Hierarchy problem

$$m_H^2 = m_{H,bare}^2 + \Delta m^2_{radiative}$$

One loop: $\Delta m^2_{radiative} \sim \frac{\lambda^2 \Lambda^2}{16\pi^2}$

λ: coupling of H to particles in loop

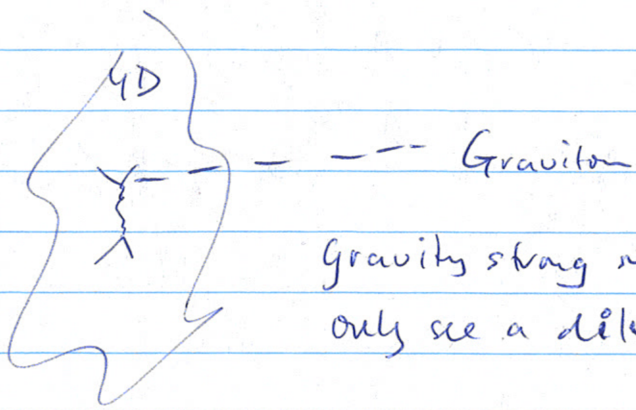
Λ: arbitrary cut-off (otherwise: diverges!)

What is Λ? GUT scale $\sim 10^{14} - 10^{16}$ GeV?
Planck scale 10^{19} GeV?
$\Rightarrow$ huge effects.

- But what if $\Lambda \sim$ a few TeV?
Radiative corrections "acceptable"

Planck scale defined by 'gravity becoming strong'.
What if gravity already becomes strong at a few TeV?

Imagine: gravity can live in 5D space, but we (= fermions, gauge bosons) are confined to a 4D subspace ("brane")



Gravity strong in 5D space, but we in 4D only see a diluted version

Motivation: string theory

Particles are end points of open strings, reside on brane

Gravitons are closed strings, free to move in the bulk

Extra dimensions are of Planck-size $\sim \frac{1}{M_{pl}} \sim 10^{-35}$ m

"Compactified"

Open problem: how are ED compactified?

Extra dimensions of 10^{-35} m do not solve hierarchy problem, let's see if we can make them larger.

"Large extra dimensions" or ADD model

Consider Gauss' law:

field E caused by a source M

$$\oint E \cdot ds = \text{constant } (\sim M)$$

gravity in 4D: $\oint E \cdot ds = 4\pi G_N M$

$$E = \frac{G_N M}{r^2}$$

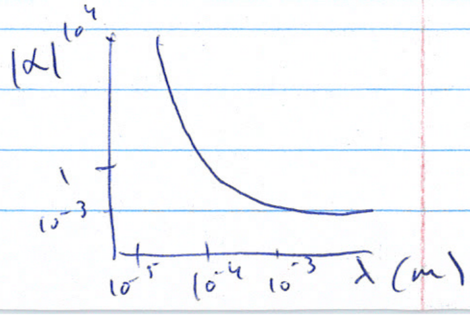
The same in 5D: $\oint_V E \cdot dV = \text{constant}$

$$E = \frac{G^* M}{r^3}$$

In $4+\delta$ dimensions, gravity falls off like $\frac{1}{r^{(2+\delta)}}$

Experiment: $\frac{1}{r^2}$ holds certainly for $100\,\mu m < r < R_{\text{solar system}}$

$$F_g \to F_g\left(1 + \alpha e^{-r/\lambda}\right)$$



So let's assume size of extra dimensions is R ("small")

For $r \gg R$ we do not see ED : gravity $\frac{1}{r^2}$

$r \ll R$ we need to consider ED effects

Matching the expressions at $r = R$: $G^* = G_N \cdot R$ $\quad (G_N = G^*/R)$

Equivalently in terms of the Planck scale:

4D: $M_{Pl}^2 \sim \frac{1}{G_N}$ $\qquad$ 5D $(\delta = 1)$: $M_{Pl}^{*(2+\delta)} \sim \frac{1}{G^*}$

$$M_{Pl}^2 = M_{Pl}^{*(2+\delta)} \, V_\delta$$

V_δ ↑ volume extra dimensions $(\sim R^\delta)$

M_{Pl}^* ↑ "True" Planck scale in $4+\delta$ dimensions.

By a suitable choice of R (or V) we can make M_{Pl}^* of order TeV, while the "diluted" or "apparent" Planck scale is 10^{19} GeV.

So, in the hierarchy problem we take Λ to be M_{Pl}^*
Gravity will become strong at M_{Pl}^*, so we do need some form of new physics to save us!
(quantum gravity!)

To make this work:

$M_{Pl} \sim 10^{19}$ GeV $\quad M^*_{Pl} \sim 1$ TeV

$\delta = 1$: $R \sim$ solar system ← ruled out

$\delta = 2$ $\quad R \sim$ mm ← ?

$\delta = 3$ $\quad R$ few nm ← feasible

etc.

Suppose $\delta = 1$: particle described by a 5-vector

$$E = \sqrt{\bar{p}^2 + p_5^2 + m^2} \equiv \underbrace{\sqrt{\bar{p}^2 + m^2_{KK}}}_{\text{in 4D}}$$

Particles have an effective mass due to p_5

If $R \to \infty$ p_5 is continuous.

But if R is finite, can consider it to be compactified on a circle of radius R

⇒ only discrete states are possible in extra dimension but infinitely many

(like a particle in a box in QM)

In fact: $m^2 = m_0^2 + \sum_{n=1,\infty} \frac{n^2}{R^2}$

"Kaluza-Klein tower" or "KK tower"

In string theory: $R = R_{Pl} \to m_1 = \mathcal{O}(M_{Pl})$

But in ADD scenario the mass splittings in the tower could be of order 10^{-4} eV to a few MeV

⇒ One can (typically) treat it as a continuum

⇒ gravity becomes strong because there are so many graviton states!
(coupling of each graviton still small)

Constraints on this model:

Astrophysical:

supernova emission : $M^* > 4$ TeV $\delta = 3$
1 TeV $\delta = 4$

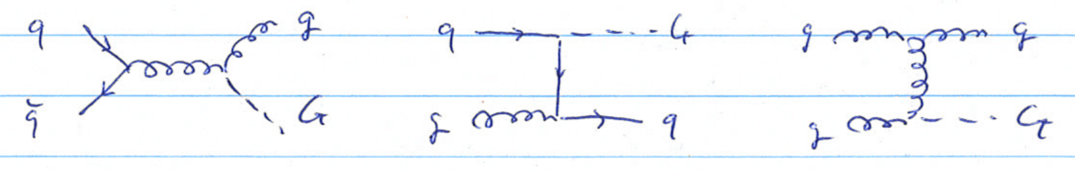
neutron stars: if KK particles decay into photons, may heat neutron stars $M^* > 70$ TeV $\delta = 3$

early universe : cooling down by gravitons disappearing (can be accomodated in current models)

$KK \to \gamma\gamma$ contributes to diffuse cosmic gamma ray spectrum
$M^* > 5$ TeV for $\delta = 3$

Collider constraints

Graviton production at LHC for example

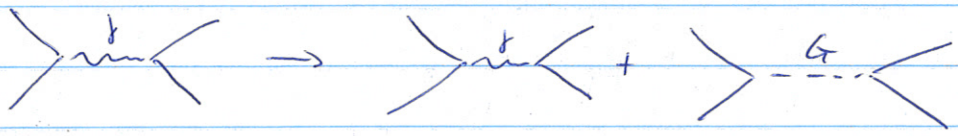


each individual diagram very small,
but σ enhanced by the large size of KK tower!

Signatures: $q\,G$ or $g\,G$ with G disappearing in ED
$\Rightarrow$ "monojet"

or at LEP: $e^+e^- \to \gamma G$ "monophoton"

Or: virtual graviton exchange

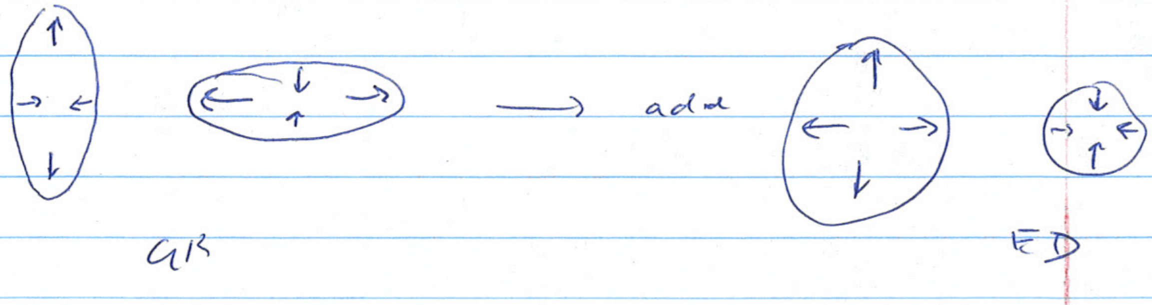


Effects are small
→ look for effects at high p_T, E_T^{miss} end of spectra
→ make use of spin-2 character graviton
(compare spin 1: $1+\cos^2\theta$ peaking forward, spin-2 more central)

Recently in the news:
Gravitational waves are sensitive to ED

Small extra dimension: additional high-frequency waves (probably hard to detect, LIGO sensitive to few hundred kHz)

also: additional "breathing mode" of space time



Alternative model: "warped extra dimensions"
(Randall-Sundrum)

Previous scenario: extra dimensions are intrinsically flat

In RS: only 1 extra dimension, but intrinsically curved

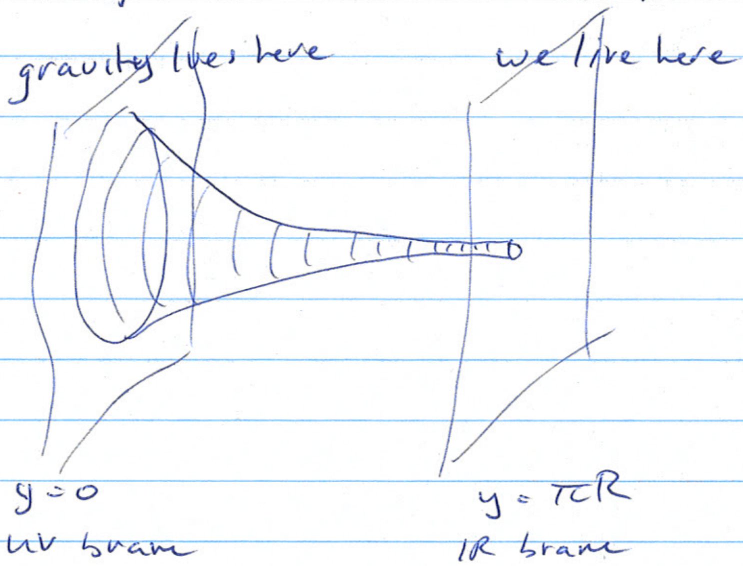
Imagine 5^{th} dimension, coordinate in that dimension y
5^{th} dimension only filled with vacuum energy

Metric: $ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$

Each slice in 5D space at constant y is a 4D space with rapidly changing metric as a function of y

e^{-2ky} is called "warp factor"
k is a free parameter, taken to be of order M_{Pl}

Imagine 2 branes in 5D space, each at certain y



Energy of fields at $y = \pi R$ are red-shifted by a factor $e^{-2k\pi R}$ with respect to those at $y = 0$

In this scenario: gauge theories are confined to 4D branes, our brane is the IR brane, gravity originates in UV brane, can propagate through 5D space, but arrives red-shifted at our brane.

$$M_{Pl}^* = M_{Pl}\, e^{-2k\pi R}$$

M_{Pl}^* can be $\mathcal{O}(1\,\text{TeV})$ if kR is $\mathcal{O}(10)$, k $\mathcal{O}(M_{Pl})$
$\left(R \sim \frac{\mathcal{O}(10)}{\mathcal{O}(M_{Pl})}\right.$, i.e. very small!$\left.\right)$

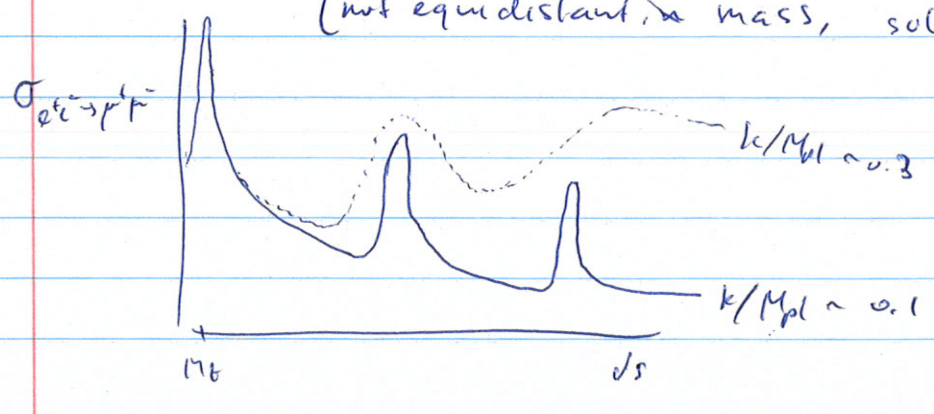
(Model also known as RS1 model.
In RS2, some tricks are applied so that $y_{IR} \to \infty$)

Closer scrutiny of behaviour of gravitational fields in bulk lead to predictions of 1 extra scalar particle that stabilizes the size of ED → "radion"
(or "modulus field")

Interestingly: radion could mix with Higgs
(and modify Higgs phenomenology)

In RS model, gravitational coupling strengths of individual gravitons becomes strong
(G^* instead of G_N)

Also: KK modes exist, but typically $\Delta m \sim \mathcal{O}(\text{TeV})$
⇒ clearly visible resonances ($\gamma\gamma$, $\ell\bar{\ell}$, $q\bar{q}$, gg, ...)
(not equidistant in mass, solutions of Bessel functions)



Giving up on the assumption that only gravity propagates in the bulk: allow all fields to enter ED

⇒ "Universal extra dimensions"

KK modes of all fields: → lots of resonances.

Not observed: KK modes $\Delta m > 1$ TeV

ED size $(1\,\text{TeV})^{-1}$ (10^{-18} m or smaller)

In principle, KK modes can mix with SM particles and affect precision EW observables
To prevent this, KK-parity $(-1)^n$ is introduced, and assumed to be conserved.
(Lightest KK $n=1$ mode must then be stable!)

If a mass M is confined within a radius smaller than the Schwarzschild radius $R_S = \frac{2 M G_N}{c^2}$, a black hole can form.

A black hole radiates like a black body with temperature T_H

$$k_B T_h = \frac{\delta + 1}{4\pi R_S} \hbar c$$

A reasonable number for the formation cross section σ is

$$\sigma \sim \pi R_S^2$$

For SM particles, without ED, R_S is very small

However, ED change R_S:

$$R_S \sim \frac{1}{M^*}\left(\frac{M_{BH}}{M^*}\right)^{\frac{1}{1+\delta}}$$ for $M_{BH} > M^*$ and $R_S < R_{ED}$.

Numerically: $R_S \sim \left(\frac{M_{BH}}{1\,TeV}\right)^{\frac{1}{1+\delta}} \times$
10^{-26} cm $\delta = 1$
10^{-18} cm $\delta = 2$
10^{-14} cm $\delta = 3$
⋮

and cross section $\propto R_S$ could be $\mathcal{O}$(pb) or $\mathcal{O}$(nb)
(e.g. σ_H @ LHC!)

For a mass of 1 TeV, its lifetime $\sim 10^{-27} - 10^{-25}$ s
It will decay democratically to all particle species

Signature: high multiplicity, high p_T particles

For $M_{BH} \simeq M^*$ the picture is more complicated
Quantum effects become important.
No theory, only a few models.
"Quantum Black Holes": resonance decaying non-thermally in small nr of SM particles
(e.g. dijet resonance)

If model is correct, BH's should be made in large numbers in ultra high energy CR interactions.