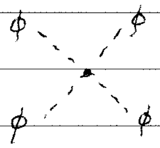
Part 3: Gauge extensions of the Standard Model (chapter 3 of the textbook)

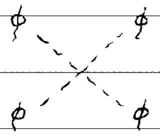
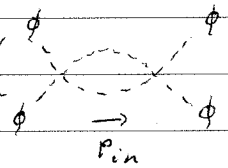
Next we study gauge group extensions of the Standard Model, motivated by what we have seen in the neutrino sector. In fact also the Standard Model itself is possibly giving us hints of new, beyond the Standard Model physics. To this end we first have a look at how interactions change with energy.

§3.1 Renormalization group equations (RGE's)

Consider ϕ^4-theory: $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4$ ($\phi \in \mathbb{R}$ scalar field)

$\Rightarrow$ interaction vertex  $= -6i\lambda$.

At 1-loop order the matrix element for the process $\phi\phi \to \phi\phi$ reads (neglecting m^2 for convenience): s = center-of-mass energy squared p_{in}^2 of the initial state

$i\mathcal{M}(s) =$  $+$ ( + two more) $= -6i\lambda + \frac{36\lambda^2}{2(2\pi)^4}\left(\int \frac{d^4\ell}{\ell^2(\ell+p_{in})^2} + \text{two more}\right)$

use UV cutoff Λ_{UV} for large loop momenta ℓ

$$= -6i\lambda + \frac{9i\lambda^2}{8\pi^2}\left(3\log(\Lambda_{UV}^2/s) + \text{terms independent of } s, \Lambda_{UV}\right).$$

logarithmic divergence (LD) for $\Lambda_{UV} \to \infty$

- The Lagrangian parameter λ ("bare coupling") is not an observable quantity. The quantum corrections are an integral part of the effective coupling, which can be measured through $|\mathcal{M}|^2$.
- $\mathcal{M}$ seems to depend logarithmically on the UV cutoff Λ_{UV} at $\mathcal{O}(\lambda^2)$, but ...
- $|\mathcal{M}|^2$ is observable and should therefore be independent of Λ_{UV}. After all, Λ_{UV} can be chosen arbitrarily $\Rightarrow$ an observable should not depend on it. Consequence: $\lambda(\Lambda_{UV})$ such that

$$0 = \frac{d\mathcal{M}}{d\Lambda_{UV}} = -6\frac{d\lambda}{d\Lambda_{UV}} + \frac{9\lambda}{4\pi^2}\frac{d\lambda}{d\Lambda_{UV}}(\ldots) + \frac{27\lambda^2}{4\pi^2}\frac{1}{\Lambda_{UV}} + \mathcal{O}(\lambda^3)$$

$$\Rightarrow \Lambda_{UV}\frac{d\lambda}{d\Lambda_{UV}} = \boxed{\frac{d\lambda}{d\log(\Lambda_{UV})} = \frac{9\lambda^2}{8\pi^2}} + \mathcal{O}(\lambda^3) \quad \text{or} \quad \frac{d(1/\lambda)}{d\log(\Lambda_{UV})} = -\frac{9}{8\pi^2} + \mathcal{O}(\lambda)$$

this is an example of a renormalization-group equation (RGE)

$$\Rightarrow \text{solution:} \quad \boxed{\lambda(\Lambda_{UV}) = \frac{\lambda(\mu)}{1 - \frac{9\lambda(\mu)}{8\pi^2}\log(\Lambda_{UV}/\mu)}}$$

Suppose we measure the effective coupling at $p_{in}^2 = \mu^2$:

$$-6\lambda(\mu^2) \equiv \mathcal{M}(\mu^2) = -6\lambda + \frac{9\lambda^2}{8\pi^2}\left[3\log(\Lambda_{UV}^2/\mu^2) + \text{terms independent of } \Lambda_{UV}, \mu\right] + \mathcal{O}(\lambda^3)$$

$$\Rightarrow -\lambda = -\lambda(\mu^2) - \frac{3\lambda^2(\mu^2)}{16\pi^2}\left[3\log(\Lambda_{UV}^2/\mu^2) + \dots\right] + \mathcal{O}(\lambda^3(\mu^2)).$$

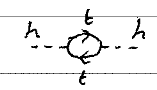
At an arbitrary $p_{in}^2 = s$ we then find $-6\lambda(s) \equiv \mathcal{M}(s) = -6\lambda(\mu^2) - \frac{27\lambda^2(\mu^2)}{8\pi^2}\log(s/\mu^2)$.

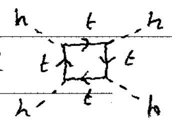
Hence, the RGE connects the (renormalized) effective couplings at different energies!

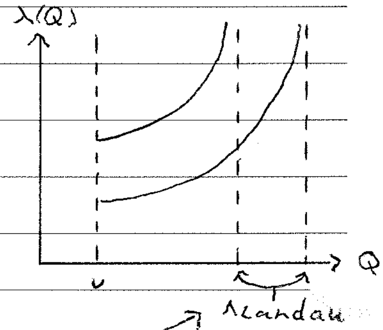
(by taking into account radiative corrections) running coupling

Let's now turn to the Standard Model Higgs sector: the RGE for λ reads

$$\frac{d\lambda(Q)}{d\log(Q)} \approx \frac{3}{8\pi^2}\left[4\lambda^2 + \underbrace{2\lambda f_t^2 - f_t^4}\right] \quad \text{with initial condition } \lambda(v) = m_h^2/2v^2 \equiv \lambda_0.$$

from top-quark loops 

and 



Upper bound: if m_h, λ_0 are large, then $\frac{d\lambda}{d\log(Q)} \approx \frac{3\lambda^2}{2\pi^2}$

$$\Rightarrow \lambda(Q) \approx \frac{\lambda_0}{1 - \frac{3\lambda_0}{2\pi^2}\log(Q/v)}.$$

Implication: m_h larger $\Rightarrow \Lambda_{Landau}$ smaller (i.e. new physics closer by).

scale where new/non-pert. physics sets in!

Lower bound: if m_h, λ_0 are small, $m_t = v f_t/\sqrt{2}$ effectively large $\Rightarrow \frac{d\lambda}{d\log(Q)} \approx -3f_t^4/8\pi^2 \propto (m_t/v)^4$, at $Q = \Lambda_{vac}$ the value of $\lambda(Q)$ will become negative $\rightarrow$ no stable ground state.

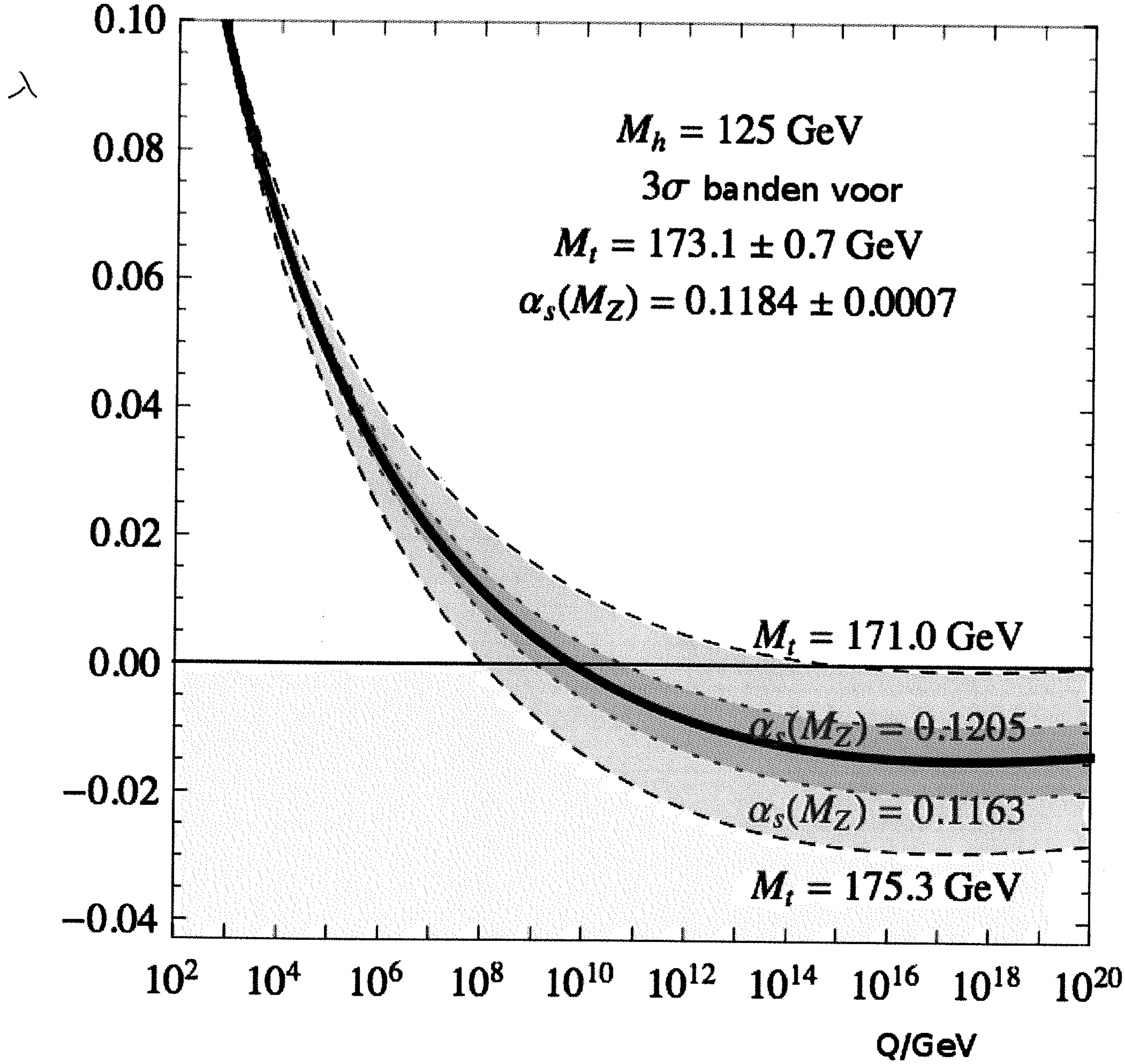
- Light Higgs ($m_h \leq 125$-130 GeV) signals new physics below the Planck scale,
- Standard Model Higgs sector can be extended all the way up to the Planck scale if the Higgs boson mass is in the window 130-170 GeV.

Present experimental bounds from direct Higgs searches:

ATLAS experiment @ LHC $\rightarrow m_h = 124.98 \pm 0.19\,(\text{stat.}) \pm 0.21\,(\text{syst.})$ GeV,

CMS experiment @ LHC $\rightarrow m_h = 125.26 \pm 0.20\,(\text{stat.}) \pm 0.08\,(\text{syst.})$ GeV.

Running of the Higgs parameter λ



Glimpse of new physics ?

Do we see a first glimpse of new physics or will $\lambda(Q)$ vanish near the Planck scale? If the latter happens slow enough, the Higgs boson might have a role to play in inflation, which requires a scalar potential with a sufficiently flat minimum. Either way, this deserves more scrutiny! slow-roll condition

§3.2 Grand Unified Theories (GUT's)

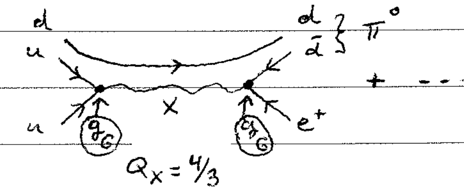
In the strict sense the Standard Model is not a genuine unification theory, in spite of the fact that it combines strong, weak and electromagnetic interactions. In the $SU(2) \times U(1)$ theory the couplings g_W and g_B are not related. Such a relation can be achieved only by means of a gauge theory if $SU(2) \times U(1)$ is embedded in a larger group $G \supset SU(2) \times U(1)$.

A grand unified theory aims at a unification of all Standard Model forces and gauge couplings. This will involve an embedding of $SU(3)_c \times SU(2)_L \times U(1)_Y$ in a single larger simple gauge group G, such that the matter fermions live in larger multiplets (irreducible representations) of this GUT-group G that involve both quarks and leptons: $G \neq G_1 \times G_2$

$$\underset{g_G}{G} \xrightarrow[M_X \gg m_W]{\text{symmetry breaking}} \underset{g_s}{SU(3)_c} \times \underset{g_W}{SU(2)_L} \times \underset{g_B}{U(1)_Y} \xrightarrow{\langle\Phi\rangle_0} \underset{g_s}{SU(3)_c} \times \underset{e}{U(1)_{e.m.}}$$

Here M_X represents the symmetry-breaking scale of the GUT theory (= unification scale) $\Rightarrow$ massless gauge bosons (X bosons, called leptoquarks) acquire masses $\sim M_X$ and mediate interactions between quarks and leptons! This means that the SM concepts of lepton- and baryon-number conservation have been abandoned, which opens up the possibility of proton decay:

$p \to e^+ \pi^0$



$$+ \; \cdots \Rightarrow \Gamma_p = 1/\tau_p = C \Big(\underbrace{\frac{g_G^2}{M_X^2}}_{\text{effective coupling like } G_F} \Big)^2 m_p^5 .$$

This proportionality could have been guessed from dimensional considerations, since $[\Gamma_p] = [m_p] = 1$ and $[g_G^2/M_X^2] = -2$. The coefficient C is small, i.e. typically $\mathcal{O}(10^{-5})$.

"mass dimension of Γ_p"

Take the single unified GUT coupling $g_G \sim 1$ and $M_X \sim 10^{15}$ GeV

$\Rightarrow \tau_p \sim 10^{32}$ yr vs $\tau_p^{exp} > 10^{34}$ yr, i.e. $M_X > 10^{15}$ GeV is required for being compatible with the experimental lower bound on the proton lifetime. This unification scale could therefore be substantially below the Planck scale $\Lambda_{Pl} = \mathcal{O}(10^{19}\,\text{GeV})$ where gravity becomes strong ($\Rightarrow$ no need to unify with gravity at the same time!).

Coupling unification: in the GUT-scenario the running gauge couplings of the Standard Model should approach a single unified coupling g_G at the unification scale M_X

$$\alpha_i^{-1}(Q) \equiv \frac{4\pi}{g_i^2(Q)} \quad (i=1,2,3) \xrightarrow{RGE} \alpha_G^{-1} = \frac{4\pi}{g_G^2} \quad \text{at } Q = M_X .$$

$$g_1^2 = \frac{5}{3} g_B^2 \,, \quad g_2^2 = g_W^2 \,, \quad g_3^2 = g_S^2$$

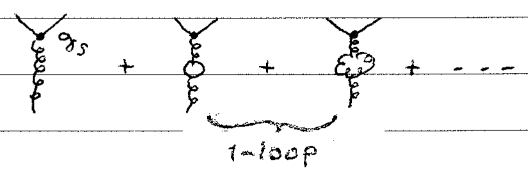
Why the funky factor 5/3 for the $U(1)_Y$ gauge coupling?

The embedding in fact involves $S(U(3)\times U(2)) \subset G$, which has the same Lie algebra as $SU(3)_c \times SU(2)_L \times U(1)_Y$. The group elements in the fundamental representation are constrained by $\det\begin{pmatrix} U_3 & \emptyset \\ \emptyset & U_2 \end{pmatrix} = \det(U_3)\det(U_2) = 1$, which gives rise to the usual $SU(3)$ and $SU(2)$ constraints AND a residual $U(1)_Y$ phase freedom corresponding to group elements $\begin{pmatrix} e^{-i\varphi/3} I_3 & \emptyset \\ \emptyset & e^{i\varphi/2} I_2 \end{pmatrix}$

$= e^{i\varphi\, \mathrm{diag}(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2})}$ with $\det = 1$. The generator $T_Y = q\, \mathrm{diag}(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2})$ of these group elements should satisfy the normalization condition of G: $Tr(T^a T^b) = \frac{1}{2}\delta^{ab}$ fund. repr.

$\Rightarrow \frac{1}{2} = \underbrace{3*(-q/3)^2 + 2*(q/2)^2}_{5q^2/6} \Rightarrow q = \sqrt{3/5}$, resulting in $\underline{g_B^2 = \frac{3}{5} g_G^2 \text{ at } Q = M_X}$.

The running of the gauge couplings is caused by loop corrections:

e.g. 

,

and is governed by RGE's: $\dfrac{d\,\alpha_i^{-1}(Q)}{d\log(Q)} = -\dfrac{b_i}{2\pi} + \mathcal{O}(\alpha_i)$. (2 loops)

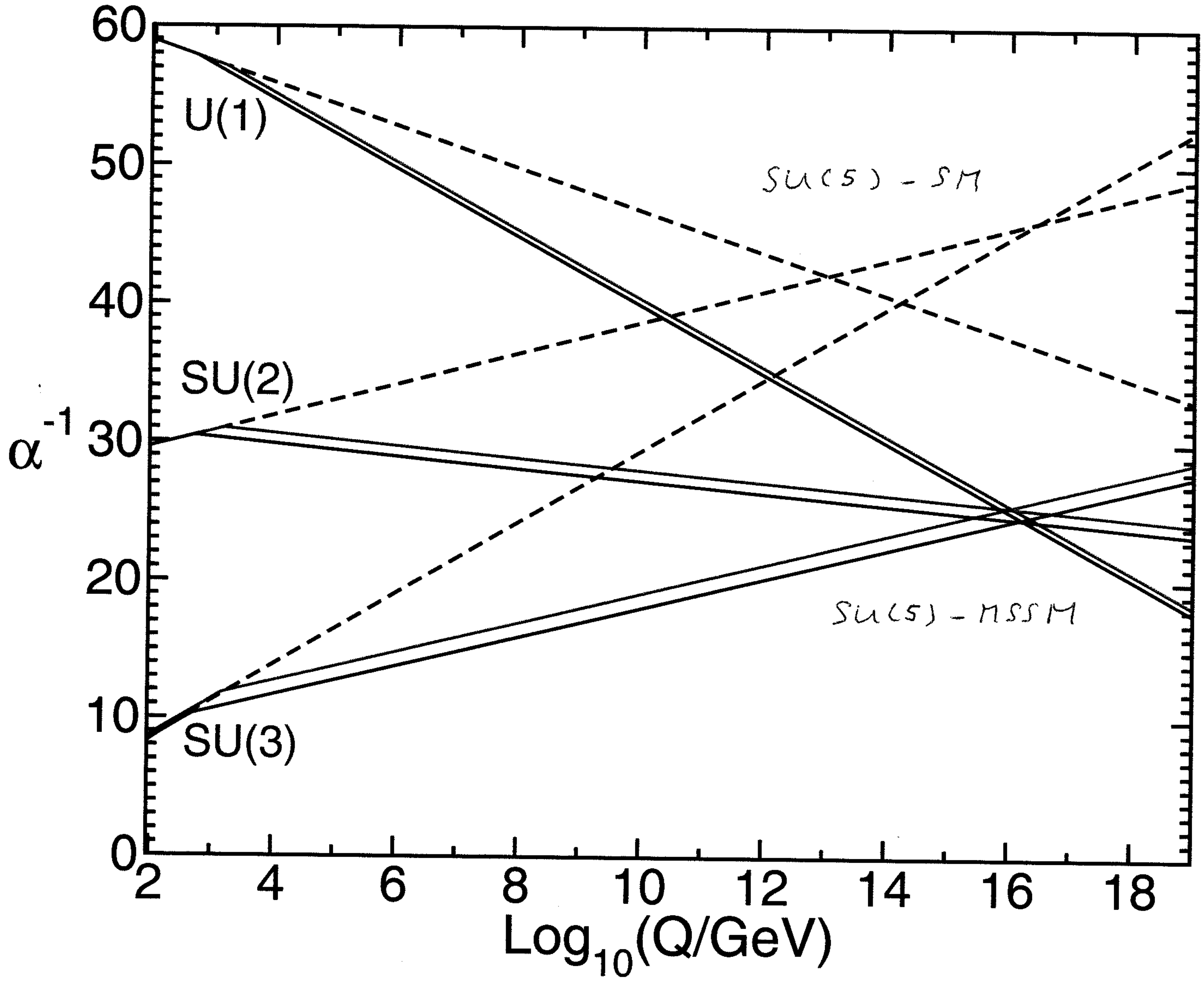
The coefficients b_i differ from model to model! A very popular model has been the $SU(5)$ embedding of the Standard Model and its minimal supersymmetric counterpart (MSSM):

unification of the gauge couplings ?

$\alpha_1(Q) = 5\, g_B^2(Q) / 12\pi$

$\alpha_2(Q) = g_W^2(Q) / 4\pi$

$\alpha_3(Q) = g_S^2(Q) / 4\pi$



compatible with unif. at 2×10^{16} GeV asymptotically free

SM $\rightarrow b_1 = 41/10$, $b_2 = -19/6$, $b_3 = -7$;

MSSM $\rightarrow b_1 = 33/5$, $b_2 = 1$, $b_3 = -3$.

Measurements reveal: *) <u>a GUT scenario seems feasible if supersymmetry is imposed!</u>

*) <u>$U(1)_Y$ in SM requires high-scale new physics</u>

(either gravity or something else)!

<u>GUT gauge-group generalities</u> : rank of G = maximum number of independent mutually commuting (simultaneously diagonalizable) generators of G

$\Rightarrow$ the corresponding quantum numbers are the simultaneously measurable independent charges of the particles.

*) $SU(N) \rightarrow$ rank $= N-1 =$ number of independent diagonal generators,

follows from $Tr(T^a) = 0$ and $(T^a)^\dagger = T^a$

*) $SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow$ rank $= 2 + \underbrace{1 + 1}_{I_3,\, Y} = 4$ = rank of the SM.

We want $G \supset SU(3)_c \times SU(2)_L \times U(1)_Y$ embedding $\Rightarrow$

- at least 4 independent charges $\rightarrow$ rank $G \geq 4$.
- G must allow complex representations, just like $SU(3)_c$ $\leftarrow$ c.c. of fund. repr. is a different repr.
- G should have a single unified gauge coupling $\Rightarrow$ G is a simple group (i.e. not a product group) or a product group of identical simple groups, the couplings of which are required to be equal by some discrete symmetry.

<u>The building blocks for GUT's</u> :

group	rank	# generators	complex repr.
$SU(N)$	$N-1$	N^2-1	$N \geq 3$
$SO(2N+1)$	N	$N(2N+1)$	X
$SO(2N)$	N	$N(2N-1)$	$N = 5, 7, 9, \ldots$
G_2	2	14	X
F_4	4	52	X
$E6$	6	78	V
E_7	7	133	X
E_8	8	248	X

Popular GUT groups have been $SU(5)$ [rank 4], $SO(10)$ [rank 5], $E6$ [rank 6].

Within multiplets (representations) of these groups the charges of the particles should add up to zero, since the charge is a generator of the group and should therefore be traceless. By the way, the same should hold for hypercharge.

In order to be able to lump together fermionic states of the same chirality, charge conjugation is used to turn right-handed states into left-handed ones. This means that we have 15 non-sterile left-handed states per fermion generation: $u_{1_L}, u_{2_L}, u_{3_L}$; $(u_{1_R})^c, (u_{2_R})^c, (u_{3_R})^c$; $d_{1_L}, d_{2_L}, d_{3_L}$; $(d_{1_R})^c, (d_{2_R})^c, (d_{3_R})^c$; ν_L; e_L; $(e_R)^c$.

colour indices

SU(5): this is the smallest unification group that requires no additional matter fermions to be added to the theory, as all left-handed states can be accomodated in precisely two representations of SU(5)!

sterile $(\nu_R)^c$ can be added as a singlet

$$(\psi^i_{5^*_L}) = \begin{pmatrix} (d_{1R})^c \\ (d_{2R})^c \\ (d_{3R})^c \\ \nu_L \\ e_L \end{pmatrix} = (\psi^i_{5_R})^c, \quad (\psi^i_{5_R}) = \begin{pmatrix} d_{1R} \\ d_{2R} \\ d_{3R} \\ (e_L)^c \\ -(\nu_L)^c \end{pmatrix} \longleftarrow \text{fundamental representation}$$

$$\sum_i Q_i = \boxed{-3Q_d + Q_e = 0} \Rightarrow \text{charge quantization!}$$

because quarks come in three colours, the associated charge is fractional in terms of the unit charge: $Q_d = Q_e/3$.

Note: it is not an option to replace $(d_{i_R})^c$ by up-quark fields since $\pm 3Q_u + Q_e \neq 0$!

$$\sum_i Y_i = 3\,[\underbrace{-Y(d_R)}_{2/3}] + 2\underbrace{Y(L_{i_L})}_{-1} = 2 - 2 = 0 \quad \text{cf. } T_Y \text{ on p.42}$$

$$(\psi^{ij}_{10_L}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & (u_{3_R})^c & -(u_{2_R})^c & -u_{1L} & -d_{1L} \\ -(u_{3_R})^c & 0 & (u_{1_R})^c & -u_{2L} & -d_{2L} \\ (u_{2_R})^c & -(u_{1_R})^c & 0 & -u_{3L} & -d_{3L} \\ u_{1L} & u_{2L} & u_{3L} & 0 & -(e_R)^c \\ d_{1L} & d_{2L} & d_{3L} & (e_R)^c & 0 \end{pmatrix} \longleftarrow \text{antisymmetric 10 representation}$$

$$\sum Q = 3(-Q_u + Q_u + Q_d) - Q_e = 3Q_d - Q_e = 0, \text{ again due to charge quantization}$$

$$\sum Y = 3\,[-Y(u_R) + 2Y(Q_{i_L})] - Y(e_R) = 3*(-\tfrac{4}{3} + \tfrac{2}{3}) + 2 = 0$$

Drawbacks: – M_X too low: $\tau_p^{SU(5)} = 10^{30\pm2}$ yr, ruled out by experiment

– no experimental support for SU(5) gauge-coupling unification

↳ minimal SU(5) ruled out! ← true for most GUT groups

But ... supersymmetric version might work!