Ex. 7 Consider a SUSY theory with $\{\hat{Q}_\alpha, \hat{\bar{Q}}_\beta\} = 2(\gamma^\mu)_{\alpha\beta}\hat{P}_\mu$ for its generators $\hat{Q}$ and $\hat{\bar{Q}} = \hat{Q}^\dagger\gamma^0$. The indices α and β are spinor indices and $\hat{P}_\mu$ is the 4-momentum operator.

(i) Project the right-hand side of the operator identity onto $\hat{P}_0 = \hat{H}$ and derive in this way that $\hat{P}_0 = \frac{1}{8}\{\hat{Q}_\alpha, \hat{Q}_\alpha^\dagger\}$.

(ii) Prove that all physical states $|\psi\rangle$ (with non-negative norm) have $\langle\psi|\hat{H}|\psi\rangle \geq 0$ in this theory.

(iii) Consider the ground state $|\Omega\rangle$ of the theory, with $\hat{H}|\Omega\rangle = E_0|\Omega\rangle$. Prove that $E_0 = 0 \Leftrightarrow \hat{Q}|\Omega\rangle = \hat{\bar{Q}}|\Omega\rangle = 0$.