The interaction terms between the square brackets have been obtained by requiring

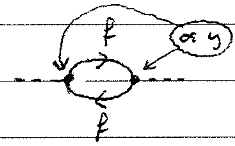
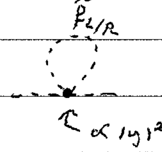
*) renormalizability ⇒ no couplings with negative mass dimension
⇒ maximally 2 fermionic fields, 2 auxiliary fields, 4 scalar fields;
*) supersymmetry ⇒ many terms are eliminated.

Using the equations of motion for the non-propagating fields F_L and F_L^*:

$$\frac{\partial \mathcal{L}_{WZ}}{\partial F_L} - \partial_\mu \frac{\partial \mathcal{L}_{WZ}}{\partial(\partial_\mu F_L)} = 0 \Rightarrow F_L^* = i W'(\tilde{\psi}_L) \text{ and similarly } F_L = -i\,[W'(\tilde{\psi}_L)]^*$$

⇒ <u>F-terms</u>: $F_L^* F_L + (-i W'(\tilde{\psi}_L) F_L + h.c.) \rightarrow -|W'(\tilde{\psi}_L)|^2 \equiv \mathcal{L}_F$.

Most general form of the superpotential: $W(\tilde{\psi}_L) = \underbrace{L}_{[L]=2}\tilde{\psi}_L + \frac{1}{2}\underbrace{M}_{[M]=1}\tilde{\psi}_L^2 + \frac{1}{6}\underbrace{y}_{[y]=0}\tilde{\psi}_L^3$, where
- the M-term gives the fermions and sfermions the same mass,
- the y-term links Yukawa interactions and quartic scalar interactions:

cancellation between  and  that solves the hierarchy problem,

- all three terms feature in the scalar potential: $V_F = |W'(\tilde{\psi}_L)|^2 = |L + M\tilde{\psi}_L + \frac{1}{2} y \tilde{\psi}_L^2|^2 \geq 0$
⇒ • $L \neq 0 \rightarrow \langle \tilde{\psi}_L \rangle_0 \neq 0$ possible, breaking SUSY in view of the absence of $\langle \psi_L \rangle_0 \neq 0$
• $L = 0 \rightarrow$ minimal for $\langle \tilde{\psi}_L \rangle_0 = 0$ EWSB
• $y \neq 0 \rightarrow$ triple and quartic scalar interactions!

SU(N)-style

Next we <u>add gauge interactions and the associated vector (gauge) supermultiplet</u>:
$\partial_\mu \rightarrow D_\mu = \partial_\mu + i g W_\mu^a T^a$ in the matter Lagrangian + add "kinetic" terms for the gauge supermultiplet

$$\mathcal{L}_V = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} + \frac{i}{2} \bar{\tilde{W}} \not{D} \tilde{W} + \frac{1}{2} D^a D^a, \text{ with } W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g f^{abc} W_\mu^b W_\nu^c$$
$$\text{and } (D_\mu \tilde{W})^a = \partial_\mu \tilde{W}^a \underbrace{- g f^{abc}}_{+ i g (T^b)^{ac}} W_\mu^b \tilde{W}^c.$$

In addition we need extra gauge interactions to make everything supersymmetric:

$$\mathcal{L}_{int} = -\sqrt{2} g (\bar{\tilde{W}}^a \tilde{\psi}_L^* T^a \psi_L + h.c.) + g \tilde{\psi}_L^* T^a \tilde{\psi}_L D^a.$$

<u>Extra requirement</u>: <u>the superpotential should be a gauge singlet (i.e. gauge inv.)</u>
⇒ $L\tilde{\psi}_L$ only allowed if $\tilde{\psi}_L$ is a singlet!

Using the equations of motion for the non-propagating fields D^a:

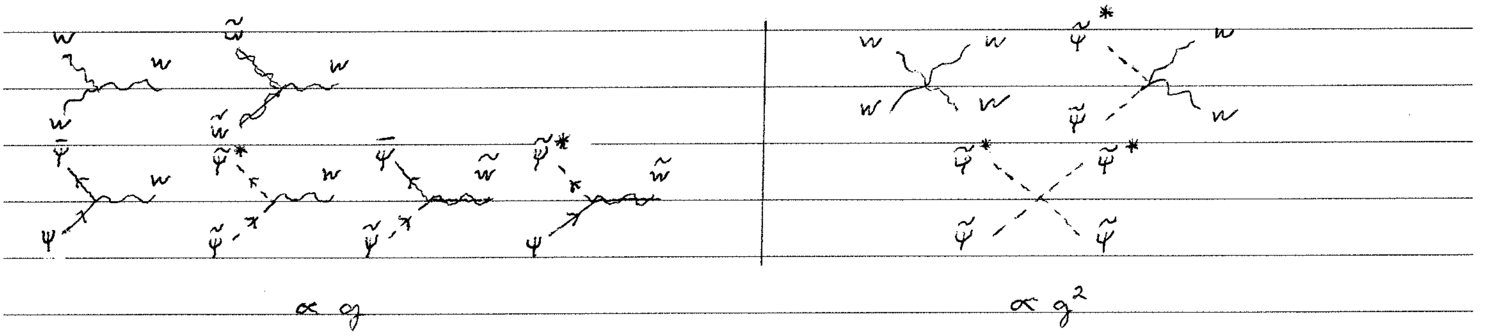
$$\frac{\partial}{\partial D^a}(\mathcal{L}_V + \mathcal{L}_{int}) = 0 \Rightarrow D^a = -g\,\tilde{\psi}_L^* T^a \tilde{\psi}_L \leftarrow$$ this becomes $-g[\tilde{\psi}_L^* T^a \tilde{\psi}_L - \tilde{\psi}_R^* T^a \tilde{\psi}_R]$ for a non-chiral SU(N) theory

$\Rightarrow$ D-terms: $-\frac{g^2}{2}(\tilde{\psi}_L^* T^a \tilde{\psi}_L)^2 \equiv \mathcal{L}_D$, giving rise to quartic couplings of scalar fields that are fixed by gauge couplings! This will have important phenomenological consequences in the Higgs sector: Higgs masses cannot all be arbitrarily large! Contribution to the scalar potential:
$V_D = -\mathcal{L}_D = \frac{g^2}{2}(\tilde{\psi}_L^* T^a \tilde{\psi}_L)^2 \geq 0 \Rightarrow V = V_F + V_D$ is bounded from below, with minimum at $\tilde{\psi}_L = 0$ if $L = 0$!

↑ (quartic int. $\propto g^2, |y|^2$)

↑ (bad for EWSB)

- the gauginos are massless, since the gauge bosons have to be massless,
- no scalar-gaugino-gaugino interactions $\Rightarrow$ no EWSB source for gaugino masses!
- all new interactions are gauge interactions, i.e. $\propto g$ or g^2



§4.3 The ingredients for a Minimal Supersymmetric Standard Model (MSSM)

As mentioned before, all leptons, quarks and Higgs bosons live in chiral supermultiplets. They will have superpartners with the same gauge quantum numbers. The particles can only acquire mass through the Higgs mechanism, otherwise gauge invariance is broken $\Rightarrow$ before electroweak symmetry breaking (EWSB) all particles, including superpartners, will be massless. In the SM one Higgs doublet Φ was able to do the job by means of the conjugate doublet $\Phi^c = i\tau^2\Phi^*$. This is not allowed in the SUSY version of the SM, since in an analytic superpotential of scalar fields not both Φ and Φ^* can feature

$\Rightarrow$ we need at least two Higgs doublets $H_1 = \binom{h_1^0}{h_1^-}$ with $y(H_1) = -1$ and $H_2 = \binom{h_2^+}{h_2^0}$ with $y(H_2) = +1$ in order to give masses to down- and up-type fermions
[By the way: this is also needed for the cancellation of the chiral anomalies].

Two complex Higgs doublets: 8 d.o.f. $\xrightarrow{EWSB}$ 8 - 3 = 5 physical d.o.f.,
i.e. 5 physical Higgs states.

Now we can list the particle content of the minimal SUSY extension of the SM

supermultiplet	bosons	fermions	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	
gauge multiplets	spin-1	spin-1/2	repr.	repr.	Y	
gluons + gluinos	g	$\tilde{g}$	octet	singlet	0	non-chiral, self-conjugate
W-bosons + winos	$W^{\pm}, W^3$	$\tilde{W}^{\pm}, \tilde{W}^3$	singlet	triplet	0	
B-boson + bino	B	$\tilde{B}$	singlet	singlet	0	
matter multiplets	spin-0	spin-1/2				
squarks + quarks	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	$(u, d)_L$	triplet	doublet	+1/3	
(3 families)	$\tilde{U}^c = \tilde{u}_R^*$	$(u_R)^c$	antitriplet	singlet	-4/3	
	$\tilde{D}^c = \tilde{d}_R^*$	$(d_R)^c$	antitriplet	singlet	+2/3	
sleptons + leptons	$\tilde{L} = (\tilde{\nu}_{e_L}, \tilde{e}_L)$	$(\nu_e, e)_L$	singlet	doublet	-1	chiral
(3 families)	$\tilde{E}^c = \tilde{e}_R^*$	$(e_R)^c$	singlet	singlet	+2	
	$\tilde{N}^c = \tilde{\nu}_{e_R}^*$	$(\nu_{e_R})^c$	singlet	singlet	0	
Higgs bosons	H_1	$(\tilde{h}_1^0, \tilde{h}_1^-)_L$	singlet	doublet	-1	
+ higgsinos	H_2	$(\tilde{h}_2^+, \tilde{h}_2^0)_L$	singlet	doublet	+1	

No supermultiplets can be defined in the SM, bearing in mind that H_1 is not part of the SM. For the supermultiplets of the MSSM we introduce the concept of R-parity

$$\boxed{R = (-1)^{3(B-L)} (-1)^{2s} = \begin{cases} +1 & \text{for SM particles + Higgses} \\ -1 & \text{for } \underset{\uparrow\ \Delta s = \pm 1/2}{\text{superpartners}} \end{cases}}$$

This multiplicative quantum number contains:
S = spin (which differs by ± 1/2 between SM particles and superpartners),
L = lepton number ($L(\ell, \tilde{\ell}) = +1$, $L(\bar{\ell}, \tilde{\ell}^*) = -1$, rest 0),
B = baryon number ($B(q, \tilde{q}) = 1/3$, $B(\bar{q}, \tilde{q}^*) = -1/3$, rest 0).

In the MSSM the superpotential terms that violate R-parity (indicated by $W_{\not R}$) are usually excluded in order to prevent rapid proton decay $p^+ \to \pi^0 e^+$.

supported by various classes of models, such as string theory

(B=1, L=0) (B=0, L=-1)

$\Rightarrow$ B+L violated!

R-parity conservation has large repercussions on SUSY phenomenology:
- any interaction involves an even number of superpartners (sparticles),
- in collider experiments sparticles are produced in pairs!
  - ↳ two incoming SM particles $\Rightarrow R_{in} = +1 = R_{out}$
- each sparticle decays into an odd number of other sparticles,
- the lightest sparticle (LSP) is stable!
  - $\Rightarrow$ if the LSP is colourless and neutral, then
    - the LSP escapes as missing energy in collider experiments;
    - the LSP is a candidate for the cold dark matter in the universe!

The superpotential of the MSSM : $W^{MSSM} = W_R$, conserving R-parity.

Rules of thumb for constructing a gauge invariant (singlet) W^{MSSM}:

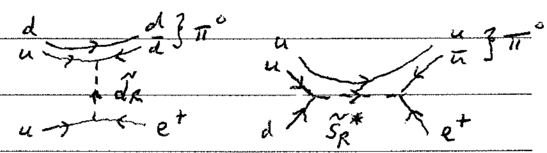
- only chiral sector scalars can be used,
- doublets have to be combined into a singlet without conjugation:
  e.g. $H_1 \times H_2 \equiv \epsilon_{lm} H_{1_l} H_{2_m}$ with $(\epsilon_{lm}) = i\tau^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$,
- $\Sigma y = 0$ without conjugation $\Rightarrow$ look at quantum numbers,
- colour triplets require antitriplets.

($|\Delta L| = 2$, excluded in MSSM) (i,j,k are family indices)

	L	M	Y					
$W_R \rightarrow$	–	$H_1 \times H_2$, $\tilde{N}_i^c \tilde{N}_j^c$	$\tilde{E}_i^c\, H_1 \times \tilde{L}_j$	$\tilde{N}_i^c\, H_2 \times \tilde{L}_j$	$\tilde{D}_i^c\, H_1 \times \tilde{Q}_j$	$\tilde{U}_i^c\, H_2 \times \tilde{Q}_j$	← (Yukawa int.)
$W_{\not R} \rightarrow$	$\tilde{N}_i^c$	$\tilde{L}_i \times H_2$	$\tilde{N}_i^c\, H_1 \times H_2$	$\tilde{N}_i^c \tilde{N}_j^c \tilde{N}_k^c$	$\tilde{E}_i^c\, \tilde{L}_j \times \tilde{L}_k$	$\tilde{D}_i^c\, \tilde{L}_j \times \tilde{Q}_k$	$\tilde{U}_i^c \tilde{D}_j^c \tilde{D}_k^c$	← ($W_{\not R}$ excluded in MSSM)
	($\lvert\Delta L\rvert = 1$)	($\lvert\Delta L\rvert = 1$)	($\lvert\Delta L\rvert = 1$)	($\lvert\Delta L\rvert = 3$)	($\lvert\Delta L\rvert = 1$)	($\lvert\Delta L\rvert = 1$)	($\lvert\Delta B\rvert = 1$)	

W_R has $\Delta L = \Delta B = 0 \Rightarrow$ no proton decay

$W_{\not R}$ has $\Delta(B \pm L) \neq 0 \Rightarrow$ proton decay possible, e.g.



$$W_R = -\mu H_1 \times H_2 + \tilde{E}^c y_e H_1 \times \tilde{L} + \tilde{D}^c y_d H_1 \times \tilde{Q} - \tilde{U}^c y_u H_2 \times \tilde{Q} - \tilde{N}^c y_\nu H_2 \times \tilde{L}$$

(less standard)

*) $y_{e,d,u,\nu}$ are complex 3x3 Yukawa matrices in family space $\rightarrow$ masses and mixing of quarks, masses and mixing of leptons after EWSB;

*) μ is the only new SUSY parameter; ← (what determines its size?)

*) there are no direct mass terms (like $M \tilde{\psi} \tilde{\psi}$) for the (s)quarks and (s)leptons in W_R, just as in the SM; ← (e.g. $\tilde{Q}_i \times \tilde{Q}_i = 0$ and $\tilde{Q}_i \times \tilde{Q}_j$, $\tilde{U}_i^c \tilde{Q}_j$ not gauge invariant)

*) there is only one class of pure singlet chiral scalars, $\tilde{N}_i^c$ ⇒ the only $L\tilde{\varphi}$ term in the superpotential could in principle be $L_i \tilde{N}_i^c$, potentially leading to $\langle \tilde{N}_i^c \rangle_0 \neq 0$. However, $\tilde{N}_i^c$ does not couple to the electroweak gauge bosons and can therefore not give mass to the $W^\pm$, Z bosons
⇒ EWSB will not work in an unbroken SUSY-version of the SM!

Real life: the superpartners of the SM particles have not been detected yet
⇒ sparticle masses cannot be the same as particle masses.

Supersymmetry must be broken without spoiling its desired properties
→ soft SUSY breaking:
- gauge and yukawa couplings unchanged,
- SUSY masses ≠ SM masses, with mass difference of $\mathcal{O}$(few TeV) in order to stay natural,
- extra interactions between scalar fields, which are needed for EWSB,
- R-parity conserved.

use Lagrangian terms with couplings of positive mass dimension
⇒ hierarchy problem not re-introduced, as the impact of the interactions is suppressed at high energies in that case

Note: gauginos have gauge interactions only ⇒ there is no scalar-gaugino-gaugino term present in the MSSM that could turn into a gaugino mass term when the scalar gets a vev ⇒ look outside the MSSM for SUSY breaking.

"hidden sector"

Many ideas exist about the origin of SUSY breaking and the way it translates to the MSSM: mediated by gravity, gauge interactions, anomalies, extra dimensions, ... ← very small couplings

$[F] = 2$!

Interesting option: F-term SUSY breaking at high scale M ⇒ scale hierarchy becomes possible, $m_{soft} \doteq \frac{\langle F \rangle_0}{M}$ ⇒ $\sqrt{\langle F \rangle_0} = \mathcal{O}(\sqrt{M * m_{soft}})$, i.e. $M = M_{pl}$, $m_{soft} = \mathcal{O}(1\,\text{TeV})$ yields $\sqrt{\langle F \rangle_0} = \mathcal{O}(10^{11}\,\text{GeV})$.

F- and D-terms

SUSY breaking ⇒ ground state has $\langle \Omega | \hat{H} | \Omega \rangle = \langle \Omega | \hat{V} | \Omega \rangle > 0$
⇒ $\langle \Omega | F_i | \Omega \rangle \neq 0$ and/or $\langle \Omega | D^a | \Omega \rangle \neq 0$ for some i and/or a.

usually leads to bad spectrum

Soft SUSY breaking is represented generically by an effective Lagrangian

$$\mathcal{L}_{soft} = -m_{H_1}^2 |H_1|^2 - m_{H_2}^2 |H_2|^2 + b\,(H_1 \times H_2 + h.c.) - \tfrac{1}{2}(M_1 \bar{\tilde{B}}\tilde{B} + M_2 \bar{\tilde{W}}\tilde{W} + M_3 \bar{\tilde{g}}\tilde{g})$$

$$\left.\begin{array}{l} - \tilde{Q}^{+} M_{\tilde{Q}}^2 \tilde{Q} - \tilde{L}^{+} M_{\tilde{L}}^2 \tilde{L} - \tilde{u}_R^{*} M_{\tilde{u}}^2 \tilde{u}_R - \tilde{d}_R^{*} M_{\tilde{d}}^2 \tilde{d}_R - \tilde{e}_R^{*} M_{\tilde{e}}^2 \tilde{e}_R \\ + (\tilde{u}_R^{*} a_u H_2 \times \tilde{Q} - \tilde{d}_R^{*} a_d H_1 \times \tilde{Q} - \tilde{e}_R^{*} a_e H_1 \times \tilde{L} + h.c.) \end{array}\right\} \text{coefficients are } 3\times 3 \text{ family matrices}$$

$\mathcal{L}_{soft}$ contains: - mass terms for sfermions and gauginos $\Rightarrow$ superpartners get masses with generic mass scale $m_{soft} \equiv M_S = \mathcal{O}(\text{TeV})$,
- interactions between 2 and 3 scalar particles (not 4!)

$\Rightarrow$ unconstrained MSSM has 105 extra parameters, parametrizing our ignorance ... but

*) underlying theory might reduce this considerably (e.g. due to parameter unifications or a flavour-blind SUSY-breaking mechanism)

$\Rightarrow$ so-called constrained models : - handful of (unification) parameters
- highly correlated mass spectra ← RGE's
- highly model-dependent signatures

early-LHC approach

strongly constrained by LHC data

*) experimental (CP, flavour, ...) constraints might reduce this

$\Rightarrow$ one typically works with a version of the MSSM with a maximum of 20 independent parameters, called the phenomenological MSSM (pMSSM):
- much less model dependent
- much richer features $\Rightarrow$ less constrained by LHC data

present-day approach

The MSSM Higgs sector :

- quartic Higgs couplings originate from D-terms and are fixed by gauge couplings : $V_D^{Higgs} = \frac{1}{8}(g_W^2 + g_B^2)\,(H_1^{+}H_1 - H_2^{+}H_2)^2 + \frac{1}{2} g_W^2 \,(H_1^{+}H_2)(H_2^{+}H_1)$

$\Rightarrow$ the Higgs masses will not be completely free parameters of the theory.

Note : there are no contributions from F-terms, since no singlet can be constructed out of three doublets.

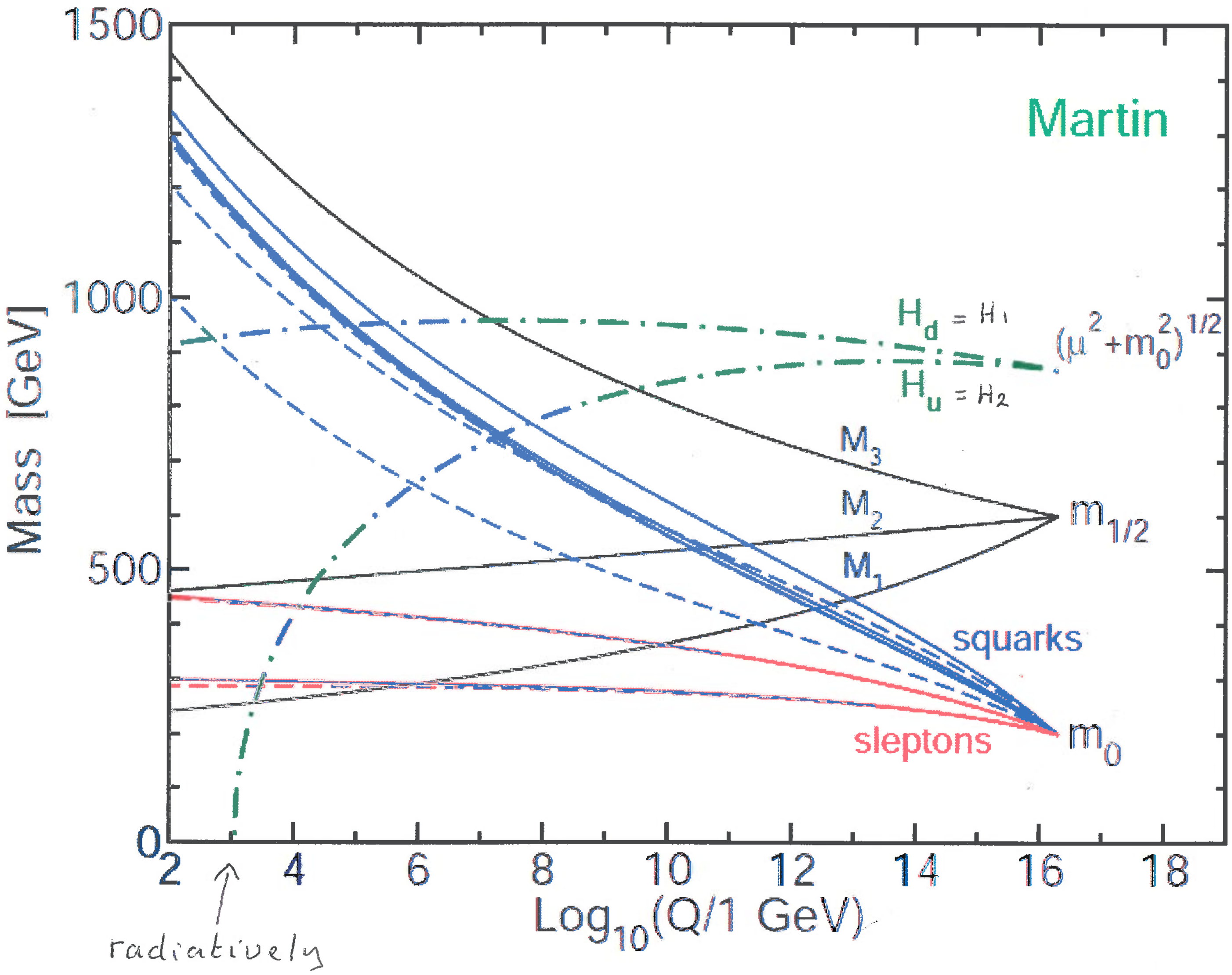
- bilinear contributions from the superpotential (from F-terms) :
$V_F^{Higgs} = |\mu|^2 \,(H_1^{+}H_1 + H_2^{+}H_2)$

- bilinear contributions from soft SUSY breaking:
$V_{soft}^{Higgs} = m_{H_1}^2 H_1^{+}H_1 + m_{H_2}^2 H_2^{+}H_2 - b\,(H_1 \times H_2 + h.c.)$ $\quad (b>0,\ m_{H_i}^2 > 0 \text{ for } i=1,2)$

$\Rightarrow V^{Higgs} = V_D^{Higgs} + V_F^{Higgs} + V_{soft}^{Higgs}$ and define $\mu_1^2 \equiv |\mu|^2 + m_{H_1}^2$, $\mu_2^2 \equiv |\mu|^2 + m_{H_2}^2$.

constrained

Renormalization group evolution in the CMSSM

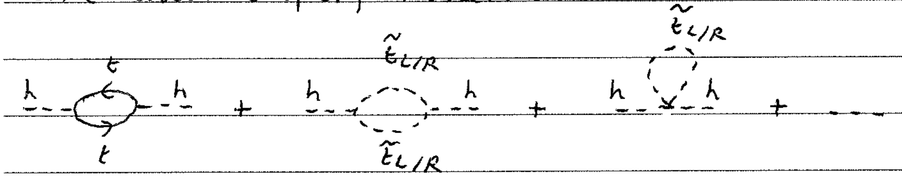


radiatively induced EWSB

We recognize the 2HDM potential introduced on p.29 with $\mu_3^2 = b$

$\Rightarrow$ <u>the unbroken MSSM $(m_{H_1}^2 = m_{H_2}^2 = 0)$ has no EWSB</u> !

<u>Prominent feature</u> : <u>unlike the SM, the MSSM predicts the existence of a light Higgs boson</u> ! At tree level the mass m_h is predicted to be below the Z-boson mass, $m_h < m_Z |\cos(2\beta)|$. This very tight mass bound is relaxed by quantum corrections (the dominant source being the heaviest fermions and their superpartners) : t, b



$\longrightarrow \delta m_h^2 = \mathcal{O}(y_t^2 m_t^2)$

$\Rightarrow$ <u>$m_h \approx 100 - 140$ GeV if $m_A = \mathcal{O}(\text{TeV})$</u>.

Experimental constraints on m_h strongly affect the allowed MSSM parameter space through the MSSM quantum corrections

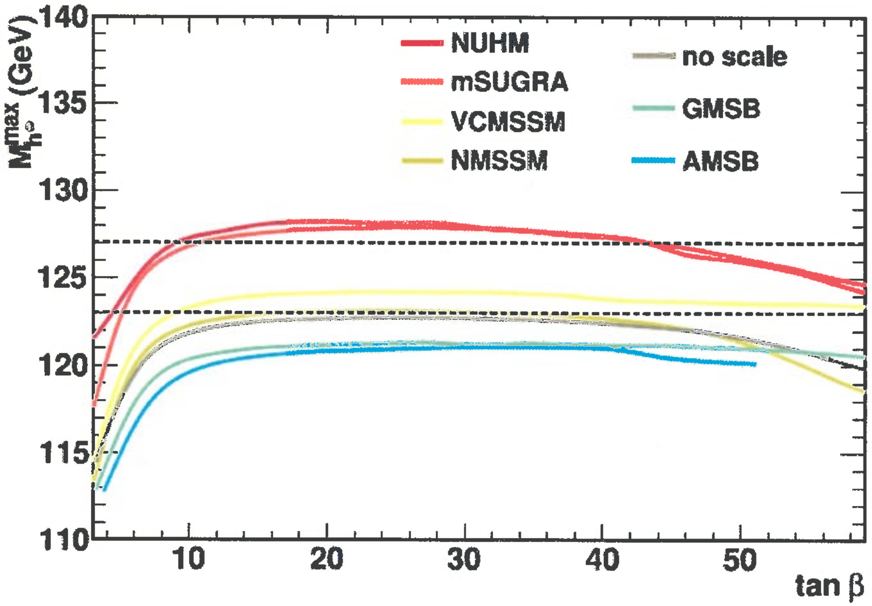
$\Rightarrow$ <u>measurement of m_h = indirect SUSY search</u> !

$m_h \approx 125$ GeV has a big impact, since it is in the upper range

Consequence: some simplified SUSY models have been ruled out by m_h.

Implications of a 125 GeV Higgs on constrained models

Arbey et al.



model	AMSB	GMSB	mSUGRA	no-scale	cNMSSM	VCMSSM	NUHM
$M_{h^0}^{max}$	121.0	121.5	128.0	123.0	123.5	124.5	128.5

Minimal models are being excluded!