- where does the Higgs potential come from?
- what is the origin of all Standard Model parameters (18 + ν-parameters)?
- what has caused the matter-antimatter asymmetry in the universe?
- where does gravity (and its associated spin-2 graviton) fit in and why is the electroweak scale $[\Lambda_{ew} = \mathcal{O}(100\,\text{GeV})] \ll$ Planck scale of strong (quantum) gravity $[\Lambda_{Pl} = \mathcal{O}(10^{19}\,\text{GeV})]$?

The Standard Model itself provides some hints that there might be "physics beyond the Standard Model" (see later). More profoundly, some cracks have emerged in the foundations of our understanding of nature, originating from non-collider experiments. Large-scale astronomical observations have revealed that

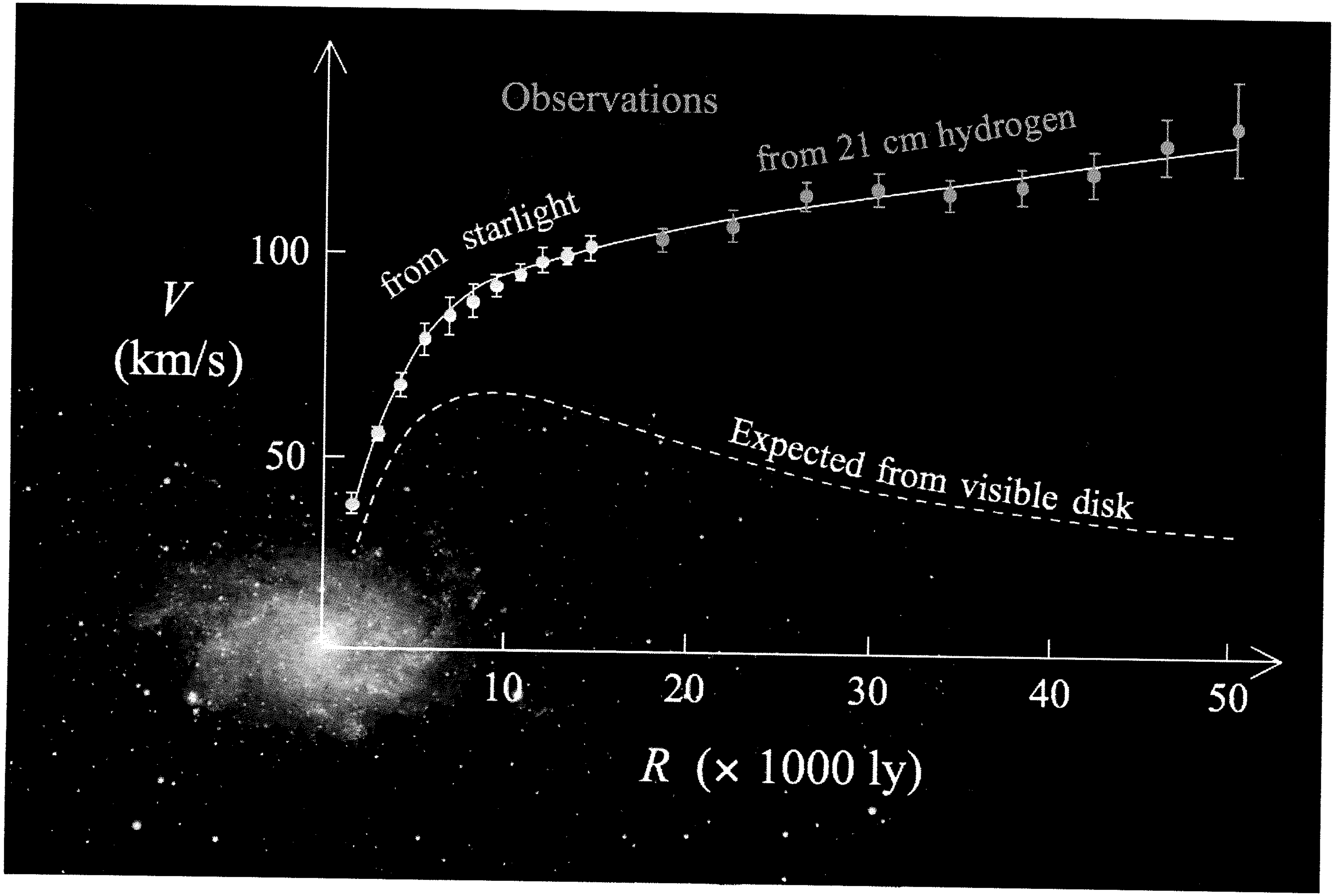
*) there seems to be 5 times more (non-luminous) dark matter than (luminous) ordinary matter that we observe electromagnetically.
Sources: rotation curves of galaxies, motion of galaxies in large galaxy clusters, gravitational lensing, anisotropies in the cosmic microwave background, ...

*) 70% of the energy density in the universe seems to bear the characteristics of a cosmological constant, which stays constant while the universe expands. This type of energy is referred to as dark energy.
Sources: accelerated expansion of the universe and anisotropies in the cosmic microwave background

↳ the Standard Model cannot provide explanations for this!

Moreover, the Standard Model also has limitations, mostly related to the Higgs sector. Before we go into specific details about this, we first focus on straightforward matter extensions of the Standard Model. Some of these extensions can actually be regarded as formally belonging to the Standard Model (but traditionally not considered as such). The observation of neutrino oscillations implies massive neutrinos, therefore we will have to add such mass terms to the Standard Model one way or another!

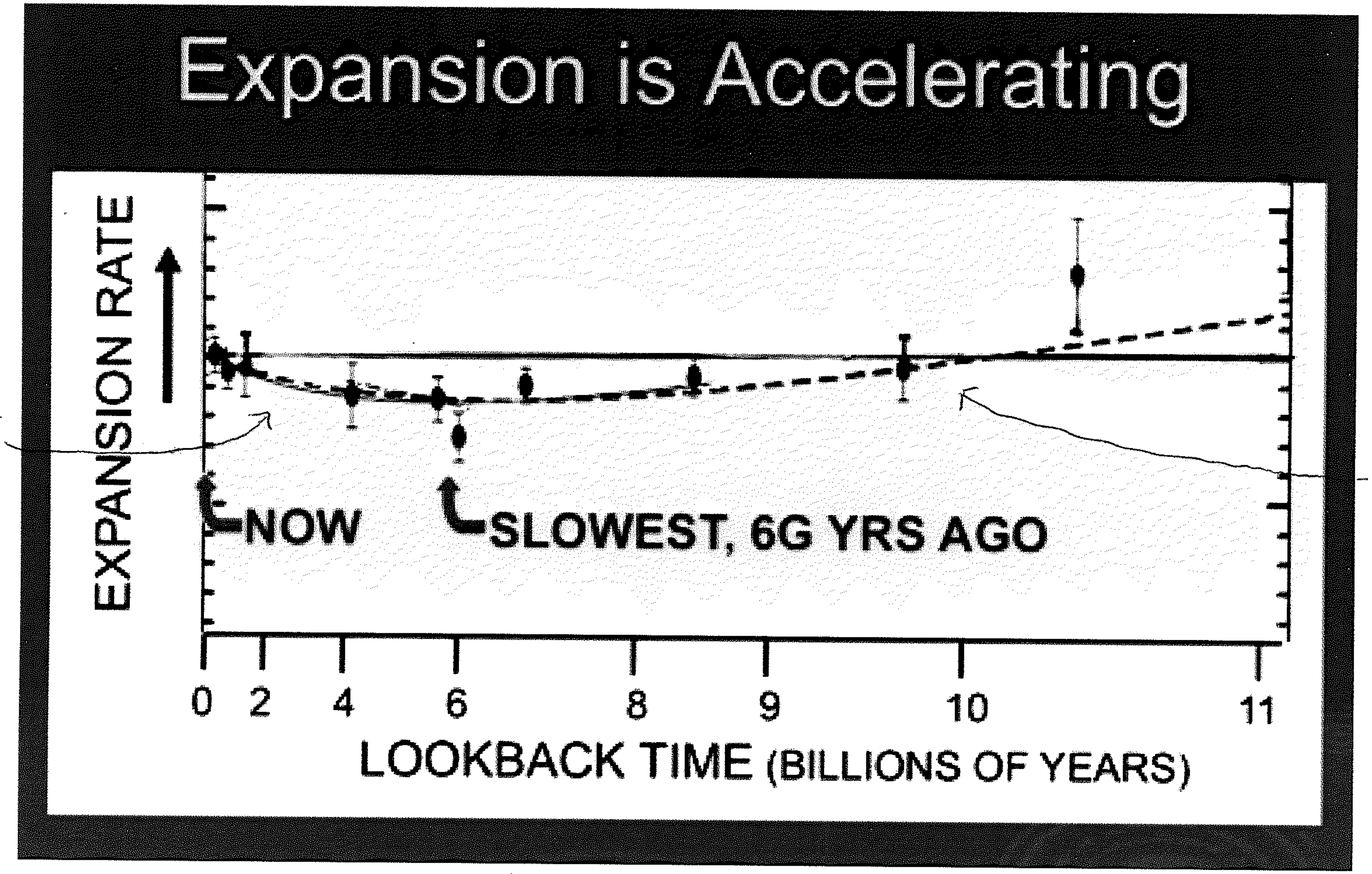
The case for dark matter : galaxy rotation curves



Accelerated expansion of the Universe

from Supernova 1a observations

acceleration due to dark energy



deceleration due to gravity from matter

Supernovae have larger luminosity than expected based on the observed redshift (since they are actually closer by)

Part 2: Matter extensions of the Standard Model (Chapter 1+2 of the textbook)

The first natural extension of the Standard Model is to consider a non-minimal Higgs sector, without changing the gauge aspects of the model.

§2.1 Non-minimal Higgs sector

Consider a Higgs multiplet Φ with weak isospin I and hypercharge Y, with neutral vev $v/\sqrt{2} \Rightarrow I_3^{vev} = -Y/2$, $Q^{vev} = 0$ (e.g. $I_3^{vev} = \mp 1/2 = -Y/2$ for Φ_{SM} and Φ_{SM}^c).
As before the Higgs Lagrangian is given by

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) , \quad D_\mu \Phi = (\partial_\mu + \frac{i g_W}{\sqrt{2}}[T^+ W_\mu^+ + T^- W_\mu^-] + i g_W T^3 W_\mu^3 + \frac{i g_B}{2} Y B_\mu) \Phi ,$$

where for the vev part we can effectively write $D_\mu \langle\Phi\rangle_0 = (\frac{i g_W}{\sqrt{2}}[T^+ W_\mu^+ + T^- W_\mu^-] + i T^3 \underbrace{[g_W W_\mu^3 - g_B B_\mu]}_{Z_\mu \sqrt{g_W^2 + g_B^2}}) \langle\Phi\rangle_0$.

Then $(D_\mu \Phi)^\dagger (D^\mu \Phi)$ leads to the following gauge-boson mass terms:

$$g_W^2 \underbrace{\langle\Phi\rangle_0^\dagger \tfrac{1}{2}(T^- T^+ + T^+ T^-) \langle\Phi\rangle_0}_{\frac{v^2}{2}[I(I+1) - (I_3^{vev})^2] = \frac{v^2}{2}[I(I+1) - Y^2/4]} W^{\mu,-} W_\mu^+ + (g_W^2 + g_B^2) \underbrace{\langle\Phi\rangle_0^\dagger 2 (T^3)^2 \langle\Phi\rangle_0}_{v^2 (I_3^{vev})^2 = v^2 Y^2/4} \tfrac{1}{2} Z^\mu Z_\mu ,$$

using that $\langle\Phi\rangle_0^\dagger (T^\pm)^2 \langle\Phi\rangle_0 = \langle\Phi\rangle_0^\dagger T^\pm T^3 \langle\Phi\rangle_0 = \langle\Phi\rangle_0^\dagger T^3 T^\pm \langle\Phi\rangle_0 = 0$ and
$T^+ T^- + T^- T^+ = 2 (T^1)^2 + 2 (T^2)^2 = 2 (\underbrace{\vec{T}^2}_{\text{with eigenvalue } I(I+1)} - (T^3)^2)$.

This implies that $m_W^2 = \frac{g_W^2 v^2}{2} (I(I+1) - Y^2/4)$ and $m_Z^2 = (g_W^2 + g_B^2) v^2 Y^2/4$ for the considered Higgs multiplet. Next suppose that we have a number of scalar multiplets with isospin I_i and hypercharge Y_i. If in each multiplet the ground state is neutral (such that $I_{3,i}^{vev} = -Y_i/2$ and vev $= v_i/\sqrt{2}$), then

$$\boxed{\rho = \frac{m_W^2}{m_Z^2 c_W^2} = \frac{m_W^2 (g_W^2 + g_B^2)}{m_Z^2 g_W^2} = \frac{\sum_i v_i^2 [I_i(I_i+1) - Y_i^2/4]}{\sum_i \frac{1}{2} v_i^2 Y_i^2}} \longleftarrow \text{Veltman } \rho\text{-parameter}$$

This low-energy neutral-current to charged-current interaction ratio G_N/G_F is strongly constrained by experiment: $\rho^{exp} = 1.0008^{+0.0017}_{-0.0007}$.
This ratio can be achieved automatically by only using $(I = 1/2, Y = \pm 1)$ and $(I = 3, Y = \pm 4)$ scalar multiplets in the electroweak symmetry breaking part of the extended Higgs sector.

§2.2 The two-Higgs-doublet model (2HDM)

The minimal extension along this line is the so-called 2HDM, which involves using two distinct Higgs doublets rather than the single Higgs doublet that is used in the Standard Model. The textbook version of the 2HDM involves two $(I=1/2,\, Y=1)$ doublets $\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}$ $(i=1,2)$, with a generic scalar potential of the form

$$V_H^{2HDM} = m_{11}^2\, \Phi_1^\dagger \Phi_1 + m_{22}^2\, \Phi_2^\dagger \Phi_2 - (m_{12}^2\, \Phi_1^\dagger \Phi_2 + h.c.) + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)$$

$$+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left[\tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \Phi_1^\dagger \Phi_2 \,(\lambda_6\, \Phi_1^\dagger \Phi_1 + \lambda_7\, \Phi_2^\dagger \Phi_2) + h.c. \right].$$

Here $m_{11}^2, m_{22}^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}$ and $m_{12}^2, \lambda_5, \lambda_6, \lambda_7 \in \mathbb{C}$, so in principle the model has 14 independent parameters. Since this generic model is too complicated to work out analytically, we focus here on a special version of this model that will feature prominently in a supersymmetric extension of the Standard Model. Anticipating that one of the Higgs doublets should occur in charge conjugated form in order to give mass to both up- and down-type fermions, we write

$(I=1/2,\, Y=-1)$ $\qquad\qquad$ $(I=1/2,\, Y=+1)$

$$H_1 = \Phi_1^c = i\tau^2 \Phi_1^* = \begin{pmatrix} \phi_1^{0*} \\ -\phi_1^- \end{pmatrix} \quad \text{and} \quad H_2 = \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$$

$$\Rightarrow \quad \Phi_1^\dagger \Phi_1 = H_1^\dagger H_1 \quad \text{and} \quad \Phi_1^\dagger \Phi_2 = \phi_1^- \phi_2^+ + \phi_1^{0*} \phi_2^0 = H_1 \times H_2 ,$$

where $H_1 \times H_2 \equiv \epsilon_{\ell m} H_{1\ell} H_{2m}$ for $(\epsilon_{\ell m}) = i\tau^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is the standard way to combine two SU(2) doublets into a singlet without conjugation (cf. the 2-electron spin-singlet state $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$).

Furthermore we substantially simplify the scalar potential to

$$\boxed{V_H = \mu_1^2\, H_1^\dagger H_1 + \mu_2^2\, H_2^\dagger H_2 - \mu_3^2\,(H_1 \times H_2 + h.c.) + \overbrace{\tfrac{1}{8}(g_W^2 + g_B^2)}^{m_Z^2/2v^2}\,(H_1^\dagger H_1 - H_2^\dagger H_2)^2 + \overbrace{\tfrac{1}{2} g_W^2}^{2m_W^2/v^2}\,(H_1^\dagger H_2)(H_2^\dagger H_1)}\,.$$

Here μ_3^2 is the only parameter that can contain a phase. However, this phase can be absorbed in the Higgs doublets $H_{1,2}$ in such a way that μ_3^2 may be taken real and positive.

Next we impose the requirements for having electroweak symmetry breaking (see exercise):

1) the Higgs potential should be bounded from below $\Rightarrow$ $2\mu_3^2 \le \mu_1^2 + \mu_2^2$

2) there should be a local minimum away from $|H_1| = |H_2| = 0$ $\Rightarrow$ $\mu_3^4 > \mu_1^2 \mu_2^2$ ①

length of the 2-D vector

The corresponding minimalization conditions are given by

$$2\mu_1^2 |H_1| + \frac{2m_Z^2}{v^2} |H_1| \left(|H_1|^2 - |H_2|^2\right) - 2\mu_3^2 |H_2| = 0 ,$$
$$2\mu_2^2 |H_2| - \frac{2m_Z^2}{v^2} |H_2| \left(|H_1|^2 - |H_2|^2\right) - 2\mu_3^2 |H_1| = 0 .$$

From ① it follows that no electroweak symmetry breaking occurs when

- μ_1^2 and μ_2^2 are too large w.r.t. μ_3^2,
- $\mu_1^2 = \mu_2^2$, which will have important implications in the supersymmetric case.

For the neutral non-zero vev's we write: $\langle H_1\rangle_0 = \begin{pmatrix} v_1/\sqrt{2} \\ 0 \end{pmatrix}$, $\langle H_2\rangle_0 = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}$, with $v_1^2 + v_2^2 \overset{\text{p.28}}{=} v^2 \Rightarrow$ take $v_1 = v\cos\beta$, $v_2 = v\sin\beta$, $\operatorname{tg}\beta \equiv v_2/v_1$ for $0 < \beta < \pi/2$.

From the minimalization conditions we then obtain

$$\left.\begin{aligned} &\mu_1^2 + \frac{m_Z^2}{2} \underbrace{\left(\cos^2\beta - \sin^2\beta\right)}_{\cos(2\beta)} - \mu_3^2 \operatorname{tg}\beta = 0 \\ &\mu_2^2 - \frac{m_Z^2}{2} \left(\cos^2\beta - \sin^2\beta\right) - \mu_3^2/\operatorname{tg}\beta = 0 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} m_Z^2 &= 2\,\frac{\mu_1^2 - \mu_2^2 \operatorname{tg}^2\beta}{\operatorname{tg}^2\beta - 1} \\ \mu_1^2 + \mu_2^2 &= \frac{\mu_3^2}{\sin\beta\cos\beta} \equiv m_A^2 \end{aligned}}$$

<u>Mass eigenstates and masses</u> (without quantum corrections): in order to find the Higgs mass eigenstates we write V_H in component form

$$V_H = \mu_1^2 \left(|\phi_1^0|^2 + |\phi_1^-|^2\right) + \mu_2^2 \left(|\phi_2^0|^2 + |\phi_2^+|^2\right) - \mu_3^2 \left(\phi_1^{0*}\phi_2^0 + \phi_1^-\phi_2^+ + \text{h.c.}\right)$$
$$+ \frac{m_Z^2}{2v^2}\left(|\phi_1^0|^2 + |\phi_1^-|^2 - |\phi_2^0|^2 - |\phi_2^+|^2\right)^2 + \frac{2m_W^2}{v^2} \left|\phi_1^0 \phi_2^+ - \phi_1^+ \phi_2^0\right|^2$$

and write $\phi_i^0 = \phi_i'^0 + v_i/\sqrt{2}$ $(i = 1,2)$. Collecting quadratic terms in the scalar fields one finds:

*) <u>Charged scalars</u> $\rightarrow$ $\Big(\underbrace{\mu_3^2 \frac{v^2}{v_1 v_2}}_{m_A^2} + m_W^2\Big) \left(\phi_1^-, \phi_2^-\right) \overbrace{\begin{pmatrix} \sin^2\beta & -\sin\beta\cos\beta \\ -\sin\beta\cos\beta & \cos^2\beta \end{pmatrix}}^{\text{real, symmetric} \Rightarrow \text{rotate to diagonalize}} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$

$\Rightarrow$ mass eigenvalues $\sqrt{m_A^2 + m_W^2} \equiv m_{H^\pm}$, 0

mass eigenstates $H^\pm = -\sin\beta\,\phi_1^\pm + \cos\beta\,\phi_2^\pm$, $G^\pm = \cos\beta\,\phi_1^\pm + \sin\beta\,\phi_2^\pm$

Goldstone bosons, eaten by $W^\pm$

*) <u>Neutral pseudoscalars</u> $\rightarrow \frac{1}{2}\mu_3^2\,(\sqrt{2}\,\mathrm{Im}\phi_1^0, \sqrt{2}\,\mathrm{Im}\phi_2^0)\begin{pmatrix} tg\beta & -1 \\ -1 & 1/tg\beta \end{pmatrix}\begin{pmatrix}\sqrt{2}\,\mathrm{Im}\phi_1^0 \\ \sqrt{2}\,\mathrm{Im}\phi_2^0\end{pmatrix}$

(<u>CP-odd</u>)

$\Rightarrow$ mass m_A, state $A^0 = -\sqrt{2}\,\mathrm{Im}\phi_1^0 \sin\beta + \sqrt{2}\,\mathrm{Im}\phi_2^0 \cos\beta$

no CP phase in V_H

mass 0, state $G^0 = \sqrt{2}\,\mathrm{Im}\phi_1^0 \cos\beta + \sqrt{2}\,\mathrm{Im}\phi_2^0 \sin\beta$

Goldstone boson, eaten by Z

*) <u>Neutral scalars</u> $\rightarrow \frac{1}{2}(\sqrt{2}\,\mathrm{Re}\phi_1'^0, \sqrt{2}\,\mathrm{Re}\phi_2'^0)\begin{pmatrix} \mu_3^2\, tg\beta + m_Z^2\cos^2\beta & -\mu_3^2 - m_Z^2 \sin\beta\cos\beta \\ -\mu_3^2 - m_Z^2\sin\beta\cos\beta & \mu_3^2/tg\beta + m_Z^2\sin^2\beta \end{pmatrix}\begin{pmatrix}\sqrt{2}\,\mathrm{Re}\phi_1'^0 \\ \sqrt{2}\,\mathrm{Re}\phi_2'^0\end{pmatrix}$

(<u>CP-even</u>)

$\Rightarrow$ squared masses: $m_{h/H}^2 = \frac{1}{2}\left\{m_A^2 + m_Z^2 \mp \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2\cos^2(2\beta)}\right\}$,

states: $h^0 = -\sqrt{2}\,\mathrm{Re}\phi_1'^0 \sin\alpha + \sqrt{2}\,\mathrm{Re}\phi_2'^0\cos\alpha$ and

$H^0 = \sqrt{2}\,\mathrm{Re}\phi_1'^0\cos\alpha + \sqrt{2}\,\mathrm{Re}\phi_2'^0 \sin\alpha$,

where $\frac{tg(2\alpha)}{tg(2\beta)} = \frac{m_A^2+m_Z^2}{m_A^2-m_Z^2}$, $\frac{\sin(2\alpha)}{\sin(2\beta)} = -\frac{m_H^2+m_h^2}{m_H^2-m_h^2}$ $(-\pi/2 < \alpha < 0)$.

<u>Hence the following expressions for $H_{1,2}$ diagonalize the system</u>:

$$H_1 \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}[v\cos\beta + H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta - iG^0\cos\beta] \\ H^-\sin\beta - G^-\cos\beta \end{pmatrix}$$

$$H_2 \equiv \begin{pmatrix} H^+\cos\beta + G^+\sin\beta \\ \frac{1}{\sqrt{2}}[v\sin\beta + H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta + iG^0\sin\beta] \end{pmatrix}$$

in terms of the mass eigenstates $H^0, h^0, A^0, G^0, H^\pm, G^\pm$

5 physical Higgs states

<u>Decoupling limit</u>: $m_A \gg m_Z \Rightarrow m_A, m_H, m_{H^\pm} \gg m_Z$, $m_h = \mathcal{O}(m_Z)$,

corresponding to $\alpha \simeq \beta - \pi/2$ (i.e. $\sin\alpha \simeq -\cos\beta$, $\cos\alpha \simeq \sin\beta$)

↳ in this limit h^0 will behave like the Standard Model Higgs boson (see below).

$\sqrt{m_W^2+m_A^2}$ $\qquad$ $\sqrt{m_Z^2+m_A^2-m_h^2}$

<u>More general</u>: $m_{H^\pm} \geq \max(m_A, m_W)$, $m_H \geq \max(m_A, m_Z)$ @ tree level

$m_h \leq \min(m_A, m_Z)\,|\cos(2\beta)|$

<u>very tight mass bound for m_h</u>, but will be modified by quantum corrections (predominantly from heavy fermions)

<u>Complementarity of the couplings to gauge bosons</u>: consider the kinetic terms $(D_\mu H_1)^\dagger(D^\mu H_1) + (D_\mu H_2)^\dagger(D^\mu H_2)$, with $D_\mu H_i = (\partial_\mu + \frac{i}{2}g_W\vec{\tau}\cdot\vec{W}_\mu + \frac{i}{2}g_B Y(H_i)B_\mu)H_i$ $(i=1,2)$.

Write $V \equiv W^\pm, Z$ and $V' \equiv W^\mp, Z$, then we obtain the following interaction strengths

$Y(H_1) = -1,\ Y(H_2) = +1$

for the triple interactions between scalars and gauge bosons:

(1 in decoupling limit)

$$\left.\begin{aligned} g_{VVh^\circ} &= (\sin\beta\cos\alpha - \cos\beta\sin\alpha)\, g^{SM}_{VVh} = \sin(\beta-\alpha)\, g^{SM}_{VVh} \\ g_{VVH^\circ} &= (\sin\beta\sin\alpha + \cos\beta\cos\alpha)\, g^{SM}_{VVh} = \cos(\beta-\alpha)\, g^{SM}_{VVh} \end{aligned}\right\} \Rightarrow g^2_{VVh^\circ} + g^2_{VVH^\circ} = (g^{SM}_{VVh})^2$$

(0 in decoupling limit)

this so-called complementarity relation is required for a good high-energy behaviour of the 2HDM (i.e. the combined effect of h°, H° should be the same as for the SM Higgs)

$g_{ZA^\circ h^\circ} \propto \cos(\beta-\alpha)$, $g_{ZA^\circ H^\circ} \propto \sin(\beta-\alpha)$,

$g_{W^\pm H^\mp h^\circ} \propto \cos(\beta-\alpha)$, $g_{W^\pm H^\mp H^\circ} \propto -\sin(\beta-\alpha)$,

other couplings are absent.

(for a given mass)

Conclusion: at least one of the states $h^\circ, H^\circ, A^\circ$ has a detection probability similar to that of the Standard Model Higgs; if the heavy states H°, A° decouple ($m_A \gg m_Z$), then the light state h° resembles the Standard Model Higgs boson!

Coupling to fermions originating from the Yukawa interactions, with H_1 responsible for giving mass to down-type fermions (int. strength y_d) and H_2 responsible for giving mass to up-type fermions (int. strength y_u).

(1 in decoupling limit)

Up-type fermions: $g_{u\bar{u}h^\circ} = \dfrac{\cos\alpha}{\sin\beta}\, g^{SM}_{u\bar{u}h}$, $g_{u\bar{u}H^\circ} = \dfrac{\sin\alpha}{\sin\beta}\, g^{SM}_{u\bar{u}h}$

($y_u v_2 = f_u v \Rightarrow y_u = \dfrac{f_u}{\sin\beta}$)

$g_{u\bar{u}A^\circ} \propto \dfrac{m_u}{v\, tg\beta}$;

(1 in decoupling limit)

Down-type fermions: $g_{d\bar{d}h^\circ} = -\dfrac{\sin\alpha}{\cos\beta}\, g^{SM}_{d\bar{d}h}$, $g_{d\bar{d}H^\circ} = \dfrac{\cos\alpha}{\cos\beta}\, g^{SM}_{d\bar{d}h}$

$g_{d\bar{d}A^\circ} \propto \dfrac{m_d}{v}\, tg\beta$ ← (compensates m_d/m_u factors, such as $m_b/m_t \approx 1/35$, if $tg\beta$ is sufficiently large)

Note: in the decoupling limit ($\cos\alpha \approx \sin\beta$, $\sin\alpha \approx -\cos\beta$) the light state h° resembles the Standard Model Higgs boson, whereas the heavy states H°, A° couple to fermions with additional factors $1/tg\beta$ for up-type fermions and $tg\beta$ for down-type fermions. If $tg\beta$ is large, the hierarchy of the H°/A°-fermion couplings can be altered drastically!