Interpretation of solar ν results trickier: matter effects

Mikheyev-Smirnov-Wolfenstein effect

ν_e ($\bar{\nu}_e$) have interactions in matter that ν_μ, ν_τ don't have

Forward scattering amplitude of ν_e in matter interferes with unscattered amplitudes
$\Rightarrow$ effective change in mass & mixing angle

flavour basis

$$i \frac{d}{dt} \Psi_f(t) = H \Psi_f(t)$$

mass basis

$$i \frac{d}{dt} \Psi_m(t) = H \Psi_m(t)$$

$$H_m = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \qquad \Psi_f(t) = e^{-iH_f t} \qquad \Psi_m(t) = e^{-iH_m t}$$

$$\Psi_f(t) = U^{-1} e^{-iH_m t} U \qquad U: \text{PMNS matrix}$$

In matter with electron density N_e this changes:

$$H_f = U^{-1} \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} U + \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Solution (assuming 2 flavours)

$$\Delta_m m^2 \simeq \Delta m^2 \sqrt{(a - \cos 2\theta)^2 + \sin^2 2\theta}$$

$$\sin^2 2\theta_m \simeq \sin^2 2\theta \Big/ \left((a - \cos 2\theta)^2 + \sin^2 2\theta \right)$$

with $a = \dfrac{2\sqrt{2}\, E G_F N_e}{\Delta m^2}$ (inverted hierarchy: $-\Delta m^2$)

resonant effect if $a = \cos 2\theta$

Plays a major role in the sun: produced ν_e come out as mixture ν_e, ν_μ, ν_τ at surface.

In this solution: $\Delta m^2_{solar} = 7 \cdot 10^{-5}\ eV^2 \qquad \theta_{solar} \sim 32°$

Matter effects also play a role in long-baseline oscillation in the earth! (say $L > 300$ km or so)

Extending to 3 flavours:

$$|\nu_\alpha\rangle = U_{PMNS}\,|\nu_i\rangle$$

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} e^{\frac{i\alpha_1}{2}} & 0 & 0 \\ 0 & e^{\frac{i\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv UV$$

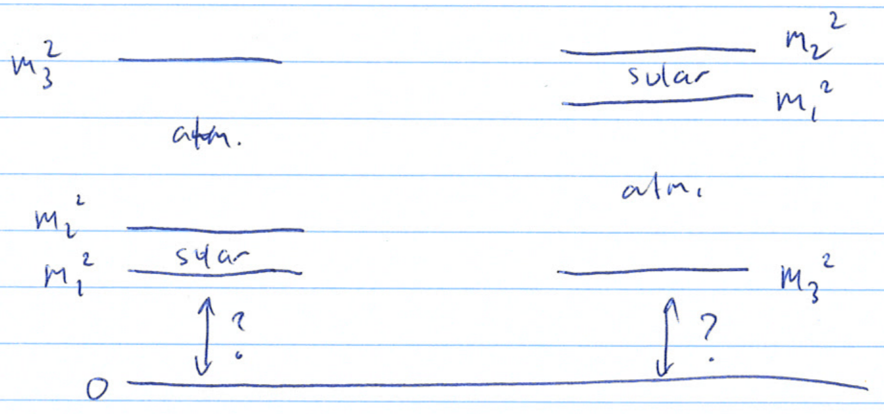
$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

α: Majorana phases (can be transformed away for Dirac neutrino's)

δ: CP-violation phase

3 angles $\theta_{12}, \theta_{13}, \theta_{23}$, 3 mass differences



normal hierarchy

inverted hierarchy

solar: $\Delta m_{12}^2 = 7 \cdot 10^{-5}$ eV

atm: $\Delta m \sim 2 \cdot 10^{}$

$\Rightarrow \Delta m_{13}^2 \approx \Delta m$

CPT invariance: $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = P(\nu_\beta \to \nu_\alpha)$

In general, however: $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$

CP violation

CP-violation occurs if U is truely complex: $\delta \neq 0$ or π

θ_{13} finally measured around 2010

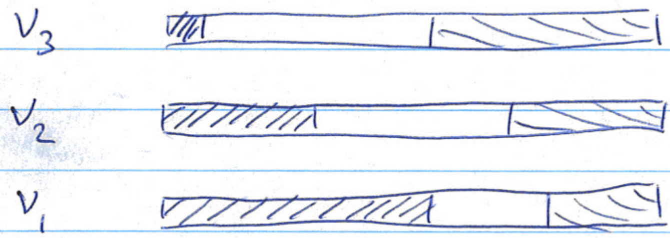
reactor ν, short baseline. Daya Bay, Reno, Chooz

$\theta_{13} \sim 8^\circ$

Fitting it all together

$\theta_{12} \sim 32^\circ$

$\theta_{23} \sim \pi/4$



$c_{12} = 0.85$

$s_{12} = 0.54$

$s_{13} = 0.14$

$$U \simeq \begin{pmatrix} c_{12} & s_{12} & s_{13} e^{-i\delta} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\frac{s_{12}}{\sqrt{2}} = 0.38$

$\frac{c_{12}}{\sqrt{2}} = 0.60$

Tri-bimaximal mixing: $\theta_{23} = \frac{\pi}{4}$ $\theta_{13} = 0$ $\theta_{12} = 35^\circ$

$(s_{12} = 1/\sqrt{3})$

$$\begin{pmatrix} \sqrt{2/3} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\frac{1}{\sqrt{3}} = 0.57$

$\frac{1}{\sqrt{6}} = 0.41$

$\sqrt{2/3} = 0.82$

Remaining unknown: mass hierarchy and δ

δ: level of CP-violation $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$

asymmetry: $$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)}$$

$$\simeq \frac{\Delta m^2_{21}}{4E_\nu} L \frac{\sin 2\theta_{12}}{\sin \theta_{13}} \sin \delta_{CP}$$

T2K, NOVA ($L = 295$ km, ~~[illegible]~~ 800 km)

Hyper K: upgraded beam and bigger detector
$\Rightarrow$ tuned to measure δ

DUNE: beam FNAL $\to$ Homestake 1300 km
long distance: matter effects need to be taken into account
$\to$ DUNE can measure hierarchy as well as δ

But earlier: use atmospheric ν and matter effects:
KM3NeT (ORCA) and IceCube (PINGU)

Challenges to the 3-flavour paradigm?

- LSND experiment @ Los Alamos
  claimed observing $\bar{\nu}_\mu \to \bar{\nu}_e$ $E_\nu = 50$ MeV $l = 30$ m
  $\Delta m \sim 1$ eV ?
  MiniBoone experiment @ FNAL could not disprove it.
  Now MicroBoone @ FNAL will try again.

- Reactor ν flux : observed rate $\bar{\nu}_e$ few percent lower than calculated.
  (ν_e disappearance, also $\Delta m \sim 1$ eV)
  Now Daya Bay: Pu neutrino rate ok, U^{235} low
  Modelling issue?

- Radioactive sources ^{51}Cr, ^{37}Ar for calibration of Ga solar ν
  $\nu_e + {}^{71}Ga \to {}^{71}Ge + e$
  $R \sim 0.86 \pm 0.05$? ν_e disappearance ?

- Big Bang nucleosynthesis and CMB anisotropies count the effective number of degrees of freedom N_{eff}.
  Light sterile ν mixing with normal ν would contribute.
  For a while $N_{eff} = 4$ seemed preferred.
  Latest data (Planck) more consistent with $N_{eff} = 3$
  Anyway, $m \sim 1$ eV not easy to accomodate

A fourth, sterile neutrino, would be a solution.
However, latest data disfavours it.

Neutrino mass :

Oscillations only give us Δm^2, not m_i

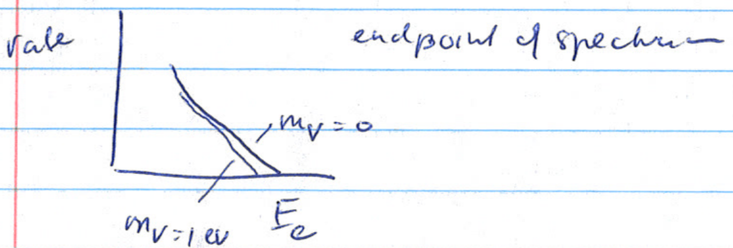
Before oscillations: measurement with ν from a dominant flavour

$m_{\nu_e} < 2$ eV   endpoint tritium β decay

$m_{\nu_\mu} < 170$ keV

$m_{\nu_\tau} < 18.2$ MeV

In β decay, one really measures $|m_\beta| = \sum_i |U_{ei}| m_i$



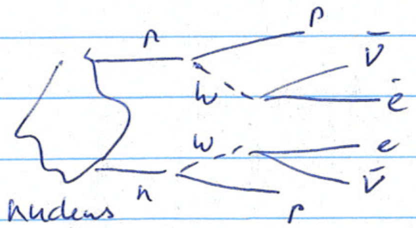
From cosmology :

Neutrino contribution to Ω (structure formation) $\sum_i m_{\nu_i} < 11$ eV

CMB + BAO + ... $\sum_i m_i <$ 0.12 eV (aggressive)
0.7 eV (less aggressive)

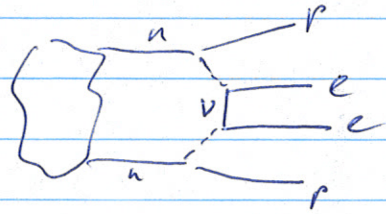
Note: if largest $\Delta m^2 = 2.3 \cdot 10^{-3}$ eV^2
then there must be a ν_i with $m_{\nu_i} \geq 0.05$ eV

Double β decay:



$2n \to 2p + 2e + 2\bar{\nu}_e$

Neutrinoless double β decay

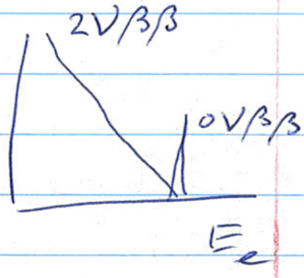


$2n \to 2p + 2e$

$0\nu\beta\beta$ decay: only if ν is Majorana $\nu = \bar{\nu}$

Decay rate $\sim |M|^2 \langle m_{\beta\beta} \rangle^2$ M nuclear matrix elements

$\langle m_{\beta\beta} \rangle = \sum_i U_{ei}^2 m_i$



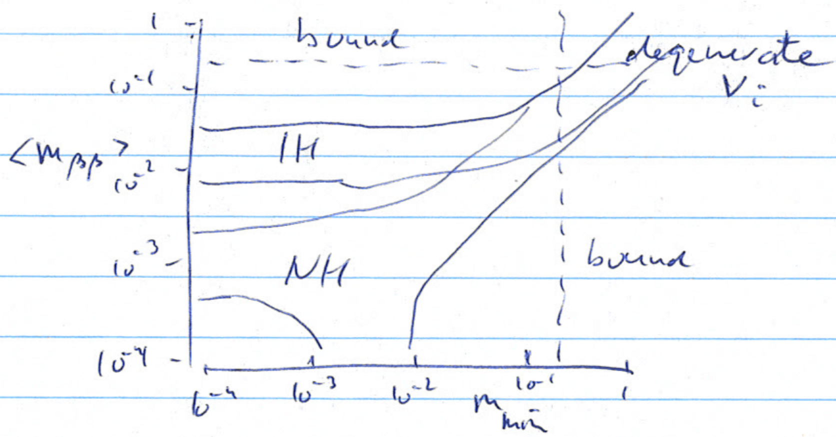
Experimental measurement:

Needs an appropriate isotope

$2\nu\beta\beta$ has been observed, $t_{1/2} \sim 10^{18} - 10^{21}$ year

$0\nu\beta\beta$ only ~~upper~~ lower limits $t_{1/2} > 10^{24} - 10^{25}$ years

$m_{\beta\beta} < 0.2 - 0.4$ eV (uncertainty!) @ 90% CL



Experiments demand very low backgrounds!