: $q^2 \ll M^2$ $\quad$ G_F equivalent to $\frac{ig^2}{M^2}$

to be precise: $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$

but at high energy $q^2 \gg M^2$ $\quad \sim \frac{ig^2}{q^2}$ non-divergent

- Charged weak current is maximally P-violating

$^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$



polarized nucleus, asymmetry follows nuclear polarization

$\psi_L = \frac{1}{2}(1-\gamma^5)\psi \qquad \psi_R = \frac{1}{2}(1+\gamma^5)\psi$

$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$

$\psi^\dagger \gamma^0 \psi \equiv \bar{\psi}\psi$ $\quad$ Lorentz invariant

$\psi^\dagger$ adjoint spinor = row vector of complex conjugates

$\bar{\psi} = \psi^\dagger \gamma^0 \qquad \psi^\dagger = \bar{\psi}\gamma^0 \qquad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

We have seen: fundamental difference in behaviour of Ψ_L and Ψ_R in charged weak interactions.

ν_L and $\bar{\nu}_R$ exist, ν_R and $\bar{\nu}_L$ do not.
(OR: the W does not interact with ~~them~~)

Left handed particles form doublets $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$, $\begin{pmatrix} u \\ d \end{pmatrix}_L$
right handed: singlets e_R, u_R, d_R

Weak isospin: $\begin{pmatrix} \nu \\ e \end{pmatrix} \begin{matrix} I_3 = +1/2 \\ I_3 = -1/2 \end{matrix}$ singlet: $I_3 = 0$

The gauge group of the SM is $SU(2)_L \times U(1)_Y$ ↪ hyperch

weak isospin for L-handed particles

Gell-Nishima relation: $Q = I_3 + \frac{Y}{2}$

$$Y_{\nu_L} = -1 \qquad Y_{\nu_R} = 0$$

$$Y_{e_L} = -1 \qquad Y_{e_R} = -2$$

Lagrangian for a free fermion: $\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$

$e_L = \frac{1}{2}(1-\gamma^5)e \qquad e_R = \frac{1}{2}(1+\gamma^5)e$ <u>projection operato</u>

$$m\bar{e}e = m\bar{e}\left(\frac{1}{2}(1-\gamma^5) + \frac{1}{2}(1+\gamma^5)\right)e$$

$$= m(\bar{e}_R e_L + \bar{e}_L e_R)$$

which is <u>not</u> invariant under $SU(2)_L$

Physical gauge bosons of SM:

triplet $W_\mu^{1,2,3}$ $\big(SU(2)\big)$ singlet B (Y)

mix into: $$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \pm W_\mu^2\right)$$

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

$$\cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g_B^2}} = \frac{g_W}{g_Z}$$

g_W: SU(2) coupling cons

g_B: U(1) coupling const

$$g_W \sin\theta_W = e \qquad g_Z = \frac{g_W}{\cos\theta_W} = \frac{e}{\sin\theta_W \cos\theta_W}$$

Higgs: add a complex scalar field $\begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} = \begin{pmatrix}(\phi_1 + i\phi_2)/\sqrt{2} \\ (\phi_3 + i\phi_4)/\sqrt{2}\end{pmatrix}$

a SU(2) doublet ! With $Y = 1$

$$\mathcal{L}_{EW} = \bar{\Psi} i\gamma^\mu D_\mu \Psi - \frac{1}{4} F_{\mu\nu}\cdot F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ (D_\mu\phi)^\dagger (D_\mu \phi) - V(\phi) - \underbrace{y_e\left[\bar{e}_R(\phi^\dagger \Psi_L) + (\bar{\Psi}_L \phi) e_R\right]}_{\text{for electrons, for exa}}$$

$$D_\mu = \partial_\mu + i g_W W_\mu \cdot \tau + i\frac{g_B}{2} Y$$

This is gauge invariant, because $\bar{\Psi}_L \phi e_R$ and $\bar{e}_R \phi^\dagger \Psi_L$ are !

Introducing a very special form of $V(\phi)$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4 \qquad \mu^2 < 0 \quad \lambda > 0$$

will not spoil gauge invariance.

$V(\phi)$ has a minimum at $\phi^0 = \phi_3/\sqrt{2} = v$

we can choose $\phi_1 = \phi_2 = \phi_4 = 0$

The minimum is at v, with $v^2 = \frac{-\mu^2}{\lambda}$

h is the Higgs field around the minimum

$$D_\mu \Phi \to \left[\partial_\mu + i g_W W_\mu \cdot T + \frac{i g_B}{2} B_\mu\right]\begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L}_h \to \frac{1}{2}\left(\partial_\mu h \partial^\mu h - m_h^2 h^2\right) - \lambda v h^3 + \frac{\lambda}{4} h^4$$
$$+ 2\sqrt{\sqrt{2} G_F}\, h \left(m_W^2 W_\mu^+ W_\mu^- + \frac{m_Z^2}{2} Z_\mu Z^\mu\right)$$
$$+ \sqrt{2} G_F h^2 \left(m_W^2 W_\mu^+ W_\mu^- + \frac{m_Z^2}{2} Z_\mu Z^\mu\right)$$
$$+ \sum_f \left(m_f + \sqrt{2} G_F m_f h\right) f\bar{f}$$

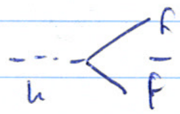
$$g_W = \frac{e}{\sin\theta_W} = 2\sqrt{\sqrt{2} G_F}\, m_W \qquad m_W = \frac{1}{2} v g_W$$

$$g_Z = \frac{e}{\sin\theta_W \cos\theta_W} = 2\sqrt{\sqrt{2} G_F}\, m_Z \qquad m_Z = \frac{1}{2} v \sqrt{g_W^2 + g_B^2}$$

$$\frac{1}{v} = \frac{g_W}{2 m_W} = \sqrt{\sqrt{2} G_F} = \frac{1}{246\ \text{GeV}}$$

$$m_h^2 = 2\lambda v^2 \qquad m_f = \frac{y_f v}{\sqrt{2}}$$

and interaction terms



$$\frac{-i y_f}{\sqrt{2}} = \frac{-i m_f}{v} = -i \frac{1}{2} \frac{g_W}{m_W} m_f$$

$$2i \frac{m_W^2}{v} g^{\mu\nu} = i g_W m_W g^{\mu\nu}$$ (idem for Z)

$$-i\lambda v = -i \frac{1}{4} \frac{g_W}{m_W} m_h^2$$

+ quartic couplings WWhh ZZhh hhhh

A priori $m_h = \sqrt{2\lambda v^2}$ is unknown, since λ is unknown.

From the LHC we know now: $m_h = 125$ GeV $\rightarrow$ $\lambda \approx 0.13$

Also, at lowest order: $\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$ (Veltman

However, the ρ parameter has loop-corrections

$$\rho = \rho_0 (1 + \Delta\rho) \qquad \Delta\rho = \delta\rho_{top} + \delta\rho_{Higgs} + \dots$$

$$\delta\rho_{top} = \frac{3 G_F m_t^2}{8\sqrt{2}\pi^2} \simeq 0.0096 \left(\frac{m_t}{173 \text{ GeV}} \right)^2$$

$$\delta\rho_{Higgs} = \frac{-3 G_F m_Z^2 \sin^2\theta_W}{8\sqrt{2}\pi^2} \left(\ln \frac{m_h^2}{m_W^2} - \frac{5}{6} \right)^2$$

Quadratic dependence on m_t, but logarithmically on m_h!

Equations above give mass to down-type fermions. (down-type quarks, charged leptons).

For up-type quarks, we need a different Higgs doublet $\begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$ Y:

In the minimal SM we can be economical:

$$\phi_c = -i\tau_2 \phi^* = \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix} \xrightarrow{SB} \frac{1}{\sqrt{2}} \begin{pmatrix} v + h \\ 0 \end{pmatrix}$$

However, more generally this could be a different doublet!

e.g. down-type quarks vev $= v_1$

up-type quarks vev $= v_2$

$v_1^2 + v_2^2 = v^2$, and $\frac{v_1}{v_2}$ is a free parameter (known as $\tan\beta$)

Higgs production:

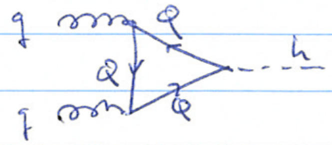
at e^+e^- colliders: $e^+e^- \to Z^* \to ZH$ $\quad \sqrt{s} > m_Z$

very clean, can study H in detail

also $e^+e^- \to e^+e^- V^* V^* \to e^+e^- H$ $\quad$ VBF

at LHC:

gluon fusion



$$\sigma \sim \frac{\pi^2}{m_h^3} \Gamma(h \to gg)\, \tau \frac{dL}{d\tau} \qquad \tau = \frac{m_h^2}{s}$$

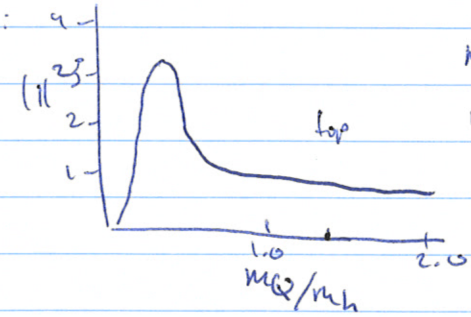
$$\frac{dL}{d\tau} = \int dx_1\, dx_2\, \delta(x_1 x_2 - \tau)\, g(x_1) g(x_2)$$

gluon distribution in proton

$$\Gamma(h \to gg) = \frac{\sqrt{2}\, G_F \alpha_s^2}{8\pi^3} \frac{m_h^3}{9} |I|^2$$

I is the loop integral of fermions in the diagram

it has the following shape:



$m_b/m_h \sim 0.03$

$m_t/m_h \sim 1.4$

for small m_Q

$|I|^2 \sim \left(\frac{m_Q}{m_h}\right)^4$

Big consequence: heavier generations Q, $m_Q > m_t$

would contribute similarly to $|I|^2$

$\Rightarrow \sigma_{Higgs}$ sets limits on # generations quarks

Beware: only chiral families

not in vector-like quarks

Other production: vector boson fusion
Higgs-strahlung
ttH

ggF and ttH dominated by fermionic couplings
VH and VBF dominated by bosonic couplings

Decay modes

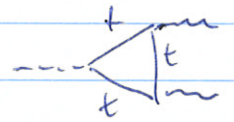
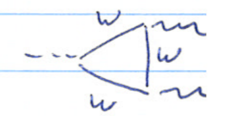
$$\Gamma(h \to f\bar{f}) = \frac{N_c G_F m_f^2 m_h}{4\sqrt{2}\pi} \left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2}$$

For $m_h < 2M_W$, $h \to b\bar{b}$ dominates

$$\Gamma(h \to WW) = \frac{G_F m_h^3}{8\sqrt{2}\pi}\left(1 - 4\gamma_W^2 + 12\gamma_W^4\right)\left(1 - 4\gamma_W^2\right)^{1/2} \quad \gamma_W = \frac{m}{m}$$

dominated by $W_L W_L$ for $m_h \gg m_W$

$$\Gamma(h \to ZZ) = \frac{G_F m_h^3}{16\sqrt{2}\pi}\left(1 - 4\gamma_Z^2 + 12\gamma_Z^4\right)\left(1 - 4\gamma_Z^2\right)^{1/2} \quad \gamma_Z = \frac{m}{m}$$

$H \to \gamma\gamma$ via loops: top and W  

Weak interactions define a scale $m_W \sim m_Z \sim m_h \sim v$

energy scale also implies length scale, temperature, time

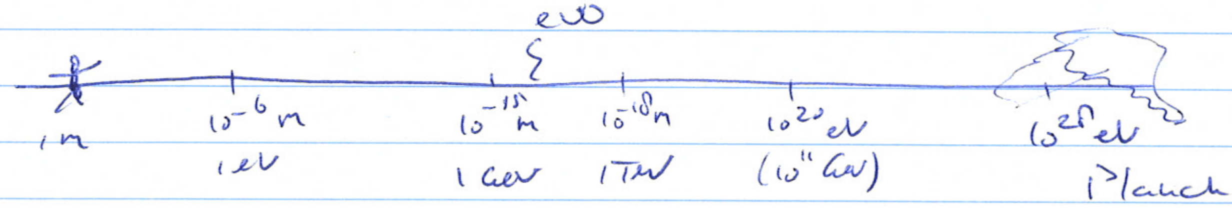
photons: $E = h\nu = \frac{hc}{\lambda}$ $\qquad$ $1\,\text{eV} \sim 1.2\,\mu\text{m}$

$1\,\text{GeV} \sim 10^{-15}\,\text{m}$

particles: $\lambda = h/p$ $\quad E^2 = p^2c^2 - m^2c^4$

for $p \gg m$ $\quad E = pc$ $\quad \lambda = \frac{hc}{E}$

temperature $E \sim kT$ (Planck's law) $\qquad k = 8.6 \cdot 10^{-5}\,\text{eV}/\text{K}$



highest energy cosmic rays

LHC: $\sim 10\,\text{TeV} \sim 10^{-19}\,\text{m}$

Schwarzschild radius of a black hole $r_s \sim \frac{2Gm}{c^2}$ $\qquad$ G: newton

Compton wavelength $\lambda = \frac{hc}{E} = \frac{hc}{mc^2} = \frac{h}{mc}$

Planck scale: $\frac{\text{Compton } \lambda}{\pi} \sim r_s$ $\qquad \frac{h}{\pi mc} \sim \frac{2Gm}{c^2}$

$$m = \sqrt{\frac{\hbar c}{G}} \sim 1.2 \cdot 10^{19}\,\text{GeV}$$

Planck length $h/M_{pl}c \sim 10^{-35}\,\text{m}$ $\qquad$ Planck time $\sim 10^{-44}\,\text{s}$

for $E \ll M_{pl}$ $\quad \lambda \gg r_s$ $\quad$ quantum nature dominates

$E > M_{pl}$ $\quad r_s > \lambda$ $\quad$ particle is effectively a black hole

Can we expect our theories to still hold beyond M_{pl}?

EW scale $\sim 10^{15}\,\text{K} \sim 10^{-10}\,\text{s}$ after big bang

GUT scale $\sim 10^{15}\,\text{GeV} \sim 10^{28}\,\text{K} \sim 10^{-37}\,\text{s}$

QCD scale $\sim 100\,\text{MeV} \sim 10^{12}\,\text{K} \sim 10^{-5}\,\text{s}$

Searches for extra generations of fermions

Normal extra quarks: ruled out by $\sigma(gg \to h)$
Before Higgs discovery:
searches $b' \to Wt$
$t' \to Wb$ limits ~ 700 GeV
(ie mass $\lesssim$ 700 GeV excluded

But: Higgs does not exclude vector-like quarks
= quarks where L and R transform the same
A $m\bar{\Psi}\Psi$ term in the Lagrangian is not forbidden

Searches: pair production cross section = SM quarks
(production = QCD : mass, colour, α_s)
Limits do depend on decay mode Wt, Zb, bh
~ 600 GeV Wb, Zt, ht

Neutrino's: limits from Z production and decay

$$\sigma_{Z \to f\bar{f}} = \sigma_{f\bar{f}}^{peak} \frac{s\Gamma_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

Breit-Wigner with width Γ_Z

$$\sigma_{f\bar{f}}^{peak} = \frac{1}{R_{QED}} \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

$\Gamma_{e\bar{e}}$: partial width of $Z \to e\bar{e}$ $\quad \left(Br(Z \to e\bar{e}) = \frac{\Gamma_{ee}}{\Gamma_Z} \right)$
$\Gamma_{f\bar{f}}$ = " " " $Z \to f\bar{f}$

$$\Gamma_{f\bar{f}} = \frac{N_c^f G_F M_Z^3}{6\sqrt{2}\pi}\left(|g_{Af}|^2 + |g_{Vf}|^2\right) + \text{radiative corrections}$$

g_{Vf} and g_{Af} are vector and axial-vector couplings of Z to fermions:

$$-\frac{1}{2}\frac{g_W}{\cos\theta_W}\bar{\Psi}_f \gamma^\mu (g_{Vf} - g_{Af}\gamma^5)\Psi_f Z_\mu$$

$$g_{Vf} = I_3^f - 2Q_f \sin^2\theta_W$$
$$g_{Af} = I_3^f$$

e.g for neutrino's ; $I_3^f = \frac{1}{2}$, $Q_f = 0 \Rightarrow g_V = g_A = \frac{1}{2}$

$\Gamma(Z \to \nu\bar{\nu}) \approx 167$ MeV for 1 generation

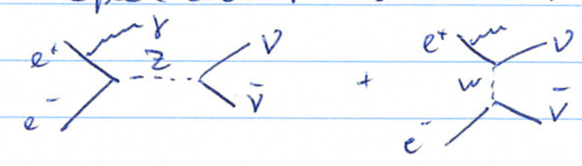
$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \underbrace{\sum_{q=u,d,c,s,b}\Gamma_{qq}}_{\text{do not forget } N_c} + \underbrace{\Gamma_{\text{invisible}}}_{N_\nu \Gamma_{\nu\nu}}$$

Γ_Z follows from $\sigma_{Z\to f\bar{f}}$: 2.4952 ± 0.0023 GeV

$\Gamma_{\text{invisible}} \sim 500$ MeV (20 % of Γ_{tot})

$$\frac{\Gamma_{\text{inv}}}{\Gamma_{ll}} = N_\nu \left(\frac{\Gamma_{\nu\nu}}{\Gamma_{ll}}\right)_{SM} \qquad N_\nu = 2.9840 \pm 0.0082$$

Independent measurement : from $e^+e^- \to \nu\bar{\nu}\gamma$



$N_\nu = 3.0 \pm 0.1$

$\Rightarrow$ There are 3 generations of ν with mass $< \frac{M_Z}{2}$

Neutrino's heavier than $\frac{M_Z}{2}$: at odds with cosmology

Or: neutrino's not coupling to Z (Sterile)

$\Gamma_{invisible} - 3\Gamma_{\nu\nu} = -2.7 \pm 1.3$ MeV

$\Rightarrow$ There is no room for further invisible Z decays.

In the SM, there are no further invisible decays: ok.
In BSM: SUSY has lightest neutralino $\tilde{\chi}^0$

$\Gamma_{invisible}$ implies: no room for $Z \rightarrow \tilde{\chi}^0 \tilde{\chi}^0$

$\Rightarrow$ either: $\tilde{\chi}^0$ does not exist

or $m_{\tilde{\chi}^0} > \frac{m_Z}{2}$

or $\tilde{\chi}^0$ does not couple to Z

4^{th} generation charged lepton: $L^{\pm}$

if no 4^{th} generation ν: $L^{\pm}$ not in a SU(2) doublet

Disregarding that: people still search for $L^{\pm}$

$L^{\pm} \rightarrow \nu_L W^{\pm}$ weak decay $\Gamma \sim \frac{G_F^2 m_L^5}{192\pi^3}$

similar to τ discovery $\tau\tau \rightarrow e\mu + X$

Limits from LEP ~ 100 GeV

If couplings are very small, $L^{\pm}$ could be long-lived
$\Rightarrow$ stable charged heavy lepton > 100 GeV (LEP

Another question one can ask:

Are quarks and leptons really fundamental particles?

point-like, without substructure

Consider the sequence e, μ, τ

Suppose e were not fundamental, but built of something "sm

Could it be that e = ground state, μ = first excited state?

(Think of hydrogen atom, $J/\psi, \psi', \psi''$ etc)

In that case, one would expect radiative decay to ground state

$\mu \to e\gamma$

experiment: $Br < 5.7 \cdot 10^{-13}$ @ 90% CL

(MEG detector @ PSI, μ^+ beam)

Nevertheless, the idea of substructure is still appealing.

Would lead to excited leptons and quarks: e^*, ν^*, q^*

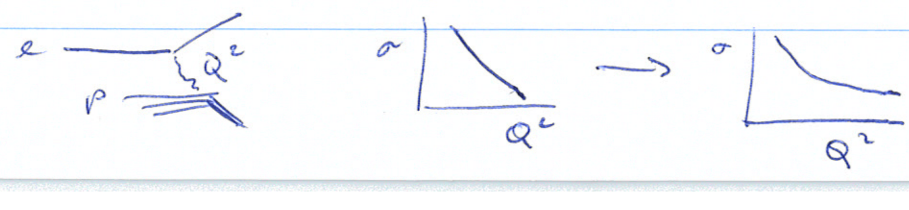
Also: at the scale of constituent binding energies, there should appear new interactions. At energies much below the compositeness scale Λ, these interactions are suppressed by inverse powers of Λ.

So: At $E \ll \Lambda$, we won't notice.

But if E approaches Λ, we will start to see deviations in cross sections: $\sigma \to \sigma_{SM}\left(1 + \mathcal{O}\left(\frac{\hat{s}}{\Lambda^2}\right)\right)$

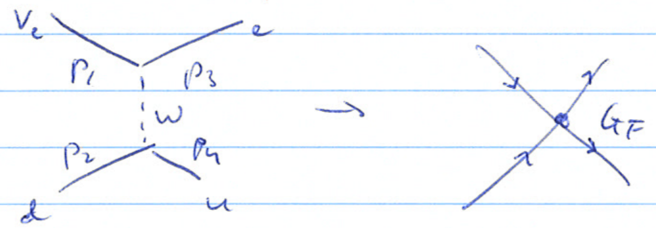
for example $qq \to qq$

(Note the analogy with Rutherford's discovery of the atomic nucleus in α scattering, or the discovery of partons in deep inelastic scattering



We can write down a Lagrangian in terms of contact interactions.

Again, in analogy with weak interactions:



$$\frac{g_W}{\sqrt{2}}\bar{\Psi}_3 \frac{1}{2}\gamma^\mu(1-\gamma^5)\Psi_1 \quad \frac{g_{\mu\nu}}{q^2-M_W^2} \quad \frac{g_W}{\sqrt{2}}\bar{\Psi}_4 \frac{1}{2}\gamma^\nu(1-\gamma^5)\Psi_2$$

$$=$$

$$\frac{G_F}{\sqrt{2}} g_{\mu\nu}\left[\bar{\Psi}_3\gamma^\mu(1-\gamma^5)\Psi_1\right]\left[\bar{\Psi}_4\gamma^\nu(1-\gamma^5)\Psi_2\right]$$

For $Q^2 \ll M_W^2$: $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$ effective description ok

$Q^2 \gtrsim M_W^2$: behaviour like $\frac{1}{q^2}$ ok.

So, for contact interactions:

$$\mathcal{L} = \frac{g_c^2}{2\Lambda^2}\Big[\eta_{LL}(\bar{\Psi}_L\gamma_\mu\Psi_L)(\bar{\Psi}_L\gamma^\mu\Psi_L)$$

$$+\eta_{RR}(\bar{\Psi}_R\gamma_\mu\Psi_R)(\bar{\Psi}_R\gamma^\mu\Psi_R)$$

$$+\eta_{LR}(\bar{\Psi}_L\gamma_\mu\Psi_L)(\bar{\Psi}_R\gamma^\mu\Psi_R)$$

$$+\eta_{RL}(\bar{\Psi}_R\gamma_\mu\Psi_R)(\bar{\Psi}_L\gamma^\mu\Psi_L)\Big]$$

by convention a vector interaction (chiral invariance)