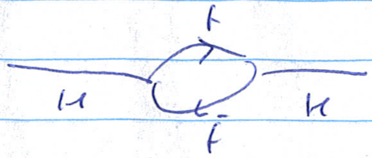
Coming back to hierarchy problem:
radiative corrections to m_H:

$$m_H^2 = m_0^2 + \frac{\lambda^2}{16\pi^2}\left(-\Lambda^2 + m_f^2 \ln\left(\Lambda^2/m_H^2\right) + \dots\right)$$

for fermion loops



which diverges for $\Lambda \to \infty$, so we apply a cutoff $\mathcal{O}(M_{Pl})$

Can we protect m_H by some symmetry?

> Little Higgs: could H be the Nambu-Goldstone boson of a broken global symmetry?

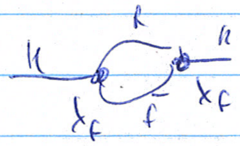
$\Rightarrow$ H would be massless at tree level

It would get a mass from radiative corrections, but by clever model building, not at one loop, but only at higher loops.

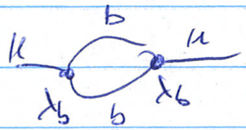
My opinion: bit contrived model
hierarchy problem not really solved, only shifted to higher loops

> Supersymmetry: fermion loops have opposite sign compared to boson loops

$\Rightarrow$ Cancel fermionic loops by bosonic loops



$$\delta m^2 = \frac{\lambda_f^2}{16\pi^2}\left(-\Lambda^2 + m_f^2 \ln\left(\Lambda^2/m_H^2\right) + \dots\right)$$



$$\delta m^2 = \frac{\lambda_b^2}{16\pi^2}\left(\Lambda^2 - m_b^2 \ln\left(\Lambda^2/m_H^2\right) + \dots\right)$$

If $\lambda_f = \lambda_b$: quadratic divergences cancel.

If $m_f = \frac{\lambda_f v}{\sqrt{2}} = m_b = \frac{\lambda_b v}{\sqrt{2}}$, everything cancels.

If $\lambda_f = \lambda_b$, but $m_f \neq m_b$:

$$\delta m^2 \sim \left(m_f^2 - m_b^2\right) \ln\left(\Lambda^2/m_H^2\right)$$

which may be acceptable if $m_f^2 - m_b^2$ is not too huge

(say, no more than few TeV^2)

In this case, f or b (or both) get (part of) their mass not from the Higgs mechanism.

Within SM: no appropriate fermion - boson match to achieve this.

Instead, we postulate the existence of a bosonic partner of every SM fermion, and a fermionic partner of every SM boson.

This partner has <u>the same quantum numbers</u> as SM particle

$\Rightarrow$ The same colour $(SU(3))$

weak isospin $(SU(2))$ } same Q

hypercharge $(U(1))$

Yukawa coupling to H : $\lambda_f = \lambda_b$

Supersymmetric partners of fermions: scalar fermions (squarks, sleptons) spin-0

SM: $\begin{pmatrix} u \\ d \end{pmatrix}_L$ u_R d_R SUSY: $\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$ $\tilde{u}_R$ $\tilde{d}_R$

L and R represent SU(2) quantum numbers.

Supersymmetric partners of bosons: spin-1/2 "bosinos"

gluon → gluino

B → bino

$W^{1,2,3}$ → wino

~~H~~ → higgsino

$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{\phi}^+ \\ \tilde{\phi}^0 \end{pmatrix} \begin{pmatrix} \tilde{\phi}^{0*} \\ -\tilde{\phi}^- \end{pmatrix}$

SM particles and SUSY particles form "supermultiplets"

$m_f^2 - m_b^2$ not too large: no large mass splitting within supermultiplet

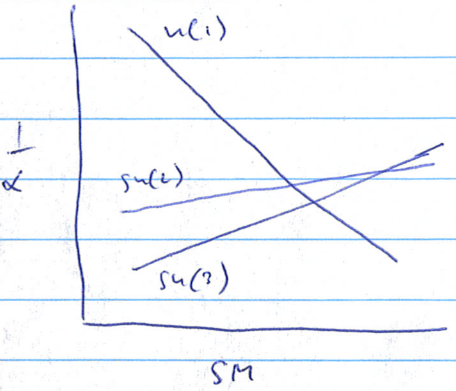
Since SUSY particles have not yet been observed, we already know $m_f \neq m_b$: SUSY is broken.

Higgs sector in SUSY: specific realisation of 2HDM

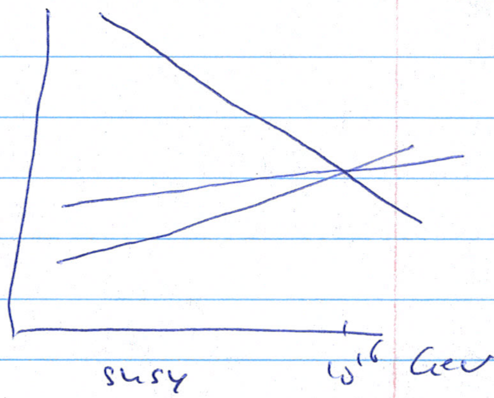
What is good about SUSY?

- cancellation of divergences (loop corrections) in Higgs mass.
  Exact if $m_f = m_b$
  reduced to "soft breaking" if $m_f - m_b$ "not too large"

- unification of gauge couplings works better in SUSY



$\longleftrightarrow$



SUSY: extra contributions in β function from SUSY particles.
Works as long as new particles have $m \lesssim 10$ TeV or so.

- Supersymmetry provides an excellent dark matter candidate.
  Under the assumption of "R-parity conservation"

$$R \equiv (-1)^{3(B-L)+2S}$$

B = baryon number
L = lepton number
S = spin.

SM particles have $R = 1$
SUSY particles have $R = -1$

R is a <u>multiplicative</u> quantum number

R-parity conservation: $R_{in} = R_{out}$

R-parity conservation implies :
- SUSY particles are always produced in pairs
- the decay of a SUSY particle must lead to an odd number of new SUSY particles

↳ the lightest SUSY particle (LSP) is stable

In many SUSY models, the LSP is $\tilde{\chi}_1^0$
= lightest mass eigenstate of mixture $\tilde{B}, \tilde{W}, \tilde{H}$

It would have been made in large quantities in big bang.
Could still be around today → dark matter.

After big bang: probably too many $\tilde{\chi}_1^0$, overcloses universe

To get the right relic density today $\Omega h^2 \sim 0.1$
$\tilde{\chi}_1^0$ must have disappeared (but cannot decay!)

$\tilde{\chi}_1^0$ is Majorana : $\tilde{\chi}_1^0 = \bar{\tilde{\chi}}_1^0$

$\tilde{\chi}_1^0 \tilde{\chi}_1^0$ annihilation can occur

For $m_{\tilde{\chi}^0} > 0$ and $m_{\tilde{\chi}^0} <$ few TeV, the properties including annihilation are <u>just right</u> ⇒ WIMP miracle.

What is <u>not good</u> about SUSY?

> Where are the SUSY particles then?

> No good model of SUSY breaking
⇒ introduction of many new free parameters

In supersymmetry, we define supermultiplets :

Chiral supermultiplets

ψ_L spin ½ left-handed Dirac fields

$\tilde{\psi}_L$ spin-0 complex scalar fields (sfermions)

F : complex scalar field : auxiliary no kinetic terms
→ does not enter in phenomenology

right-handed Dirac fields are conjugated to bring them to left-handed form : $\psi_R \to (\psi_R)^c = i\,\sigma^2 \psi_R^*$

Vector supermultiplets

W_μ^a spin-1 real gauge fields

$\tilde{W}_\mu^a$ spin-½ Majorana fermions "gaugino"

D real scalar field : auxiliary no kinetic terms
→ does not enter in phenomenology

In chiral supermultiplet, we have for example

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R \quad d_R$$

SUSY partners: $\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix} \quad \tilde{u}_R \quad \tilde{d}_R$

spin-0 : L and R have nothing to do with chirality
but everything with quantum numbers
under SU(2)

Generically, <u>mass eigenstates</u> are mixtures of <u>flavour eigenstates</u>

$\hat{q}_L, \hat{q}_R$ mix to $\tilde{q}_1, \tilde{q}_2$ $\tilde{q}_1$ being lightest

In generic "soft susy breaking Lagrangian" there are explicit mass terms for susy particles
(part of their mass not from Higgs mechanism, but from (unknown) susy breaking mechanism)

e.g. for $\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$ m_{Q_1} $\Leftarrow$ assumed to be equal for members of same SU(2) multiplet

$\tilde{u}_R$ m_U

$\tilde{d}_R$ m_D

(often assume $m_{Q_1} = m_U = m_D$)
idem for other squarks and sleptons.

Mixing: $\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_u & -\sin\theta_u \\ \sin\theta_u & \cos\theta_u \end{pmatrix} \begin{pmatrix} \hat{u}_L \\ \hat{u}_R \end{pmatrix}$

θ_u is a function of m_{Q_1}, m_U and m_u (up-quark mass) plus a few other terms in $\mathcal{L}$ that do not matter now.

It turns out: for ~~light~~ first and second generation squarks and sleptons: $\theta \simeq 0$

$\Rightarrow$ matrix is diagonal, $(\tilde{u}_1, \tilde{u}_2) \simeq (\hat{u}_L, \hat{u}_R)$

But <u>not</u> for third generation (m_t, m_b, m_τ relatively large)
e.g. θ_t can be sizable $\Rightarrow$ large mixing

This can make $m_{\tilde{t}_1}$ much lighter than $m_{\tilde{t}_2}$

In fact: $\tilde{t}_1$ probably the lightest squark
(because top is the heaviest!)
$\Rightarrow$ light stop searches important priority at LHC

To some extent, the same for $\tilde{\tau}$: the lightest slepton.

Analogous for the gauginos:
$\tilde{\chi}^{\pm}$, $\tilde{\chi}^{0}$ are mixtures of $\tilde{B}$, $\tilde{W}_\mu^{1,2,3}$, $\tilde{h}$

$\tilde{B}$: neutral; $\tilde{W}_\mu^{1,2,3}$: 1 neutral, 2 charged; $\tilde{h}$: 2 neutral, 2 charged

$\tilde{\chi}^{\pm}$: 4; $\tilde{\chi}^{0}$: 4

In the Lagrangian, $\tilde{B}$ has a mass term M_1
$\tilde{W}^{1,2,3}$ mass term M_2
($\tilde{g}$ mass term M_3, but no mixing)
$\tilde{h}$ mass term μ

Mixing matrix for charged states:

$$\begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix} \qquad \begin{pmatrix} \tilde{W}^{\pm} \\ \tilde{h}^{\pm} \end{pmatrix}$$

(remember: $\tan\beta \equiv \frac{v_u}{v_d}$, ratio of vev in 2HDM)

Lighter states: $\tilde{\chi}_1^{\pm}$ Heavier states $\tilde{\chi}_2^{\pm}$

Depending on $M_2 << \mu$ or $\mu << M_2$,
the lighter state is more "wino-like"
or more "higgsino-like"

Neutral states:

$$\begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\ -M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\cos\theta_W & 0 & -\mu \\ M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{w}^0 \\ \tilde{h}_1^0 \\ \tilde{h}_2^0 \end{pmatrix}$$

Four mass states $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$ in increasing mass.

Again, depending on values of M_1, M_2, μ

$\tilde{\chi}_1^0$ can be bino-like $M_1 \ll M_2, \mu$

wino-like $M_2 \ll M_1, \mu$

higgsino-like $\mu \ll M_1, M_2$

Or they can be mixtures.

(For example: if $\mu \ll M_1, M_2$, both lightest neutralinos $\tilde{\chi}_1^0, \tilde{\chi}_2^0$ and lightest chargino $\tilde{\chi}_1^\pm$ are determined by μ: $\Rightarrow$ degenerate in mass)

If R-parity $\left(\equiv (-1)^{3(B-L)+2S}\right)$ is conserved:
the lightest SUSY particle must be stable.

In principle, $\tilde{\chi}_1^0, \tilde{\chi}_1^\pm, \tilde{t}_1, \tilde{\tau}_1, \tilde{\nu}$ are all candidates for LSP

However, a stable LSP has cosmological consequences

In particular: <u>coloured</u> or <u>charged</u> LSPs are strongly disfavoured

That leaves $\tilde{\chi}_1^0$ and $\tilde{\nu}$ as preferred LSP candidates

$\tilde{\nu}$: couples too well to the Z : we would have observed it already in direct dark matter searches. (Also: too much self-annihilation, not enough Ωh^2)

$\tilde{\chi}_1^0$: excellent candidate for DM

Early universe : thermal equilibrium $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \leftrightarrow SM\ \overline{SM}$

where SM = SM fermion or boson.

$\tilde{\chi}_1^0$ is Majorana fermion : $\bar{\tilde{\chi}}_1^0 = \tilde{\chi}_1^0$: self-annihilation ok

As temperature drops, $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow SM\ \overline{SM}$ still possible

but $SM\ \overline{SM} \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ kinematically no longer possible

$$\frac{dN_\chi}{dt} < 0$$

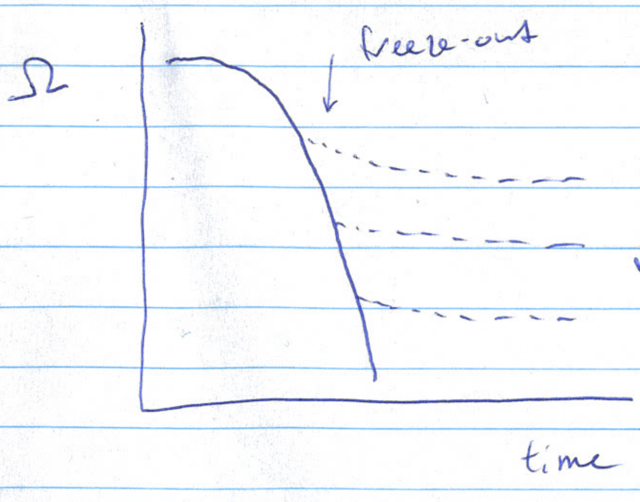
As universe expands: $\tilde{\chi}_1^0$ no longer find each other

$\Rightarrow$ freeze-out density ~ constant $\Rightarrow$ current dark matter density?

Density depends on annihilation rate $\langle \sigma \cdot v \rangle$

σ : annihilation cross section

v : velocity of χ when colliding



Rule of thumb: $\Omega h^2 = \frac{m_\chi n_\chi}{\rho_c} \simeq \frac{10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle \sigma \cdot v \rangle}$

For $\Omega h^2 \sim 0.1$ and $v \sim 1000$ km/s : $\sigma \sim 10^{-34}\,\mathrm{cm}^2$
$= 100$ pb

$\Rightarrow$ Weak cross sections for particles with masses between few GeV and few TeV

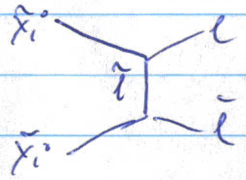
Weakly Interacting Massive Particles WIMP - miracle.

However: it does need an effective annihilation mechanism, with sizable cross section.

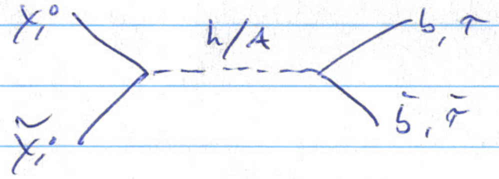
If σ too small: Ωh^2 too high

σ too large: Ωh^2 below 0.1

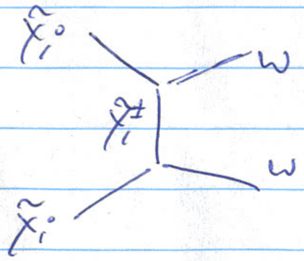
A few possibilities:



"bulk annihilation region" with light sleptons



"Higgs funnel"

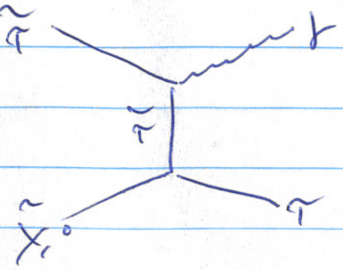


Higgsino LSP

$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_1^\pm$ degenerate

large couplings to W, Z

Also $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ and $\tilde{\chi}_1^0 \tilde{\chi}_1^\pm$ co-annihilation

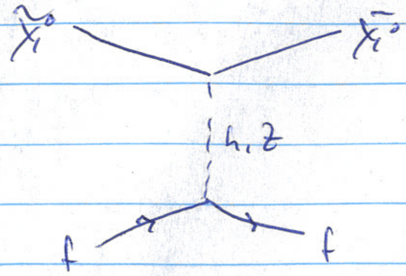


$\tilde{\chi}_1^0 \tilde{\tau}$ coannihilation

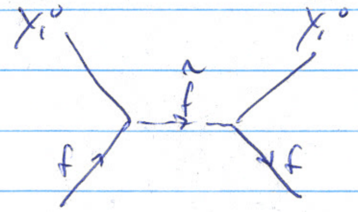
if $m_{\tilde{\tau}} \simeq m_{\tilde{\chi}_1^0}$

Discovery of SUSY DM:

a)



or

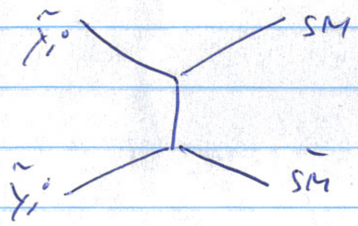


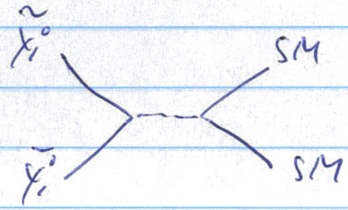
direct detection of DM scattering

Low cross sections, little recoil energy ($\mathcal{O}$(10 keV))

$\Rightarrow$ Difficult, low-background experiments

b)

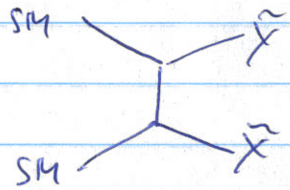


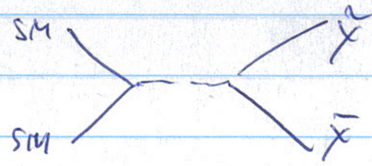


Indirect detection of $\tilde{\chi}_1^0 \tilde{\chi}_1^0$ annihilation in universe

e.g. excess of γ-rays, positrons, antiprotons, neutrinos from, for example, galactic center

c)





at LHC

Better not $\tilde{\chi}_1^0 \tilde{\chi}_1^0$: invisible

Rather: $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to Z^* \tilde{\chi}_1^0 \, W^* \tilde{\chi}_1^0$

$\tilde{\chi}_1^+ \tilde{\chi}_1^- \to W^* \tilde{\chi}_1^0 \, W^* \tilde{\chi}_1^0$

2- or 3-leptons with missing energy

Cross sections depend on M, but also on nature of gaugino.

e.g.: bino-like $\tilde{\chi}_1^0$ will <u>not</u> couple to W

Also in decay: Higgsino $\tilde{\chi}_1^0$: degenerate mass spectrum

$\Rightarrow$ soft leptons.

In the light of susy GUTs, it is sometime assumed that $M_1(M_{GUT}) = M_2(M_{GUT}) = M_3(M_{GUT})$ with $M_{GUT} = 10^{16}$ GeV.

Then, for $M_{1,2,3}$ at EW scale: solve RGE ("running parameters")

Typically leads to $M_1 : M_2 : M_3 = 1 : 2 : 7$

<u>Effects of susy on observables:</u>

In principle, superpotential has terms like

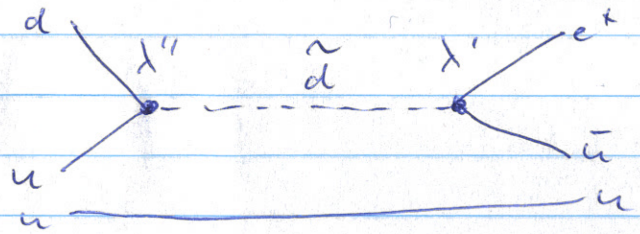
$$\lambda \hat{L}^i \hat{L}^j \hat{E}^{c k} + \lambda' \hat{L}^i \hat{Q}^j \hat{D}^{c k} + \lambda'' \hat{U}^{c i} \hat{D}^{c j} \hat{D}^{c k}$$

supermultiplet with SU(2) doublets of $\begin{pmatrix} \nu \\ \ell \end{pmatrix}_L$ and $\begin{pmatrix} q \\ q' \end{pmatrix}_L$

E, U, D: supermultiplets with $e_R, \hat{e}_R$ etc....

$\lambda, \lambda', \lambda''$ free parameters

This allows interactions like



$p \to e^+ \pi^0$

In GUTs suppressed by $\frac{1}{M_X^4}$ with $M_X \sim 10^{16}$ GeV

But $m_{\tilde{d}} \ll 10^{16}$ GeV: trouble!

Constrains λ parameters to very small values

Or: R-parity conservation: $\lambda = \lambda' = \lambda'' = 0$